

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.1-
 $a+b-x+c-x^2-p$

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [143]. This is test number [32].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (143)	% 0.00 (0)
Mathematica	% 100.00 (143)	% 0.00 (0)
Maple	% 79.02 (113)	% 20.98 (30)
Maxima	% 75.52 (108)	% 24.48 (35)
Fricas	% 79.02 (113)	% 20.98 (30)
Sympy	% 32.87 (47)	% 67.13 (96)
Giac	% 74.13 (106)	% 25.87 (37)
Mupad	% 92.31 (132)	% 7.69 (11)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

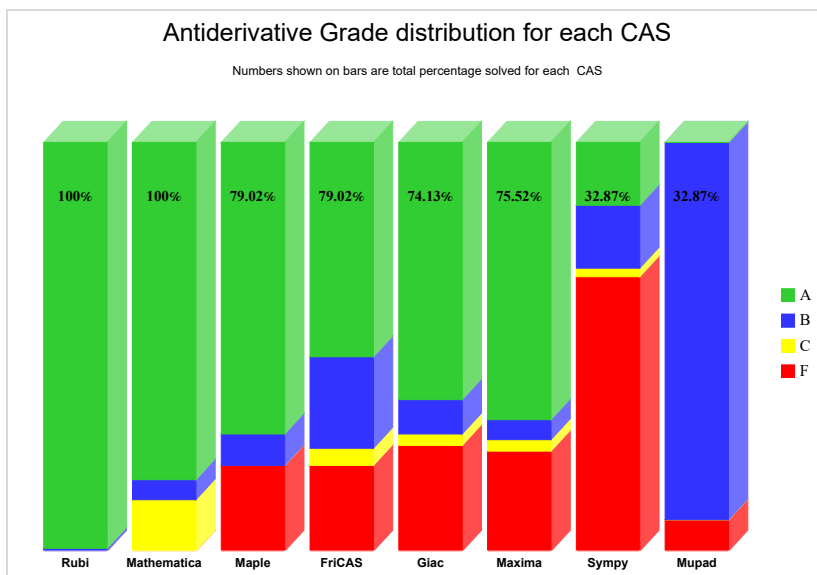
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

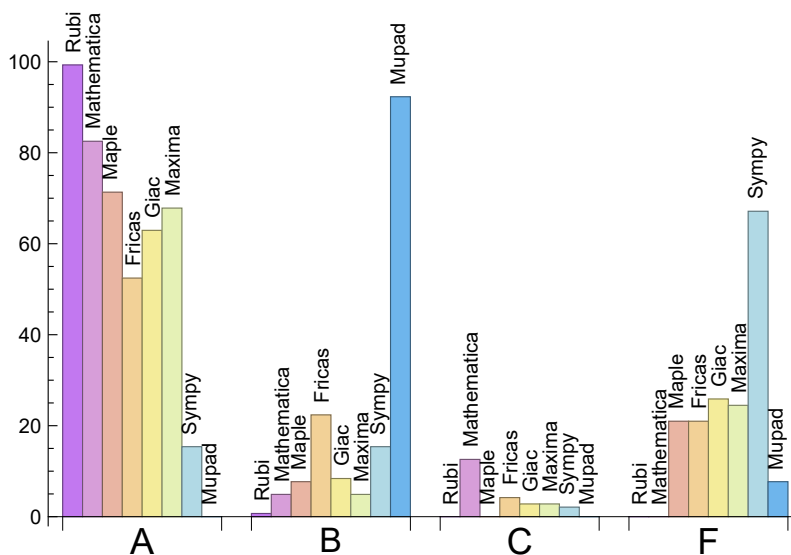
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.30	0.70	0.00	0.00
Mathematica	82.52	4.90	12.59	0.00
Maple	71.33	7.69	0.00	20.98
Maxima	67.83	4.90	2.80	24.48
Fricas	52.45	22.38	4.20	20.98
Sympy	15.38	15.38	2.10	67.13
Giac	62.94	8.39	2.80	25.87
Mupad	0.00	92.31	0.00	7.69

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	30	100.00 %	0.00 %	0.00 %
Maxima	35	85.71 %	0.00 %	14.29 %
Fricas	30	100.00 %	0.00 %	0.00 %
Sympy	96	100.00 %	0.00 %	0.00 %
Giac	37	89.19 %	0.00 %	10.81 %
Mupad	11	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

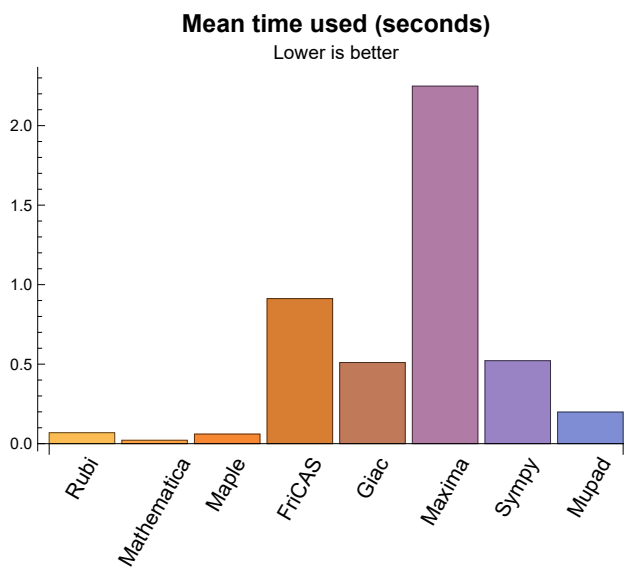
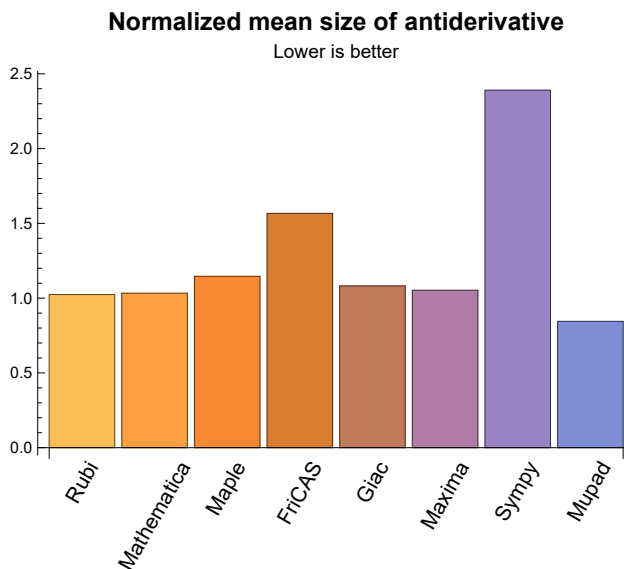
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	83.41	1.02	38.00	1.00
Mathematica	0.02	45.29	1.03	40.00	1.00
Maple	0.06	56.69	1.15	32.00	0.86
Maxima	2.25	45.49	1.05	33.50	1.03
Fricas	0.91	63.31	1.57	43.00	1.21
Sympy	0.52	118.68	2.39	70.00	1.81
Giac	0.51	45.38	1.08	33.00	0.89
Mupad	0.20	39.27	0.84	33.50	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {14, 15, 102}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

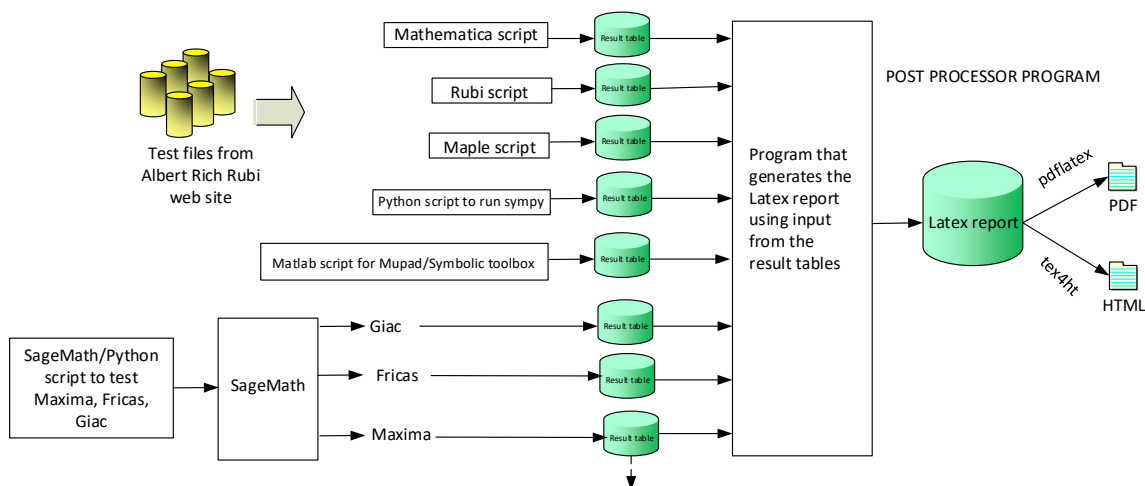
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 83 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { 17, 21, 25, 28, 29, 83, 102 }

C grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 130, 131, 138 }

B grade: { 25, 74, 75, 76, 77, 83, 101, 102, 125, 126, 127 }

C grade: { }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 78, 79, 80, 81, 82, 84, 85, 86, 87, 90, 91, 92, 93, 94, 97, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

B grade: { 17, 74, 75, 76, 77, 83, 101 }

C grade: { 71, 73, 111, 122 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 88, 89, 95, 96, 98, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 22, 23, 24, 26, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 78, 79, 81, 82, 86, 88, 89, 91, 92, 93, 94, 98, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 121, 123, 130, 131, 138 }

B grade: { 17, 18, 21, 25, 27, 74, 75, 76, 77, 80, 83, 84, 85, 87, 90, 95, 96, 97, 99, 100, 104, 114, 115, 117, 118, 120, 124, 125, 126, 127, 128, 129 }

C grade: { 70, 71, 72, 73, 111, 122 }

F grade: { 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.6 Sympy

A grade: { 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 68, 69, 78, 79, 80, 81, 82, 91, 92, 93, 94, 138 }

B grade: { 53, 54, 61, 62, 63, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 90, 95, 96, 97, 98, 99, 100 }

C grade: { 86, 101, 102 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 64, 65, 66, 67, 70, 71, 72, 73, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 27, 28, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 68, 69, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 115, 116, 118, 119, 120, 121, 123, 124, 128, 129, 130, 131, 138 }

B grade: { 64, 74, 75, 76, 77, 83, 101, 114, 117, 125, 126, 127 }

C grade: { 70, 71, 72, 73 }

F grade: { 17, 18, 19, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 67, 111, 122, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 138 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	142	173	180	258	0	132	151
normalized size	1	1.00	0.97	1.18	1.22	1.76	0.00	0.90	1.03
time (sec)	N/A	0.054	0.223	0.046	1.341	0.703	0.000	0.632	0.735
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	91	130	69	0	1	100
normalized size	1	1.00	0.98	0.75	1.07	0.57	0.00	0.01	0.83
time (sec)	N/A	0.031	0.095	0.118	3.001	0.909	0.000	0.573	0.292
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	71	103	59	0	1	80
normalized size	1	1.00	0.93	0.75	1.08	0.62	0.00	0.01	0.84
time (sec)	N/A	0.022	0.079	0.093	3.012	0.969	0.000	0.460	0.298

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	51	76	49	0	1	60
normalized size	1	1.00	1.10	0.74	1.10	0.71	0.00	0.01	0.87
time (sec)	N/A	0.015	0.064	0.118	2.985	1.173	0.000	0.610	0.162

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	64	31	49	39	0	1	39
normalized size	1	1.00	1.49	0.72	1.14	0.91	0.00	0.02	0.91
time (sec)	N/A	0.010	0.049	0.105	2.963	0.803	0.000	0.625	0.087

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	82	117	68	0	57	81
normalized size	1	1.00	0.87	0.81	1.16	0.67	0.00	0.56	0.80
time (sec)	N/A	0.028	0.067	0.044	3.058	0.722	0.000	0.471	0.166

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	64	90	58	0	47	63
normalized size	1	1.00	0.99	0.81	1.14	0.73	0.00	0.59	0.80
time (sec)	N/A	0.019	0.047	0.046	2.944	0.690	0.000	0.454	0.242

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	46	63	48	0	37	45
normalized size	1	1.00	1.19	0.81	1.11	0.84	0.00	0.65	0.79
time (sec)	N/A	0.013	0.046	0.048	2.898	0.861	0.000	0.523	0.112

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	58	28	36	38	0	27	26
normalized size	1	1.00	1.66	0.80	1.03	1.09	0.00	0.77	0.74
time (sec)	N/A	0.009	0.032	0.051	2.885	1.010	0.000	0.477	0.052

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	28	36	35	0	25	26
normalized size	1	1.00	0.91	0.80	1.03	1.00	0.00	0.71	0.74
time (sec)	N/A	0.010	0.039	0.047	2.974	0.726	0.000	0.361	0.048

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	58	28	36	38	0	27	26
normalized size	1	1.00	1.66	0.80	1.03	1.09	0.00	0.77	0.74
time (sec)	N/A	0.010	0.034	0.045	2.926	1.313	0.000	0.435	0.054

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	42	55	43	0	35	39
normalized size	1	1.00	0.86	0.82	1.08	0.84	0.00	0.69	0.76
time (sec)	N/A	0.010	0.063	0.043	2.888	0.857	0.000	0.543	0.194

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	40	33	41	32	0	33	29
normalized size	1	1.00	1.14	0.94	1.17	0.91	0.00	0.94	0.83
time (sec)	N/A	0.007	0.027	0.046	1.418	0.838	0.000	0.520	0.199

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	48	33	43	32	0	33	29
normalized size	1	1.00	1.30	0.89	1.16	0.86	0.00	0.89	0.78
time (sec)	N/A	0.007	0.036	0.045	1.314	0.710	0.000	0.517	0.109

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	33	43	36	0	37	29
normalized size	1	1.00	1.18	0.85	1.10	0.92	0.00	0.95	0.74
time (sec)	N/A	0.007	0.026	0.048	1.349	0.872	0.000	0.415	0.195

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	75	111	105	0	74	96
normalized size	1	1.00	0.84	0.90	1.34	1.27	0.00	0.89	1.16
time (sec)	N/A	0.018	0.021	0.048	1.343	0.803	0.000	0.492	0.271

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	53	10	21	19	0	0	19
normalized size	1	1.00	3.31	0.62	1.31	1.19	0.00	0.00	1.19
time (sec)	N/A	0.006	0.012	0.105	2.932	0.937	0.000	0.000	0.276

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	28	39	0	0	20
normalized size	1	1.00	0.92	0.81	1.08	1.50	0.00	0.00	0.77
time (sec)	N/A	0.003	0.005	0.102	1.372	0.813	0.000	0.000	0.045

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	42	55	62	0	0	31
normalized size	1	1.00	0.68	0.79	1.04	1.17	0.00	0.00	0.58
time (sec)	N/A	0.007	0.010	0.099	1.368	1.030	0.000	0.000	0.124

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	48	62	82	82	0	0	40
normalized size	1	1.00	0.61	0.78	1.04	1.04	0.00	0.00	0.51
time (sec)	N/A	0.012	0.012	0.101	1.370	0.811	0.000	0.000	0.286

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	9	8	19	0	8	8
normalized size	1	1.00	3.33	0.75	0.67	1.58	0.00	0.67	0.67
time (sec)	N/A	0.006	0.012	0.043	2.953	0.943	0.000	0.498	0.106

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	25	28	29	0	29	18
normalized size	1	1.00	0.95	1.14	1.27	1.32	0.00	1.32	0.82
time (sec)	N/A	0.002	0.005	0.051	1.344	0.960	0.000	0.507	0.136

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	35	55	46	0	39	28
normalized size	1	1.00	0.69	0.78	1.22	1.02	0.00	0.87	0.62
time (sec)	N/A	0.006	0.009	0.044	1.363	0.967	0.000	0.408	0.032

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	45	82	61	0	49	73
normalized size	1	1.00	0.76	0.67	1.22	0.91	0.00	0.73	1.09
time (sec)	N/A	0.012	0.013	0.047	1.366	0.923	0.000	0.504	0.195

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	47	35	21	27	0	15	42
normalized size	1	1.00	3.92	2.92	1.75	2.25	0.00	1.25	3.50
time (sec)	N/A	0.008	0.013	0.072	2.916	0.943	0.000	0.520	0.312

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	45	37	29	27	0	36	36
normalized size	1	1.00	1.88	1.54	1.21	1.12	0.00	1.50	1.50
time (sec)	N/A	0.008	0.017	0.052	1.375	0.815	0.000	0.546	0.226

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	7	8	18	0	6	6
normalized size	1	1.00	1.40	0.70	0.80	1.80	0.00	0.60	0.60
time (sec)	N/A	0.006	0.008	0.046	3.034	0.598	0.000	0.392	0.109

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	14	17	17	0	18	11
normalized size	1	1.00	2.06	0.88	1.06	1.06	0.00	1.12	0.69
time (sec)	N/A	0.004	0.006	0.041	1.311	0.914	0.000	0.540	0.511

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	14	17	17	0	18	11
normalized size	1	1.00	2.06	0.88	1.06	1.06	0.00	1.12	0.69
time (sec)	N/A	0.004	0.012	0.051	1.327	0.825	0.000	0.539	0.529

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	48	0	0	0	0	0	36
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.717	0.014	0.852	0.000	0.682	0.000	0.000	0.234

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	45	0	0	0	0	0	36
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.449	0.009	0.547	0.000	0.874	0.000	0.000	0.173

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	43	0	0	0	0	0	36
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.11
time (sec)	N/A	0.394	0.011	0.767	0.000	1.001	0.000	0.000	0.215

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	47	0	0	0	0	0	36
normalized size	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.457	0.011	0.750	0.000	0.727	0.000	0.000	0.249

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	50	0	0	0	0	0	36
normalized size	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	0.08
time (sec)	N/A	0.510	0.012	1.260	0.000	1.051	0.000	0.000	0.275

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	842	842	48	0	0	0	0	0	36
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	1.058	0.013	0.759	0.000	0.909	0.000	0.000	0.181

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	45	0	0	0	0	0	36
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.947	0.009	0.708	0.000	1.190	0.000	0.000	0.160

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	45	0	0	0	0	0	36
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.860	0.010	0.701	0.000	0.759	0.000	0.000	0.200

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	45	0	0	0	0	0	36
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05
time (sec)	N/A	0.947	0.011	0.882	0.000	0.973	0.000	0.000	0.231

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	838	838	50	0	0	0	0	0	36
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	1.036	0.012	1.336	0.000	0.923	0.000	0.000	0.245

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	48	0	0	0	0	0	36
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	0.30
time (sec)	N/A	0.046	0.014	0.625	0.000	0.898	0.000	0.000	0.190

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0	36
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.034	0.012	0.740	0.000	1.039	0.000	0.000	0.169

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	45	0	0	0	0	0	36
normalized size	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.40
time (sec)	N/A	0.033	0.009	0.570	0.000	0.962	0.000	0.000	0.172

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	0	0	0	0	0	36
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.023	0.010	0.723	0.000	1.060	0.000	0.000	0.199

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	0	0	0	0	0	36
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.61
time (sec)	N/A	0.023	0.009	0.723	0.000	0.761	0.000	0.000	0.189

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	0	0	0	0	0	36
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	0.43
time (sec)	N/A	0.032	0.011	0.918	0.000	0.968	0.000	0.000	0.224

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	50	0	0	0	0	0	36
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	0.31
time (sec)	N/A	0.044	0.011	1.370	0.000	0.856	0.000	0.000	0.256

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	50	0	0	0	0	0	36
normalized size	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	0.25
time (sec)	N/A	0.060	0.012	0.768	0.000	1.107	0.000	0.000	0.288

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	0	0	0	0	0	48
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.010	0.009	0.503	0.000	0.952	0.000	0.000	0.316

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	43	43	49	43	43
normalized size	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.017	0.002	0.041	1.313	0.725	0.070	0.389	0.026

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
normalized size	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.011	0.002	0.040	1.348	0.753	0.070	0.529	0.039

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.007	0.001	0.038	1.364	0.728	0.063	0.432	0.029

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.042	1.293	0.708	0.062	0.352	0.017

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.005	0.009	0.040	2.902	0.814	0.141	0.395	0.066

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.010	0.024	0.047	3.112	0.827	0.223	0.417	0.143

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	58	188	105	45	55
normalized size	1	1.00	0.89	0.82	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.016	0.033	0.049	2.987	0.873	0.339	0.339	0.162

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	76	66	58	146	97	63	37
normalized size	1	1.00	0.90	0.79	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.022	0.108	0.044	1.380	0.818	4.426	0.359	0.204

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	51	43	124	70	49	37
normalized size	1	1.00	1.00	0.78	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.014	0.080	0.044	1.303	0.956	3.008	0.427	0.157

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	28	94	41	37	35
normalized size	1	1.00	1.07	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.009	0.019	0.043	1.357	0.932	1.907	0.399	0.127

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	23	20
normalized size	1	1.00	1.00	0.84	0.52	2.36	0.68	0.92	0.80
time (sec)	N/A	0.005	0.006	0.042	1.390	0.994	1.051	0.406	0.187

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
normalized size	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.002	0.004	0.044	1.253	0.740	0.647	0.476	0.032

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
normalized size	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.006	0.008	0.045	1.020	0.713	0.862	0.511	0.192

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	37	46	69	413	41	44
normalized size	1	1.00	0.69	0.64	0.79	1.19	7.12	0.71	0.76
time (sec)	N/A	0.010	0.010	0.043	1.313	1.085	1.461	0.456	0.193

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	61	91	1265	55	61
normalized size	1	1.00	0.66	0.62	0.79	1.18	16.43	0.71	0.79
time (sec)	N/A	0.016	0.013	0.046	1.350	0.698	2.203	0.410	0.205

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	35	30	19	0	45	19
normalized size	1	1.00	0.87	1.52	1.30	0.83	0.00	1.96	0.83
time (sec)	N/A	0.003	0.037	0.046	2.957	1.038	0.000	0.393	0.049

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	30	9	0	26	19
normalized size	1	1.00	1.09	1.09	1.30	0.39	0.00	1.13	0.83
time (sec)	N/A	0.003	0.005	0.044	2.858	0.987	0.000	0.377	0.046

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	0	25	14
normalized size	1	1.00	0.90	0.79	0.21	0.28	0.00	0.86	0.48
time (sec)	N/A	0.005	0.005	0.048	2.830	0.791	0.000	0.286	0.283

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	17	9	14	0	0	21
normalized size	1	1.00	0.80	0.68	0.36	0.56	0.00	0.00	0.84
time (sec)	N/A	0.003	0.005	0.046	2.892	0.957	0.000	0.000	0.170

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	30	9	8	26	13
normalized size	1	1.00	1.09	1.09	1.30	0.39	0.35	1.13	0.57
time (sec)	N/A	0.003	0.009	0.045	3.010	0.960	0.075	0.346	0.113

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	6	8	7	15	14
normalized size	1	1.00	0.90	0.79	0.21	0.28	0.24	0.52	0.48
time (sec)	N/A	0.005	0.008	0.050	2.969	0.868	0.078	0.456	0.353

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	0	26	18
normalized size	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78
time (sec)	N/A	0.003	0.007	0.047	3.015	0.723	0.000	0.375	0.327

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	6	6	0	23	15
normalized size	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52
time (sec)	N/A	0.005	0.006	0.049	2.898	1.108	0.000	0.447	0.245

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	30	9	0	26	18
normalized size	1	1.00	1.17	1.17	1.30	0.39	0.00	1.13	0.78
time (sec)	N/A	0.003	0.006	0.045	2.962	0.855	0.000	0.407	0.067

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	6	6	0	23	15
normalized size	1	1.00	0.97	0.86	0.21	0.21	0.00	0.79	0.52
time (sec)	N/A	0.006	0.004	0.049	2.777	0.857	0.000	0.345	0.295

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	233	253	334	184
normalized size	1	1.00	1.90	5.83	2.15	2.14	2.32	3.06	1.69
time (sec)	N/A	0.140	0.030	0.039	1.413	0.790	0.194	0.429	0.314

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	235	250	334	184
normalized size	1	1.00	1.90	5.83	2.15	2.16	2.29	3.06	1.69
time (sec)	N/A	0.137	0.041	0.045	1.521	0.919	0.199	0.412	0.331

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	199	636	234	227	253	334	176
normalized size	1	1.00	1.83	5.83	2.15	2.08	2.32	3.06	1.61
time (sec)	N/A	0.142	0.028	0.039	1.409	0.962	0.202	0.450	0.337

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	234	235	248	334	184
normalized size	1	1.00	1.90	5.83	2.15	2.16	2.28	3.06	1.69
time (sec)	N/A	0.140	0.042	0.040	1.400	0.950	0.196	0.526	0.316

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	22	16	16
normalized size	1	1.00	1.00	0.94	0.89	0.89	1.22	0.89	0.89
time (sec)	N/A	0.011	0.009	0.042	2.955	1.055	0.114	0.369	0.139

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67
time (sec)	N/A	0.009	0.013	0.099	2.975	0.900	0.164	0.410	0.272

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	34	17	27	39	39	31	15
normalized size	1	1.00	1.79	0.89	1.42	2.05	2.05	1.63	0.79
time (sec)	N/A	0.019	0.019	0.043	2.953	0.885	0.119	0.458	0.209

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	8
normalized size	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	0.62
time (sec)	N/A	0.005	0.003	0.049	1.327	1.037	0.103	0.507	0.082

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	15	14	17	8
normalized size	1	1.00	1.00	0.76	0.71	0.71	0.67	0.81	0.38
time (sec)	N/A	0.006	0.003	0.050	1.314	1.051	0.113	0.464	0.091

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	13	13	12	15	6
normalized size	1	2.83	2.83	2.33	2.17	2.17	2.00	2.50	1.00
time (sec)	N/A	0.004	0.003	0.048	1.288	1.063	0.104	0.330	0.176

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	38	50	76	40	23
normalized size	1	1.00	1.00	0.89	1.41	1.85	2.81	1.48	0.85
time (sec)	N/A	0.019	0.008	0.056	1.314	1.034	0.223	0.346	0.359

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
normalized size	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.019	0.008	0.050	1.326	0.990	0.233	0.383	0.386

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	41	87	27	23
normalized size	1	1.00	1.00	0.90	0.87	1.32	2.81	0.87	0.74
time (sec)	N/A	0.020	0.008	0.049	1.317	0.797	0.196	0.482	0.381

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	39	51	76	45	23
normalized size	1	1.00	1.07	0.96	1.44	1.89	2.81	1.67	0.85
time (sec)	N/A	0.018	0.007	0.051	1.441	0.927	0.234	0.532	0.418

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	113	124	34	46
normalized size	1	1.00	1.00	0.92	0.00	2.97	3.26	0.89	1.21
time (sec)	N/A	0.034	0.010	0.062	0.000	0.925	0.221	0.432	0.229

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	35	0	124	100	30	33
normalized size	1	1.00	0.97	1.00	0.00	3.54	2.86	0.86	0.94
time (sec)	N/A	0.027	0.009	0.064	0.000	1.106	0.232	0.365	0.273

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	31	49	67	102	55	28
normalized size	1	1.00	1.28	0.97	1.53	2.09	3.19	1.72	0.88
time (sec)	N/A	0.025	0.010	0.072	2.910	0.899	0.255	0.445	0.227

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	36	45	39	36	33
normalized size	1	1.00	1.00	0.86	0.84	1.05	0.91	0.84	0.77
time (sec)	N/A	0.015	0.022	0.037	2.274	0.909	0.150	0.580	0.038

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	47	68	58	51	34
normalized size	1	1.00	1.44	0.86	1.09	1.58	1.35	1.19	0.79
time (sec)	N/A	0.016	0.024	0.055	2.807	0.851	0.151	0.522	0.163

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	32	34	53	31	36	34
normalized size	1	1.00	0.97	0.94	1.00	1.56	0.91	1.06	1.00
time (sec)	N/A	0.008	0.010	0.053	1.316	0.925	0.140	0.454	0.200

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	34	53	32	36	34
normalized size	1	1.00	1.00	0.76	0.81	1.26	0.76	0.86	0.81
time (sec)	N/A	0.009	0.013	0.049	1.347	1.069	0.151	0.318	0.080

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	334	265	67	119
normalized size	1	1.00	0.99	0.96	0.00	4.70	3.73	0.94	1.68
time (sec)	N/A	0.040	0.058	0.056	0.000	1.079	0.589	0.404	0.171

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	86	0	317	230	75	107
normalized size	1	1.00	1.00	1.19	0.00	4.40	3.19	1.04	1.49
time (sec)	N/A	0.036	0.044	0.048	0.000	0.948	0.548	0.514	0.320

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	84	97	171	218	90	100
normalized size	1	1.00	1.13	1.22	1.41	2.48	3.16	1.30	1.45
time (sec)	N/A	0.034	0.060	0.052	2.839	0.965	0.603	0.526	0.316

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	111	0	89	212	100	110
normalized size	1	1.00	1.05	1.79	0.00	1.44	3.42	1.61	1.77
time (sec)	N/A	0.157	0.096	0.573	0.000	1.260	0.982	0.570	0.253

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	58	34	31	55	63	56	56	38
normalized size	1	1.76	1.03	0.94	1.67	1.91	1.70	1.70	1.15
time (sec)	N/A	0.036	0.032	0.080	1.389	1.285	0.292	0.527	0.258

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	58	32	31	51	59	56	54	38
normalized size	1	1.87	1.03	1.00	1.65	1.90	1.81	1.74	1.23
time (sec)	N/A	0.029	0.029	0.049	1.322	0.855	0.282	0.546	0.135

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	27	15	165	27	27
normalized size	1	1.00	1.12	1.94	1.59	0.88	9.71	1.59	1.59
time (sec)	N/A	0.018	0.020	0.241	2.954	1.112	0.192	0.466	0.117

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	56	39	33	21	70	33	42
normalized size	1	1.00	2.43	1.70	1.43	0.91	3.04	1.43	1.83
time (sec)	N/A	0.028	0.039	0.196	3.018	0.867	0.588	0.466	0.303

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	29	38	40	0	40	39
normalized size	1	1.00	1.03	0.76	1.00	1.05	0.00	1.05	1.03
time (sec)	N/A	0.011	0.016	0.055	3.042	0.905	0.000	0.468	0.080

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	38	53	0	24	23
normalized size	1	1.00	1.00	0.83	1.27	1.77	0.00	0.80	0.77
time (sec)	N/A	0.010	0.013	0.045	2.998	0.782	0.000	1.604	0.052

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	52	40	0	41	39
normalized size	1	1.00	1.00	1.02	1.06	0.82	0.00	0.84	0.80
time (sec)	N/A	0.010	0.015	0.053	2.869	0.900	0.000	0.429	0.210

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	46	58	0	53	48
normalized size	1	1.00	1.02	0.78	1.02	1.29	0.00	1.18	1.07
time (sec)	N/A	0.015	0.017	0.049	3.004	0.913	0.000	0.456	0.191

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	46	60	0	36	35
normalized size	1	1.00	1.02	0.78	1.02	1.33	0.00	0.80	0.78
time (sec)	N/A	0.016	0.017	0.046	2.972	0.756	0.000	0.470	0.052

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	58	58	0	54	48
normalized size	1	1.00	0.89	0.81	0.94	0.94	0.00	0.87	0.77
time (sec)	N/A	0.015	0.026	0.047	2.975	0.955	0.000	0.442	0.195

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	32	41	60	0	31	30
normalized size	1	1.00	1.02	0.74	0.95	1.40	0.00	0.72	0.70
time (sec)	N/A	0.012	0.019	0.045	3.083	0.877	0.000	0.401	0.150

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	50	58	58	0	54	48
normalized size	1	1.00	0.90	0.85	0.98	0.98	0.00	0.92	0.81
time (sec)	N/A	0.013	0.019	0.047	2.980	0.972	0.000	0.489	0.193

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	46	46	84	0	0	36
normalized size	1	1.00	0.92	0.78	0.78	1.42	0.00	0.00	0.61
time (sec)	N/A	0.016	0.025	0.050	2.894	0.853	0.000	0.000	0.054

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	58	58	0	54	48
normalized size	1	1.00	0.89	0.81	0.94	0.94	0.00	0.87	0.77
time (sec)	N/A	0.013	0.025	0.046	3.003	0.967	0.000	0.445	0.221

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	32	41	60	0	31	30
normalized size	1	1.00	1.03	0.82	1.05	1.54	0.00	0.79	0.77
time (sec)	N/A	0.008	0.018	0.043	3.078	0.923	0.000	0.526	0.052

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	9	8	20	0	20	20
normalized size	1	1.00	1.00	0.64	0.57	1.43	0.00	1.43	1.43
time (sec)	N/A	0.007	0.005	0.044	2.948	0.899	0.000	0.631	0.199

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	7	8	33	0	6	6
normalized size	1	1.00	1.40	0.70	0.80	3.30	0.00	0.60	0.60
time (sec)	N/A	0.007	0.006	0.046	3.011	1.050	0.000	0.518	0.132

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	22	20	0	21	20
normalized size	1	1.00	0.96	1.20	0.88	0.80	0.00	0.84	0.80
time (sec)	N/A	0.006	0.005	0.045	2.991	0.936	0.000	0.575	0.218

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	38	0	33	26
normalized size	1	1.00	1.00	0.83	0.89	2.11	0.00	1.83	1.44
time (sec)	N/A	0.010	0.006	0.049	2.877	0.964	0.000	0.616	0.225

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	16	40	0	16	16
normalized size	1	1.00	1.00	0.79	0.84	2.11	0.00	0.84	0.84
time (sec)	N/A	0.010	0.006	0.046	2.927	0.955	0.000	0.818	0.137

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	28	38	0	34	26
normalized size	1	1.00	0.80	0.86	0.80	1.09	0.00	0.97	0.74
time (sec)	N/A	0.008	0.005	0.041	3.010	0.900	0.000	0.802	0.236

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	40	0	11	11
normalized size	1	1.00	1.00	0.71	0.65	2.35	0.00	0.65	0.65
time (sec)	N/A	0.007	0.006	0.046	2.997	0.847	0.000	0.658	0.170

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	30	28	37	0	34	26
normalized size	1	1.00	0.81	0.94	0.88	1.16	0.00	1.06	0.81
time (sec)	N/A	0.008	0.006	0.043	2.987	0.867	0.000	0.562	0.228

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	16	65	0	0	17
normalized size	1	1.00	0.85	0.79	0.48	1.97	0.00	0.00	0.52
time (sec)	N/A	0.008	0.008	0.039	2.910	0.931	0.000	0.000	0.136

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	28	38	0	34	26
normalized size	1	1.00	0.80	0.86	0.80	1.09	0.00	0.97	0.74
time (sec)	N/A	0.008	0.006	0.043	2.927	1.029	0.000	0.731	0.226

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	40	0	11	11
normalized size	1	1.00	1.00	0.92	0.85	3.08	0.00	0.85	0.85
time (sec)	N/A	0.004	0.006	0.045	3.073	1.021	0.000	0.638	0.164

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	51	16	137	0	46	40
normalized size	1	1.00	1.00	2.32	0.73	6.23	0.00	2.09	1.82
time (sec)	N/A	0.013	0.026	0.076	1.402	0.894	0.000	0.754	0.423

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	44	19	141	0	53	46
normalized size	1	1.00	1.00	1.91	0.83	6.13	0.00	2.30	2.00
time (sec)	N/A	0.013	0.026	0.088	3.084	0.996	0.000	1.085	0.406

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	44	19	143	0	53	44
normalized size	1	1.00	1.00	2.20	0.95	7.15	0.00	2.65	2.20
time (sec)	N/A	0.011	0.025	0.079	2.890	0.942	0.000	0.817	0.400

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	26	38	0	17	15
normalized size	1	1.00	1.00	1.26	1.37	2.00	0.00	0.89	0.79
time (sec)	N/A	0.002	0.005	0.047	1.340	0.879	0.000	0.828	0.053

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	30	41	0	17	17
normalized size	1	1.00	1.00	1.22	1.30	1.78	0.00	0.74	0.74
time (sec)	N/A	0.003	0.006	0.046	1.236	1.189	0.000	0.774	0.057

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	30	29	0	29	19
normalized size	1	1.00	1.00	1.13	1.30	1.26	0.00	1.26	0.83
time (sec)	N/A	0.005	0.052	0.048	1.342	0.857	0.000	0.615	0.055

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	59	49	0	36	29
normalized size	1	1.00	0.72	0.84	1.37	1.14	0.00	0.84	0.67
time (sec)	N/A	0.007	0.009	0.044	1.411	0.889	0.000	0.777	0.035

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.145	1.234	0.000	1.069	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.010	3.092	0.000	0.931	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.007	3.275	0.000	0.887	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.009	2.552	0.000	0.997	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.010	0.005	3.224	0.000	0.707	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.020	1.017	0.000	1.008	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	16	19	20	16	32
normalized size	1	1.00	0.94	0.94	0.89	1.06	1.11	0.89	1.78
time (sec)	N/A	0.002	0.007	0.043	1.246	0.876	0.063	0.624	0.390

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.012	0.006	1.126	0.000	1.059	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.007	1.116	0.000	1.012	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.009	1.076	0.000	0.954	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.008	1.035	0.000	0.889	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.010	1.050	0.000	0.885	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [.5385]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.00	13	0.231
2	A	6	3	1.00	15	0.200
3	A	5	3	1.00	15	0.200
4	A	4	3	1.00	15	0.200
5	A	3	3	1.00	15	0.200
6	A	6	3	1.00	13	0.231
7	A	5	3	1.00	13	0.231
8	A	4	3	1.00	13	0.231
9	A	3	3	1.00	13	0.231
10	A	3	3	1.00	13	0.231
11	A	3	3	1.00	13	0.231
12	A	4	3	1.00	11	0.273
13	A	3	3	1.00	11	0.273
14	A	3	3	1.00	11	0.273
15	A	3	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	13	0.154
17	A	2	2	1.00	15	0.133
18	A	1	1	1.00	15	0.067
19	A	2	2	1.00	15	0.133
20	A	3	2	1.00	15	0.133
21	A	2	2	1.00	13	0.154
22	A	1	1	1.00	13	0.077
23	A	2	2	1.00	13	0.154
24	A	3	2	1.00	13	0.154
25	A	2	2	1.00	16	0.125
26	A	2	2	1.00	15	0.133
27	A	2	2	1.00	13	0.154
28	A	2	2	1.00	11	0.182
29	A	2	2	1.00	11	0.182
30	A	6	5	1.00	13	0.385
31	A	5	5	1.00	13	0.385
32	A	4	4	1.00	13	0.308
33	A	5	5	1.00	13	0.385
34	A	6	5	1.00	13	0.385
35	A	8	7	1.00	13	0.538
36	A	7	7	1.00	13	0.538
37	A	6	6	1.00	13	0.462
38	A	7	7	1.00	13	0.538
39	A	8	7	1.00	13	0.538
40	A	5	4	1.00	13	0.308
41	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	4	4	1.00	13	0.308
43	A	3	3	1.00	13	0.231
44	A	3	3	1.00	13	0.231
45	A	4	4	1.00	13	0.308
46	A	5	4	1.00	13	0.308
47	A	6	4	1.00	13	0.308
48	A	1	1	1.00	11	0.091
49	A	2	1	1.00	9	0.111
50	A	2	1	1.00	9	0.111
51	A	2	1	1.00	9	0.111
52	A	1	0	1.00	7	0.000
53	A	1	1	1.00	9	0.111
54	A	2	2	1.00	9	0.222
55	A	3	2	1.00	9	0.222
56	A	5	3	1.00	11	0.273
57	A	4	3	1.00	11	0.273
58	A	3	3	1.00	11	0.273
59	A	2	2	1.00	11	0.182
60	A	1	1	1.00	11	0.091
61	A	2	2	1.00	11	0.182
62	A	3	2	1.00	11	0.182
63	A	4	2	1.00	11	0.182
64	A	1	1	1.00	14	0.071
65	A	1	1	1.00	14	0.071
66	A	2	2	1.00	14	0.143
67	A	1	1	1.00	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	1	1	1.00	14	0.071
69	A	2	2	1.00	14	0.143
70	A	1	1	1.00	14	0.071
71	A	2	2	1.00	14	0.143
72	A	1	1	1.00	14	0.071
73	A	2	2	1.00	14	0.143
74	A	3	2	1.00	23	0.087
75	A	3	2	1.00	23	0.087
76	A	3	2	1.00	23	0.087
77	A	3	2	1.00	23	0.087
78	A	2	2	1.00	12	0.167
79	A	2	2	1.00	15	0.133
80	A	2	2	1.00	12	0.167
81	A	3	2	1.00	12	0.167
82	A	3	2	1.00	12	0.167
83	B	3	2	2.83	10	0.200
84	A	2	2	1.00	12	0.167
85	A	2	2	1.00	12	0.167
86	A	2	2	1.00	12	0.167
87	A	2	2	1.00	12	0.167
88	A	2	2	1.00	12	0.167
89	A	2	2	1.00	13	0.154
90	A	2	2	1.00	14	0.143
91	A	3	3	1.00	12	0.250
92	A	3	3	1.00	12	0.250
93	A	4	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	4	3	1.00	12	0.250
95	A	3	3	1.00	12	0.250
96	A	3	3	1.00	13	0.231
97	A	3	3	1.00	14	0.214
98	A	2	2	1.00	40	0.050
99	A	3	2	1.76	30	0.067
100	A	3	2	1.87	31	0.065
101	A	2	2	1.00	14	0.143
102	A	2	2	1.00	16	0.125
103	A	3	3	1.00	14	0.214
104	A	3	3	1.00	14	0.214
105	A	3	3	1.00	14	0.214
106	A	3	3	1.00	14	0.214
107	A	3	3	1.00	14	0.214
108	A	3	3	1.00	14	0.214
109	A	3	3	1.00	14	0.214
110	A	3	3	1.00	14	0.214
111	A	3	3	1.00	14	0.214
112	A	3	3	1.00	14	0.214
113	A	3	3	1.00	14	0.214
114	A	2	2	1.00	14	0.143
115	A	2	2	1.00	14	0.143
116	A	2	2	1.00	14	0.143
117	A	2	2	1.00	14	0.143
118	A	2	2	1.00	14	0.143
119	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	2	2	1.00	14	0.143
121	A	2	2	1.00	14	0.143
122	A	2	2	1.00	14	0.143
123	A	2	2	1.00	14	0.143
124	A	2	2	1.00	14	0.143
125	A	2	2	1.00	27	0.074
126	A	2	2	1.00	30	0.067
127	A	2	2	1.00	28	0.071
128	A	1	1	1.00	12	0.083
129	A	1	1	1.00	14	0.071
130	A	1	1	1.00	16	0.062
131	A	2	2	1.00	14	0.143
132	A	1	1	1.00	12	0.083
133	A	2	2	1.00	12	0.167
134	A	2	2	1.00	12	0.167
135	A	2	2	1.00	12	0.167
136	A	2	2	1.00	12	0.167
137	A	1	1	1.00	10	0.100
138	A	1	1	1.00	7	0.143
139	A	2	2	1.00	12	0.167
140	A	2	2	1.00	12	0.167
141	A	2	2	1.00	12	0.167
142	A	2	2	1.00	12	0.167
143	A	2	2	1.00	12	0.167

Chapter 3

Listing of integrals

3.1 $\int (bx + cx^2)^{7/2} dx$

Optimal. Leaf size=147

$$\frac{35b^8 \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}} - \frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)(bx+cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)(bx+cx^2)^{5/2}}{384c^2} + \frac{(b+cx)^{7/2}}{16c}$$

[Out] 35/6144*b^4*(2*c*x+b)*(c*x^2+b*x)^(3/2)/c^3-7/384*b^2*(2*c*x+b)*(c*x^2+b*x)^(5/2)/c^2+1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c+35/16384*b^8*arctanh(x*c^(1/2))/(c*x^2+b*x)^(1/2))/c^(9/2)-35/16384*b^6*(2*c*x+b)*(c*x^2+b*x)^(1/2)/c^4

Rubi [A] time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 620, 206}

$$-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)(bx+cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b+2cx)(bx+cx^2)^{5/2}}{384c^2} + \frac{35b^8 \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}} + \frac{(b+cx)^{7/2}}{16c}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(7/2), x]

[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b + 2*c*x)*(b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*x + c*x^2)^(5/2))/(384*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(16*c) + (35*b^8*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^(9/2))

Rule 206

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 612

`Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]`

Rule 620

`Int[1/Sqrt[(b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]`

Rubi steps

$$\begin{aligned}
 \int (bx + cx^2)^{7/2} dx &= \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c} - \frac{(7b^2) \int (bx + cx^2)^{5/2} dx}{32c} \\
 &= -\frac{7b^2(b + 2cx)(bx + cx^2)^{5/2}}{384c^2} + \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c} + \frac{(35b^4) \int (bx + cx^2)^{3/2} dx}{768c^2} \\
 &= \frac{35b^4(b + 2cx)(bx + cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b + 2cx)(bx + cx^2)^{5/2}}{384c^2} + \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c} - \frac{(35b^6)}{16384c^4} \\
 &= -\frac{35b^6(b + 2cx)\sqrt{bx + cx^2}}{16384c^4} + \frac{35b^4(b + 2cx)(bx + cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b + 2cx)(bx + cx^2)^{5/2}}{384c^2} + \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c} \\
 &= -\frac{35b^6(b + 2cx)\sqrt{bx + cx^2}}{16384c^4} + \frac{35b^4(b + 2cx)(bx + cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b + 2cx)(bx + cx^2)^{5/2}}{384c^2} + \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c} \\
 &= -\frac{35b^6(b + 2cx)\sqrt{bx + cx^2}}{16384c^4} + \frac{35b^4(b + 2cx)(bx + cx^2)^{3/2}}{6144c^3} - \frac{7b^2(b + 2cx)(bx + cx^2)^{5/2}}{384c^2} + \frac{(b + 2cx)(bx + cx^2)^{7/2}}{16c}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 142, normalized size = 0.97

$$\sqrt{x(b + cx)} \left(\frac{105b^{15/2} \sinh^{-1}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{\frac{cx}{b}+1}} + \sqrt{c} \left(-105b^7 + 70b^6cx - 56b^5c^2x^2 + 48b^4c^3x^3 + 10880b^3c^4x^4 + 25856b^2c^5x^5 + \dots\right) \right)$$

49152c^{9/2}

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(7/2),x]

[Out] (Sqrt[x*(b + c*x)]*(Sqrt[c]*(-105*b^7 + 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b^4*c^3*x^3 + 10880*b^3*c^4*x^4 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 6144*c^7*x^7) + (105*b^(15/2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[x]*Sqrt[1 + (c*x)/b]))/(49152*c^(9/2))

fricas [A] time = 0.70, size = 258, normalized size = 1.76

$$\frac{105 b^8 \sqrt{c} \log\left(2 c x + b + 2 \sqrt{c x^2 + b x} \sqrt{c}\right) + 2\left(6144 c^8 x^7 + 21504 b c^7 x^6 + 25856 b^2 c^6 x^5 + 10880 b^3 c^5 x^4 + 48 b^4 c^4 x^3 - 56 b^5 c^3 x^2 + 70 b^6 c^2 x - 105 b^7 c\right) \sqrt{c x^2 + b x}}{98304 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="fricas")

[Out] [1/98304*(105*b^8*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 + 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 + b*x))/c^5, -1/49152*(105*b^8*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) - (6144*c^8*x^7 + 21504*b*c^7*x^6 + 25856*b^2*c^6*x^5 + 10880*b^3*c^5*x^4 + 48*b^4*c^4*x^3 - 56*b^5*c^3*x^2 + 70*b^6*c^2*x - 105*b^7*c)*sqrt(c*x^2 + b*x))/c^5]

giac [A] time = 0.63, size = 132, normalized size = 0.90

$$\frac{35 b^8 \log\left(\left|-2\left(\sqrt{c} x - \sqrt{c x^2 + b x}\right) \sqrt{c} - b\right|\right)}{32768 c^{\frac{9}{2}}} - \frac{1}{49152} \left(\frac{105 b^7}{c^4} - 2\left(\frac{35 b^6}{c^3} - 4\left(\frac{7 b^5}{c^2} - 2\left(\frac{3 b^4}{c} + 8\left(85 b^3 + 2\left(101 b^2 c + 12\left(2 c^3 x + 7 b c^2\right) x\right) x\right) x\right) \sqrt{c x^2 + b x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(7/2),x, algorithm="giac")

[Out] -35/32768*b^8*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) - b))/c^(9/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^2 - 2*(3*b^4/c + 8*(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2)*x)*x)*x)*sqrt(c*x^2 + b*x))

maple [A] time = 0.05, size = 173, normalized size = 1.18

$$\frac{35 b^8 \ln\left(\frac{c x + \frac{b}{2}}{\sqrt{c}} + \sqrt{c x^2 + b x}\right)}{32768 c^{\frac{9}{2}}} - \frac{35 \sqrt{c x^2 + b x} b^6 x}{8192 c^3} - \frac{35 \sqrt{c x^2 + b x} b^7}{16384 c^4} + \frac{35 (c x^2 + b x)^{\frac{3}{2}} b^4 x}{3072 c^2} + \frac{35 (c x^2 + b x)^{\frac{3}{2}} b^5}{6144 c^3} - \frac{7}{1024 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x)^(7/2),x)`

[Out] $\frac{1}{16}*(2*c*x+b)*(c*x^2+b*x)^{(7/2)}/c-7/192*b^2/c*(c*x^2+b*x)^{(5/2)}*x-7/384*b^3/c^2*(c*x^2+b*x)^{(5/2)}+35/3072*b^4/c^2*(c*x^2+b*x)^{(3/2)}*x+35/6144*b^5/c^3*(c*x^2+b*x)^{(3/2)}-35/8192*b^6/c^3*(c*x^2+b*x)^{(1/2)}*x-35/16384*b^7/c^4*(c*x^2+b*x)^{(1/2)}+35/32768*b^8/c^4*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x)^{(1/2)})$

maxima [A] time = 1.34, size = 180, normalized size = 1.22

$$\frac{1}{8}(cx^2 + bx)^{\frac{7}{2}}x - \frac{35\sqrt{cx^2 + bx}b^6x}{8192c^3} + \frac{35(cx^2 + bx)^{\frac{3}{2}}b^4x}{3072c^2} - \frac{7(cx^2 + bx)^{\frac{5}{2}}b^2x}{192c} + \frac{35b^8 \log\left(2cx + b + 2\sqrt{cx^2 + bx}\sqrt{c}\right)}{32768c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)^(7/2),x, algorithm="maxima")`

[Out] $\frac{1}{8}*(c*x^2 + b*x)^{(7/2)}*x - \frac{35}{8192}*\sqrt{c*x^2 + b*x}*b^6*x/c^3 + \frac{35}{3072}*(c*x^2 + b*x)^{(3/2)}*b^4*x/c^2 - \frac{7}{192}*(c*x^2 + b*x)^{(5/2)}*b^2*x/c + \frac{35}{32768}*\log(2*c*x + b + 2*\sqrt{c*x^2 + b*x}*\sqrt{c})/c^4 - \frac{35}{16384}*\sqrt{c*x^2 + b*x}*b^7/c^4 + \frac{35}{6144}*(c*x^2 + b*x)^{(3/2)}*b^5/c^3 - \frac{7}{384}*(c*x^2 + b*x)^{(5/2)}*b^3/c^2 + \frac{1}{16}*(c*x^2 + b*x)^{(7/2)}*b/c$

mupad [B] time = 0.74, size = 151, normalized size = 1.03

$$\frac{(cx^2 + bx)^{7/2} \left(\frac{b}{2} + cx\right)}{8c} - \left(\frac{7b^2}{6c} \frac{(cx^2 + bx)^{5/2} \left(\frac{b}{2} + cx\right)}{24c} - \frac{5b^2}{4c} \frac{(cx^2 + bx)^{3/2} \left(\frac{b}{2} + cx\right)}{16c} - \frac{3b^2}{8c^{3/2}} \frac{\sqrt{cx^2 + bx} \left(\frac{x}{2} + \frac{b}{4c}\right) - \frac{b^2 \ln\left(\frac{b}{2} + cx + \sqrt{cx^2 + bx}\right)}{\sqrt{c}}}{16c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*x + c*x^2)^(7/2)*(b/2 + c*x))/(8*c) - (7*b^2*((b*x + c*x^2)^(5/2)*(b/2 + c*x))/(6*c) - (5*b^2*((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)))/(32*c)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (bx + cx^2)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(7/2), x)
```

3.2 $\int (3ix + 4x^2)^{7/2} dx$

Optimal. Leaf size=121

$$\frac{1}{64}(8x+3i)(4x^2+3ix)^{7/2} + \frac{21(8x+3i)(4x^2+3ix)^{5/2}}{2048} + \frac{945(8x+3i)(4x^2+3ix)^{3/2}}{131072} + \frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304} + \dots$$

[Out] 945/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)-229635/16777216*I*arcsin(-1+8/3*I*x)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {612, 619, 215}

$$\frac{1}{64}(8x+3i)(4x^2+3ix)^{7/2} + \frac{21(8x+3i)(4x^2+3ix)^{5/2}}{2048} + \frac{945(8x+3i)(4x^2+3ix)^{3/2}}{131072} + \frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304} + \dots$$

Antiderivative was successfully verified.

[In] Int[((3*I)*x + 4*x^2)^(7/2), x]

[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8*x)*((3*I)*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/2048 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + ((229635*I)/16777216)*ArcSin[1 - ((8*I)/3)*x]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
 \int (3ix + 4x^2)^{7/2} dx &= \frac{1}{64}(3i + 8x)(3ix + 4x^2)^{7/2} + \frac{63}{128} \int (3ix + 4x^2)^{5/2} dx \\
 &= \frac{21(3i + 8x)(3ix + 4x^2)^{5/2}}{2048} + \frac{1}{64}(3i + 8x)(3ix + 4x^2)^{7/2} + \frac{945 \int (3ix + 4x^2)^{3/2} dx}{4096} \\
 &= \frac{945(3i + 8x)(3ix + 4x^2)^{3/2}}{131072} + \frac{21(3i + 8x)(3ix + 4x^2)^{5/2}}{2048} + \frac{1}{64}(3i + 8x)(3ix + 4x^2)^{7/2} + \\
 &= \frac{25515(3i + 8x)\sqrt{3ix + 4x^2}}{4194304} + \frac{945(3i + 8x)(3ix + 4x^2)^{3/2}}{131072} + \frac{21(3i + 8x)(3ix + 4x^2)^{5/2}}{2048} + \\
 &= \frac{25515(3i + 8x)\sqrt{3ix + 4x^2}}{4194304} + \frac{945(3i + 8x)(3ix + 4x^2)^{3/2}}{131072} + \frac{21(3i + 8x)(3ix + 4x^2)^{5/2}}{2048} + \\
 &= \frac{25515(3i + 8x)\sqrt{3ix + 4x^2}}{4194304} + \frac{945(3i + 8x)(3ix + 4x^2)^{3/2}}{131072} + \frac{21(3i + 8x)(3ix + 4x^2)^{5/2}}{2048} +
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 119, normalized size = 0.98

$$\frac{\sqrt{x(4x + 3i)} \left(2\sqrt{3 - 4ix} \sqrt{x} (33554432x^7 + 88080384ix^6 - 79429632x^5 - 25067520ix^4 + 82944x^3 - 72576ix^2 - 888608\sqrt{3 - 4ix} \sqrt{x}) \right)}{8388608\sqrt{3 - 4ix} \sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(7/2), x]

[Out] (Sqrt[x*(3*I + 4*x)]*(2*Sqrt[3 - (4*I)*x]*Sqrt[x]*(76545*I - 68040*x - (725 76*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x^5 + (88080384*I)*x^6 + 33554432*x^7) - 229635*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/(83 88608*Sqrt[3 - (4*I)*x]*Sqrt[x])

fricas [A] time = 0.91, size = 69, normalized size = 0.57

$$\frac{1}{268435456} (2147483648 x^7 + 5637144576i x^6 - 5083496448 x^5 - 1604321280i x^4 + 5308416 x^3 - 4644864i x^2 - 888608\sqrt{3 - 4ix} \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="fricas")

[Out] 1/268435456*(2147483648*x^7 + 5637144576*I*x^6 - 5083496448*x^5 - 1604321280*I*x^4 + 5308416*x^3 - 4644864*I*x^2 - 4354560*x + 4898880*I)*sqrt(4*x^2 + 3*I*x) - 229635/16777216*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 1165671/268435456

giac [A] time = 0.57, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="giac")

[Out] 0

maple [A] time = 0.12, size = 91, normalized size = 0.75

$$\frac{229635 \operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{16777216} + \frac{(8x + 3i)(4x^2 + 3ix)^{\frac{7}{2}}}{64} + \frac{21(8x + 3i)(4x^2 + 3ix)^{\frac{5}{2}}}{2048} + \frac{945(8x + 3i)(4x^2 + 3ix)^{\frac{3}{2}}}{131072} + \frac{25515}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*I*x+4*x^2)^(7/2),x)

[Out] 1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+945/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+25515/4194304*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+229635/16777216*arcsinh(8/3*x+I)

maxima [A] time = 3.00, size = 130, normalized size = 1.07

$$\frac{1}{8}(4x^2 + 3ix)^{\frac{7}{2}}x + \frac{3}{64}i(4x^2 + 3ix)^{\frac{7}{2}} + \frac{21}{256}(4x^2 + 3ix)^{\frac{5}{2}}x + \frac{63}{2048}i(4x^2 + 3ix)^{\frac{5}{2}} + \frac{945}{16384}(4x^2 + 3ix)^{\frac{3}{2}}x + \frac{2835}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(7/2),x, algorithm="maxima")

[Out] 1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/256*(4*x^2 + 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 945/16384*(4*x^2 + 3*I*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(3/2) + 25515/524288*sqrt(4*x^2 + 3*I*x)*x + 76545/4194304*I*sqrt(4*x^2 + 3*I*x) + 229635/16777216*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)

mupad [B] time = 0.29, size = 100, normalized size = 0.83

$$\frac{229635 \ln\left(x + \frac{\sqrt{x(4x+3i)}}{2} + \frac{3}{8}i\right)}{16777216} + \frac{945\left(4x + \frac{3}{2}i\right)\left(4x^2 + x3i\right)^{3/2}}{65536} + \frac{21\left(4x + \frac{3}{2}i\right)\left(4x^2 + x3i\right)^{5/2}}{1024} + \frac{\left(4x + \frac{3}{2}i\right)\left(4x^2 + x3i\right)^{7/2}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*3i + 4*x^2)^(7/2),x)`

[Out] $(229635 \cdot \log(x + (x(4x + 3i))^{1/2}/2 + 3i/8))/16777216 + (945(4x + 3i/2) \cdot (x \cdot 3i + 4x^2)^{3/2})/65536 + (21(4x + 3i/2) \cdot (x \cdot 3i + 4x^2)^{5/2})/1024 + ((4x + 3i/2) \cdot (x \cdot 3i + 4x^2)^{7/2})/32 + (25515 \cdot (x/2 + 3i/16) \cdot (x \cdot 3i + 4x^2)^{1/2})/262144$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4x^2 + 3ix)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*I*x+4*x**2)**(7/2),x)`

[Out] `Integral((4*x**2 + 3*I*x)**(7/2), x)`

3.3 $\int (3ix + 4x^2)^{5/2} dx$

Optimal. Leaf size=95

$$\frac{1}{48}(8x+3i)(4x^2+3ix)^{5/2} + \frac{15(8x+3i)(4x^2+3ix)^{3/2}}{1024} + \frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768} + \frac{3645i \sin^{-1}\left(1 - \frac{8ix}{3}\right)}{131072}$$

[Out] 15/1024*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)-3645/131072*I*arcsin(-1+8/3*I*x)+405/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {612, 619, 215}

$$\frac{1}{48}(8x+3i)(4x^2+3ix)^{5/2} + \frac{15(8x+3i)(4x^2+3ix)^{3/2}}{1024} + \frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768} + \frac{3645i \sin^{-1}\left(1 - \frac{8ix}{3}\right)}{131072}$$

Antiderivative was successfully verified.

[In] Int[((3*I)*x + 4*x^2)^(5/2), x]

[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*((3*I)*x + 4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + ((3645*I)/131072)*ArcSin[1 - ((8*I)/3)*x]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (3ix + 4x^2)^{5/2} dx &= \frac{1}{48}(3i + 8x)(3ix + 4x^2)^{5/2} + \frac{15}{32} \int (3ix + 4x^2)^{3/2} dx \\
&= \frac{15(3i + 8x)(3ix + 4x^2)^{3/2}}{1024} + \frac{1}{48}(3i + 8x)(3ix + 4x^2)^{5/2} + \frac{405 \int \sqrt{3ix + 4x^2} dx}{2048} \\
&= \frac{405(3i + 8x)\sqrt{3ix + 4x^2}}{32768} + \frac{15(3i + 8x)(3ix + 4x^2)^{3/2}}{1024} + \frac{1}{48}(3i + 8x)(3ix + 4x^2)^{5/2} + \dots \\
&= \frac{405(3i + 8x)\sqrt{3ix + 4x^2}}{32768} + \frac{15(3i + 8x)(3ix + 4x^2)^{3/2}}{1024} + \frac{1}{48}(3i + 8x)(3ix + 4x^2)^{5/2} + \dots \\
&= \frac{405(3i + 8x)\sqrt{3ix + 4x^2}}{32768} + \frac{15(3i + 8x)(3ix + 4x^2)^{3/2}}{1024} + \frac{1}{48}(3i + 8x)(3ix + 4x^2)^{5/2} + \dots
\end{aligned}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.93

$$\frac{\sqrt{x(4x + 3i)} \left(524288x^5 + 983040ix^4 - 497664x^3 - 6912ix^2 - 6480x - \frac{10935 \sqrt[4]{-1} \sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{3-4ix}\sqrt{x}} + 7290i \right)}{196608}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(5/2), x]

[Out] (Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (983040*I)*x^4 + 524288*x^5 - (10935*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x]))/196608

fricas [A] time = 0.97, size = 59, normalized size = 0.62

$$\frac{1}{3145728} \left(8388608x^5 + 15728640ix^4 - 7962624x^3 - 110592ix^2 - 103680x + 116640i \right) \sqrt{4x^2 + 3ix} - \frac{3645}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(5/2), x, algorithm="fricas")

[Out] 1/3145728*(8388608*x^5 + 15728640*I*x^4 - 7962624*x^3 - 110592*I*x^2 - 103680*x + 116640*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 8991/1048576

giac [A] time = 0.46, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")

[Out] 0

maple [A] time = 0.09, size = 71, normalized size = 0.75

$$\frac{3645 \operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{131072} + \frac{(8x + 3i)(4x^2 + 3ix)^{\frac{5}{2}}}{48} + \frac{15(8x + 3i)(4x^2 + 3ix)^{\frac{3}{2}}}{1024} + \frac{405(8x + 3i)\sqrt{4x^2 + 3ix}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+3*I*x)^(5/2),x)

[Out] 1/48*(8*x+3*I)*(4*x^2+3*I*x)^(5/2)+15/1024*(8*x+3*I)*(4*x^2+3*I*x)^(3/2)+405/32768*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+3645/131072*arcsinh(8/3*x+I)

maxima [A] time = 3.01, size = 103, normalized size = 1.08

$$\frac{1}{6}(4x^2 + 3ix)^{\frac{5}{2}}x + \frac{1}{16}i(4x^2 + 3ix)^{\frac{5}{2}} + \frac{15}{128}(4x^2 + 3ix)^{\frac{3}{2}}x + \frac{45}{1024}i(4x^2 + 3ix)^{\frac{3}{2}} + \frac{405}{4096}\sqrt{4x^2 + 3ix}x + \frac{1215}{32768}i\sqrt{4x^2 + 3ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")

[Out] 1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2 + 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2 + 3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)

mupad [B] time = 0.30, size = 80, normalized size = 0.84

$$\frac{3645 \ln\left(x + \frac{\sqrt{x(4x+3i)}}{2} + \frac{3}{8}i\right)}{131072} + \frac{15\left(4x + \frac{3}{2}i\right)(4x^2 + x3i)^{3/2}}{512} + \frac{\left(4x + \frac{3}{2}i\right)(4x^2 + x3i)^{5/2}}{24} + \frac{405\left(\frac{x}{2} + \frac{3}{16}i\right)\sqrt{4x^2 + 3ix}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*3i + 4*x^2)^(5/2),x)

```
[Out] (3645*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/131072 + (15*(4*x + 3i/2)*(x*
3i + 4*x^2)^(3/2))/512 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/24 + (405*(x/2
+ 3i/16)*(x*3i + 4*x^2)^(1/2))/2048
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (4x^2 + 3ix)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(5/2), x)
```

3.4 $\int (3ix + 4x^2)^{3/2} dx$

Optimal. Leaf size=69

$$\frac{1}{32}(8x + 3i)(4x^2 + 3ix)^{3/2} + \frac{27(8x + 3i)\sqrt{4x^2 + 3ix}}{1024} + \frac{243i \sin^{-1}\left(1 - \frac{8ix}{3}\right)}{4096}$$

[Out] $1/32*(3*I+8*x)*(3*I*x+4*x^2)^{(3/2)}-243/4096*I*\arcsin(-1+8/3*I*x)+27/1024*(3*I+8*x)*(3*I*x+4*x^2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {612, 619, 215}

$$\frac{1}{32}(8x + 3i)(4x^2 + 3ix)^{3/2} + \frac{27(8x + 3i)\sqrt{4x^2 + 3ix}}{1024} + \frac{243i \sin^{-1}\left(1 - \frac{8ix}{3}\right)}{4096}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3I)x + 4x^2]^{3/2}, x]$

[Out] $(27*(3I + 8*x)*\text{Sqrt}[(3I)x + 4*x^2])/1024 + ((3I + 8*x)*((3I)x + 4*x^2)^{(3/2)})/32 + ((243I)/4096)*\text{ArcSin}[1 - ((8I)/3)*x]$

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSinh}[\text{Rt}[b, 2]*x]/\text{Sqrt}[a]/\text{Rt}[b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x$ && $\text{GtQ}[a, 0]$ && $\text{PosQ}[b]$

Rule 612

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p - 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $\text{GtQ}[p, 0]$ && $\text{IntegerQ}[4*p]$

Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x$ && $\text{GtQ}[4*a - b^2/c, 0]$

Rubi steps

$$\begin{aligned}
\int (3ix + 4x^2)^{3/2} dx &= \frac{1}{32}(3i + 8x)(3ix + 4x^2)^{3/2} + \frac{27}{64} \int \sqrt{3ix + 4x^2} dx \\
&= \frac{27(3i + 8x)\sqrt{3ix + 4x^2}}{1024} + \frac{1}{32}(3i + 8x)(3ix + 4x^2)^{3/2} + \frac{243 \int \frac{1}{\sqrt{3ix+4x^2}} dx}{2048} \\
&= \frac{27(3i + 8x)\sqrt{3ix + 4x^2}}{1024} + \frac{1}{32}(3i + 8x)(3ix + 4x^2)^{3/2} + \frac{81 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}} dx, x, 3i + 8x \right)}{4096} \\
&= \frac{27(3i + 8x)\sqrt{3ix + 4x^2}}{1024} + \frac{1}{32}(3i + 8x)(3ix + 4x^2)^{3/2} + \frac{243i \sin^{-1} \left(1 - \frac{8ix}{3} \right)}{4096}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.10

$$\frac{\sqrt{x(4x + 3i)} \left(2048x^3 + 2304ix^2 - 144x - \frac{243 \sqrt[4]{-1} \sin^{-1} \left((1+i)\sqrt{\frac{2}{3}} \sqrt{x} \right)}{\sqrt{3-4ix} \sqrt{x}} + 162i \right)}{2048}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(3/2), x]

[Out] (Sqrt[x*(3*I + 4*x)]*(162*I - 144*x + (2304*I)*x^2 + 2048*x^3 - (243*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/(Sqrt[3 - (4*I)*x]*Sqrt[x]))/2048

fricas [A] time = 1.17, size = 49, normalized size = 0.71

$$\frac{1}{32768} (32768x^3 + 36864ix^2 - 2304x + 2592i) \sqrt{4x^2 + 3ix} - \frac{243}{4096} \log \left(-2x + \sqrt{4x^2 + 3ix} - \frac{3}{4}i \right) - \frac{567}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(3/2), x, algorithm="fricas")

[Out] 1/32768*(32768*x^3 + 36864*I*x^2 - 2304*x + 2592*I)*sqrt(4*x^2 + 3*I*x) - 243/4096*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768

giac [A] time = 0.61, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")

[Out] 0

maple [A] time = 0.12, size = 51, normalized size = 0.74

$$\frac{243 \operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{4096} + \frac{(8x + 3i)(4x^2 + 3ix)^{\frac{3}{2}}}{32} + \frac{27(8x + 3i)\sqrt{4x^2 + 3ix}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+3*I*x)^(3/2),x)

[Out] 1/32*(8*x+3*I)*(4*x^2+3*I*x)^(3/2)+27/1024*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+243/4096*arcsinh(8/3*x+I)

maxima [A] time = 2.98, size = 76, normalized size = 1.10

$$\frac{1}{4}(4x^2 + 3ix)^{\frac{3}{2}}x + \frac{3}{32}i(4x^2 + 3ix)^{\frac{3}{2}} + \frac{27}{128}\sqrt{4x^2 + 3ix}x + \frac{81}{1024}i\sqrt{4x^2 + 3ix} + \frac{243}{4096}\log\left(8x + 4\sqrt{4x^2 + 3ix} + 3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4*x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)

mupad [B] time = 0.16, size = 60, normalized size = 0.87

$$\frac{243 \ln\left(x + \frac{\sqrt{x(4x+3i)}}{2} + \frac{3}{8}i\right)}{4096} + \frac{\left(4x + \frac{3}{2}i\right)(4x^2 + x3i)^{3/2}}{16} + \frac{27\left(\frac{x}{2} + \frac{3}{16}i\right)\sqrt{4x^2 + x3i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*3i + 4*x^2)^(3/2),x)

[Out] (243*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/4096 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(3/2))/16 + (27*(x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/64

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4x^2 + 3ix)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(3/2), x)
```

3.5 $\int \sqrt{3ix + 4x^2} dx$

Optimal. Leaf size=43

$$\frac{1}{16}\sqrt{4x^2 + 3ix}(8x + 3i) + \frac{9}{64}i \sin^{-1}\left(1 - \frac{8ix}{3}\right)$$

[Out] $-9/64*I*\arcsin(-1+8/3*I*x)+1/16*(3*I+8*x)*(3*I*x+4*x^2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {612, 619, 215}

$$\frac{1}{16}\sqrt{4x^2 + 3ix}(8x + 3i) + \frac{9}{64}i \sin^{-1}\left(1 - \frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(3*I)*x + 4*x^2], x]

[Out] $((3*I + 8*x)*\text{Sqrt}[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*\text{ArcSin}[1 - ((8*I)/3)*x]$

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{3ix + 4x^2} dx &= \frac{1}{16}(3i + 8x)\sqrt{3ix + 4x^2} + \frac{9}{32} \int \frac{1}{\sqrt{3ix + 4x^2}} dx \\
&= \frac{1}{16}(3i + 8x)\sqrt{3ix + 4x^2} + \frac{3}{64} \text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{9}}} dx, x, 3i + 8x \right) \\
&= \frac{1}{16}(3i + 8x)\sqrt{3ix + 4x^2} + \frac{9}{64} i \sin^{-1} \left(1 - \frac{8ix}{3} \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 1.49

$$\frac{1}{32} \sqrt{x(4x + 3i)} \left(16x - \frac{9\sqrt[4]{-1} \sin^{-1} \left((1 + i)\sqrt{\frac{2}{3}} \sqrt{x} \right)}{\sqrt{3 - 4ix} \sqrt{x}} + 6i \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(3*I)*x + 4*x^2], x]

[Out] (Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*(-1)^(1/4)*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]]))/(Sqrt[3 - (4*I)*x]*Sqrt[x]))/32

fricas [A] time = 0.80, size = 39, normalized size = 0.91

$$\frac{1}{256} \sqrt{4x^2 + 3ix} (128x + 48i) - \frac{9}{64} \log \left(-2x + \sqrt{4x^2 + 3ix} - \frac{3}{4}i \right) - \frac{9}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(1/2), x, algorithm="fricas")

[Out] 1/256*sqrt(4*x^2 + 3*I*x)*(128*x + 48*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 9/256

giac [A] time = 0.63, size = 1, normalized size = 0.02

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*I*x+4*x^2)^(1/2), x, algorithm="giac")

[Out] 0

maple [A] time = 0.10, size = 31, normalized size = 0.72

$$\frac{9 \operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{64} + \frac{(8x + 3i) \sqrt{4x^2 + 3ix}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+3*I*x)^(1/2),x)`

[Out] `1/16*(8*x+3*I)*(4*x^2+3*I*x)^(1/2)+9/64*arcsinh(8/3*x+I)`

maxima [A] time = 2.96, size = 49, normalized size = 1.14

$$\frac{1}{2} \sqrt{4x^2 + 3ix} x + \frac{3}{16} i \sqrt{4x^2 + 3ix} + \frac{9}{64} \log\left(8x + 4\sqrt{4x^2 + 3ix} + 3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")`

[Out] `1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)`

mupad [B] time = 0.09, size = 39, normalized size = 0.91

$$\frac{9 \ln\left(x + \frac{\sqrt{x(4x+3i)}}{2} + \frac{3i}{8}\right)}{64} + \left(\frac{x}{2} + \frac{3i}{16}\right) \sqrt{4x^2 + x3i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*3i + 4*x^2)^(1/2),x)`

[Out] `(9*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/64 + (x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4x^2 + 3ix} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*I*x+4*x**2)**(1/2),x)`

[Out] `Integral(sqrt(4*x**2 + 3*I*x), x)`

3.6 $\int (3x - 4x^2)^{7/2} dx$

Optimal. Leaf size=101

$$-\frac{1}{64}(3-8x)(3x-4x^2)^{7/2} - \frac{21(3-8x)(3x-4x^2)^{5/2}}{2048} - \frac{945(3-8x)(3x-4x^2)^{3/2}}{131072} - \frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304} - \frac{229635}{16777216} \arcsin(-1+8/3x) - 25515/4194304$$

[Out] $-945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*x)^(5/2)-1/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*\arcsin(-1+8/3*x)-25515/4194304*(3-8*x)*(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{64}(3-8x)(3x-4x^2)^{7/2} - \frac{21(3-8x)(3x-4x^2)^{5/2}}{2048} - \frac{945(3-8x)(3x-4x^2)^{3/2}}{131072} - \frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304} - \frac{229635}{16777216} \arcsin(-1+8/3x) - 25515/4194304$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(7/2), x]

[Out] $(-25515*(3-8*x)*\text{Sqrt}[3*x-4*x^2])/4194304 - (945*(3-8*x)*(3*x-4*x^2)^(3/2))/131072 - (21*(3-8*x)*(3*x-4*x^2)^(5/2))/2048 - ((3-8*x)*(3*x-4*x^2)^(7/2))/64 - (229635*\text{ArcSin}[1-(8*x)/3])/16777216$

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (3x - 4x^2)^{7/2} dx &= -\frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2} + \frac{63}{128} \int (3x - 4x^2)^{5/2} dx \\
&= -\frac{21(3 - 8x)(3x - 4x^2)^{5/2}}{2048} - \frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2} + \frac{945 \int (3x - 4x^2)^{3/2} dx}{4096} \\
&= -\frac{945(3 - 8x)(3x - 4x^2)^{3/2}}{131072} - \frac{21(3 - 8x)(3x - 4x^2)^{5/2}}{2048} - \frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2} + \frac{25515}{64}(3 - 8x)\sqrt{3x - 4x^2} \\
&= -\frac{25515(3 - 8x)\sqrt{3x - 4x^2}}{4194304} - \frac{945(3 - 8x)(3x - 4x^2)^{3/2}}{131072} - \frac{21(3 - 8x)(3x - 4x^2)^{5/2}}{2048} - \frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2} \\
&= -\frac{25515(3 - 8x)\sqrt{3x - 4x^2}}{4194304} - \frac{945(3 - 8x)(3x - 4x^2)^{3/2}}{131072} - \frac{21(3 - 8x)(3x - 4x^2)^{5/2}}{2048} - \frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2} \\
&= -\frac{25515(3 - 8x)\sqrt{3x - 4x^2}}{4194304} - \frac{945(3 - 8x)(3x - 4x^2)^{3/2}}{131072} - \frac{21(3 - 8x)(3x - 4x^2)^{5/2}}{2048} - \frac{1}{64}(3 - 8x)(3x - 4x^2)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.87

$$\frac{2x(134217728x^8 - 452984832x^7 + 581959680x^6 - 338558976x^5 + 75534336x^4 + 41472x^3 + 54432x^2 + 102060x - 229635)\sqrt{-x(4x - 3)}}{8388608}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(7/2), x]

[Out] (2*x*(-229635 + 102060*x + 54432*x^2 + 41472*x^3 + 75534336*x^4 - 338558976*x^5 + 581959680*x^6 - 452984832*x^7 + 134217728*x^8) - 229635*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(8388608*Sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 0.72, size = 68, normalized size = 0.67

$$-\frac{1}{4194304} (33554432x^7 - 88080384x^6 + 79429632x^5 - 25067520x^4 + 82944x^3 + 72576x^2 + 68040x + 76540) \sqrt{-x(4x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(7/2), x, algorithm="fricas")

[Out] $-1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520*x^4 + 82944*x^3 + 72576*x^2 + 68040*x + 76545)*\sqrt{-4*x^2 + 3*x} - 229635/8388608*\arctan(1/2*\sqrt{-4*x^2 + 3*x}/x)$

giac [A] time = 0.47, size = 57, normalized size = 0.56

$$-\frac{1}{4194304} (8(16(8(32(8(16(8x-21)x+303)x-765)x+81)x+567)x+8505)x+76545)\sqrt{-4x^2+3x} + \frac{229635}{16777216} \arcsin\left(\frac{8x}{3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x^2+3*x)^(7/2),x, algorithm="giac")`

[Out] $-1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x + 81)*x + 567)*x + 8505)*x + 76545)*\sqrt{-4*x^2 + 3*x} + 229635/16777216*\arcsin(8/3*x - 1)$

maple [A] time = 0.04, size = 82, normalized size = 0.81

$$\frac{229635 \arcsin\left(\frac{8x}{3}-1\right)}{16777216} + \frac{945(-8x+3)(-4x^2+3x)^{\frac{3}{2}}}{131072} + \frac{21(-8x+3)(-4x^2+3x)^{\frac{5}{2}}}{2048} + \frac{(-8x+3)(-4x^2+3x)^{\frac{7}{2}}}{64} + \frac{2835(-8x+3)(-4x^2+3x)^{\frac{9}{2}}}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-4*x^2+3*x)^(7/2),x)`

[Out] $-945/131072*(3-8*x)*(-4*x^2+3*x)^{(3/2)}-21/2048*(3-8*x)*(-4*x^2+3*x)^{(5/2)}-1/64*(3-8*x)*(-4*x^2+3*x)^{(7/2)}+229635/16777216*\arcsin(-1+8/3*x)-25515/4194304*(3-8*x)*(-4*x^2+3*x)^{(1/2)}$

maxima [A] time = 3.06, size = 117, normalized size = 1.16

$$\frac{1}{8}(-4x^2+3x)^{\frac{7}{2}}x - \frac{3}{64}(-4x^2+3x)^{\frac{7}{2}} + \frac{21}{256}(-4x^2+3x)^{\frac{5}{2}}x - \frac{63}{2048}(-4x^2+3x)^{\frac{5}{2}} + \frac{945}{16384}(-4x^2+3x)^{\frac{3}{2}}x - \frac{2835}{131072}(-4x^2+3x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x^2+3*x)^(7/2),x, algorithm="maxima")`

[Out] $1/8*(-4*x^2 + 3*x)^{(7/2)}*x - 3/64*(-4*x^2 + 3*x)^{(7/2)} + 21/256*(-4*x^2 + 3*x)^{(5/2)}*x - 63/2048*(-4*x^2 + 3*x)^{(5/2)} + 945/16384*(-4*x^2 + 3*x)^{(3/2)}*x - 2835/131072*(-4*x^2 + 3*x)^{(3/2)} + 25515/524288*\sqrt{-4*x^2 + 3*x}*x - 76545/4194304*\sqrt{-4*x^2 + 3*x} - 229635/16777216*\arcsin(-8/3*x + 1)$

mupad [B] time = 0.17, size = 81, normalized size = 0.80

$$\frac{229635 \operatorname{asin}\left(\frac{8x}{3}-1\right)}{16777216} + \frac{945\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{3/2}}{65536} + \frac{21\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{5/2}}{1024} + \frac{\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{7/2}}{32} + \frac{2835\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{9/2}}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x - 4*x^2)^(7/2), x)`

[Out] $(229635 \operatorname{asin}((8x)/3 - 1))/16777216 + (945(4x - 3/2)(3x - 4x^2)^{3/2})/65536 + (21(4x - 3/2)(3x - 4x^2)^{5/2})/1024 + ((4x - 3/2)(3x - 4x^2)^{7/2})/32 + (25515(x/2 - 3/16)(3x - 4x^2)^{1/2})/262144$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4x^2 + 3x)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2+3*x)**(7/2), x)`

[Out] `Integral((-4*x**2 + 3*x)**(7/2), x)`

3.7 $\int (3x - 4x^2)^{5/2} dx$

Optimal. Leaf size=79

$$-\frac{1}{48}(3-8x)(3x-4x^2)^{5/2} - \frac{15(3-8x)(3x-4x^2)^{3/2}}{1024} - \frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{3645 \sin^{-1}\left(1 - \frac{8x}{3}\right)}{131072}$$

[Out] $-15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/2)+3645/131072*\arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{48}(3-8x)(3x-4x^2)^{5/2} - \frac{15(3-8x)(3x-4x^2)^{3/2}}{1024} - \frac{405(3-8x)\sqrt{3x-4x^2}}{32768} - \frac{3645 \sin^{-1}\left(1 - \frac{8x}{3}\right)}{131072}$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(5/2), x]

[Out] $(-405*(3-8*x)*\text{Sqrt}[3*x-4*x^2])/32768 - (15*(3-8*x)*(3*x-4*x^2)^(3/2))/1024 - ((3-8*x)*(3*x-4*x^2)^(5/2))/48 - (3645*\text{ArcSin}[1-(8*x)/3])/131072$

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (3x - 4x^2)^{5/2} dx &= -\frac{1}{48}(3 - 8x)(3x - 4x^2)^{5/2} + \frac{15}{32} \int (3x - 4x^2)^{3/2} dx \\
&= -\frac{15(3 - 8x)(3x - 4x^2)^{3/2}}{1024} - \frac{1}{48}(3 - 8x)(3x - 4x^2)^{5/2} + \frac{405 \int \sqrt{3x - 4x^2} dx}{2048} \\
&= -\frac{405(3 - 8x)\sqrt{3x - 4x^2}}{32768} - \frac{15(3 - 8x)(3x - 4x^2)^{3/2}}{1024} - \frac{1}{48}(3 - 8x)(3x - 4x^2)^{5/2} + \frac{3645 \int \sqrt{3x - 4x^2} dx}{65536} \\
&= -\frac{405(3 - 8x)\sqrt{3x - 4x^2}}{32768} - \frac{15(3 - 8x)(3x - 4x^2)^{3/2}}{1024} - \frac{1}{48}(3 - 8x)(3x - 4x^2)^{5/2} - \frac{3645 \int \sqrt{3x - 4x^2} dx}{131072} \\
&= -\frac{405(3 - 8x)\sqrt{3x - 4x^2}}{32768} - \frac{15(3 - 8x)(3x - 4x^2)^{3/2}}{1024} - \frac{1}{48}(3 - 8x)(3x - 4x^2)^{5/2} - \frac{3645 \operatorname{arcsin} \left(\frac{\sqrt{3x - 4x^2}}{\sqrt{3}} \right)}{131072}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.99

$$\frac{2x(-1048576x^6 + 2752512x^5 - 2469888x^4 + 760320x^3 + 2592x^2 + 4860x - 10935) - 10935\sqrt{3 - 4x}\sqrt{x} \sin^{-1}\left(\frac{\sqrt{3x - 4x^2}}{\sqrt{3}}\right)}{196608\sqrt{-x(4x - 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(5/2), x]

[Out] (2*x*(-10935 + 4860*x + 2592*x^2 + 760320*x^3 - 2469888*x^4 + 2752512*x^5 - 1048576*x^6) - 10935*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(196608*Sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 0.69, size = 58, normalized size = 0.73

$$\frac{1}{98304} (262144x^5 - 491520x^4 + 248832x^3 - 3456x^2 - 3240x - 3645)\sqrt{-4x^2 + 3x} - \frac{3645}{65536} \arctan\left(\frac{\sqrt{-4x^2 + 3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(5/2), x, algorithm="fricas")

[Out] 1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*sqrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)

giac [A] time = 0.45, size = 47, normalized size = 0.59

$$\frac{1}{98304} (8(16(8(32(8x-15)x+243)x-27)x-405)x-3645)\sqrt{-4x^2+3x} + \frac{3645}{131072} \arcsin\left(\frac{8}{3}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")

[Out] 1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(-4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)

maple [A] time = 0.05, size = 64, normalized size = 0.81

$$\frac{3645 \arcsin\left(\frac{8x}{3} - 1\right)}{131072} - \frac{15(-8x+3)(-4x^2+3x)^{\frac{3}{2}}}{1024} - \frac{(-8x+3)(-4x^2+3x)^{\frac{5}{2}}}{48} - \frac{405(-8x+3)\sqrt{-4x^2+3x}}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2+3*x)^(5/2),x)

[Out] -15/1024*(-8*x+3)*(-4*x^2+3*x)^(3/2)-1/48*(-8*x+3)*(-4*x^2+3*x)^(5/2)+3645/131072*arcsin(8/3*x-1)-405/32768*(-8*x+3)*(-4*x^2+3*x)^(1/2)

maxima [A] time = 2.94, size = 90, normalized size = 1.14

$$\frac{1}{6}(-4x^2+3x)^{\frac{5}{2}}x - \frac{1}{16}(-4x^2+3x)^{\frac{5}{2}} + \frac{15}{128}(-4x^2+3x)^{\frac{3}{2}}x - \frac{45}{1024}(-4x^2+3x)^{\frac{3}{2}} + \frac{405}{4096}\sqrt{-4x^2+3x}x - \frac{1215}{32768}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")

[Out] 1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 + 3*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*x - 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)

mupad [B] time = 0.24, size = 63, normalized size = 0.80

$$\frac{3645 \operatorname{asin}\left(\frac{8x}{3} - 1\right)}{131072} + \frac{15\left(4x - \frac{3}{2}\right)\left(3x - 4x^2\right)^{\frac{3}{2}}}{512} + \frac{\left(4x - \frac{3}{2}\right)\left(3x - 4x^2\right)^{\frac{5}{2}}}{24} + \frac{405\left(\frac{x}{2} - \frac{3}{16}\right)\sqrt{3x - 4x^2}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4*x^2)^(5/2),x)

```
[Out] (3645*asin((8*x)/3 - 1))/131072 + (15*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))/512
+ ((4*x - 3/2)*(3*x - 4*x^2)^(5/2))/24 + (405*(x/2 - 3/16)*(3*x - 4*x^2)^(1
/2))/2048
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (-4x^2 + 3x)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(5/2), x)
```

3.8 $\int (3x - 4x^2)^{3/2} dx$

Optimal. Leaf size=57

$$-\frac{1}{32}(3-8x)(3x-4x^2)^{3/2} - \frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{243 \sin^{-1}\left(1 - \frac{8x}{3}\right)}{4096}$$

[Out] $-1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*\arcsin(-1+8/3*x)-27/1024*(3-8*x)*(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{32}(3-8x)(3x-4x^2)^{3/2} - \frac{27(3-8x)\sqrt{3x-4x^2}}{1024} - \frac{243 \sin^{-1}\left(1 - \frac{8x}{3}\right)}{4096}$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(3/2), x]

[Out] $(-27*(3-8*x)*\text{Sqrt}[3*x-4*x^2])/1024 - ((3-8*x)*(3*x-4*x^2)^(3/2))/32 - (243*\text{ArcSin}[1-(8*x)/3])/4096$

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int (3x - 4x^2)^{3/2} dx &= -\frac{1}{32}(3 - 8x)(3x - 4x^2)^{3/2} + \frac{27}{64} \int \sqrt{3x - 4x^2} dx \\
&= -\frac{27(3 - 8x)\sqrt{3x - 4x^2}}{1024} - \frac{1}{32}(3 - 8x)(3x - 4x^2)^{3/2} + \frac{243 \int \frac{1}{\sqrt{3x - 4x^2}} dx}{2048} \\
&= -\frac{27(3 - 8x)\sqrt{3x - 4x^2}}{1024} - \frac{1}{32}(3 - 8x)(3x - 4x^2)^{3/2} - \frac{81 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{9}}} dx, x, 3 - 8x \right)}{4096} \\
&= -\frac{27(3 - 8x)\sqrt{3x - 4x^2}}{1024} - \frac{1}{32}(3 - 8x)(3x - 4x^2)^{3/2} - \frac{243 \sin^{-1} \left(1 - \frac{8x}{3} \right)}{4096}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.19

$$\frac{2x(4096x^4 - 7680x^3 + 3744x^2 + 108x - 243) - 243\sqrt{3 - 4x}\sqrt{x} \sin^{-1} \left(\sqrt{1 - \frac{4x}{3}} \right)}{2048\sqrt{-x(4x - 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(3/2), x]

[Out] (2*x*(-243 + 108*x + 3744*x^2 - 7680*x^3 + 4096*x^4) - 243*sqrt[3 - 4*x]*sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(2048*sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 0.86, size = 48, normalized size = 0.84

$$-\frac{1}{1024} (1024x^3 - 1152x^2 + 72x + 81)\sqrt{-4x^2 + 3x} - \frac{243}{2048} \arctan \left(\frac{\sqrt{-4x^2 + 3x}}{2x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(3/2), x, algorithm="fricas")

[Out] -1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)

giac [A] time = 0.52, size = 37, normalized size = 0.65

$$-\frac{1}{1024} (8(16(8x - 9)x + 9)x + 81)\sqrt{-4x^2 + 3x} + \frac{243}{4096} \arcsin \left(\frac{8}{3}x - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")

[Out] -1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcsin(8/3*x - 1)

maple [A] time = 0.05, size = 46, normalized size = 0.81

$$\frac{243 \arcsin\left(\frac{8x}{3} - 1\right)}{4096} - \frac{(-8x + 3)(-4x^2 + 3x)^{\frac{3}{2}}}{32} - \frac{27(-8x + 3)\sqrt{-4x^2 + 3x}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2+3*x)^(3/2),x)

[Out] -1/32*(-8*x+3)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(8/3*x-1)-27/1024*(-8*x+3)*(-4*x^2+3*x)^(1/2)

maxima [A] time = 2.90, size = 63, normalized size = 1.11

$$\frac{1}{4}(-4x^2 + 3x)^{\frac{3}{2}}x - \frac{3}{32}(-4x^2 + 3x)^{\frac{3}{2}} + \frac{27}{128}\sqrt{-4x^2 + 3x}x - \frac{81}{1024}\sqrt{-4x^2 + 3x} - \frac{243}{4096}\arcsin\left(-\frac{8}{3}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")

[Out] 1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^2 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)

mupad [B] time = 0.11, size = 45, normalized size = 0.79

$$\frac{243 \operatorname{asin}\left(\frac{8x}{3} - 1\right)}{4096} + \frac{\left(4x - \frac{3}{2}\right)(3x - 4x^2)^{\frac{3}{2}}}{16} + \frac{27\left(\frac{x}{2} - \frac{3}{16}\right)\sqrt{3x - 4x^2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4*x^2)^(3/2),x)

[Out] (243*asin((8*x)/3 - 1))/4096 + ((4*x - 3/2)*(3*x - 4*x^2)^(3/2))/16 + (27*(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/64

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4x^2 + 3x)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(3/2), x)
```

3.9 $\int \sqrt{3x - 4x^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x) - \frac{9}{64}\sin^{-1}\left(1 - \frac{8x}{3}\right)$$

[Out] 9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x) - \frac{9}{64}\sin^{-1}\left(1 - \frac{8x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3*x - 4*x^2], x]

[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{3x - 4x^2} \, dx &= -\frac{1}{16}(3 - 8x)\sqrt{3x - 4x^2} + \frac{9}{32} \int \frac{1}{\sqrt{3x - 4x^2}} \, dx \\
&= -\frac{1}{16}(3 - 8x)\sqrt{3x - 4x^2} - \frac{3}{64} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{9}}} \, dx, x, 3 - 8x \right) \\
&= -\frac{1}{16}(3 - 8x)\sqrt{3x - 4x^2} - \frac{9}{64} \sin^{-1} \left(1 - \frac{8x}{3} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.66

$$\frac{-2x(32x^2 - 36x + 9) - 9\sqrt{3 - 4x} \sqrt{x} \sin^{-1} \left(\sqrt{1 - \frac{4x}{3}} \right)}{32\sqrt{-x(4x - 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3*x - 4*x^2], x]

[Out] (-2*x*(9 - 36*x + 32*x^2) - 9*Sqrt[3 - 4*x]*Sqrt[x]*ArcSin[Sqrt[1 - (4*x)/3]])/(32*Sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 1.01, size = 38, normalized size = 1.09

$$\frac{1}{16} \sqrt{-4x^2 + 3x} (8x - 3) - \frac{9}{32} \arctan \left(\frac{\sqrt{-4x^2 + 3x}}{2x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(1/2), x, algorithm="fricas")

[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)

giac [A] time = 0.48, size = 27, normalized size = 0.77

$$\frac{1}{16} \sqrt{-4x^2 + 3x} (8x - 3) + \frac{9}{64} \arcsin \left(\frac{8}{3}x - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(1/2), x, algorithm="giac")

[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)

maple [A] time = 0.05, size = 28, normalized size = 0.80

$$\frac{9 \arcsin\left(\frac{8x}{3} - 1\right)}{64} - \frac{(-8x + 3) \sqrt{-4x^2 + 3x}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2+3*x)^(1/2),x)

[Out] 9/64*arcsin(8/3*x-1)-1/16*(-8*x+3)*(-4*x^2+3*x)^(1/2)

maxima [A] time = 2.88, size = 36, normalized size = 1.03

$$\frac{1}{2} \sqrt{-4x^2 + 3x} x - \frac{3}{16} \sqrt{-4x^2 + 3x} - \frac{9}{64} \arcsin\left(-\frac{8}{3}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x + 1)

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{9 \operatorname{asin}\left(\frac{8x}{3} - 1\right)}{64} + \left(\frac{x}{2} - \frac{3}{16}\right) \sqrt{3x - 4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4*x^2)^(1/2),x)

[Out] (9*asin((8*x)/3 - 1))/64 + (x/2 - 3/16)*(3*x - 4*x^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4x^2 + 3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x**2+3*x)**(1/2),x)

[Out] Integral(sqrt(-4*x**2 + 3*x), x)

3.10 $\int \sqrt{6x - x^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{6x - x^2}(3 - x) - \frac{9}{2}\sin^{-1}\left(1 - \frac{x}{3}\right)$$

[Out] 9/2*arcsin(-1+1/3*x)-1/2*(3-x)*(-x^2+6*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{2}\sqrt{6x - x^2}(3 - x) - \frac{9}{2}\sin^{-1}\left(1 - \frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[6*x - x^2], x]

[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{6x - x^2} \, dx &= -\frac{1}{2}(3 - x)\sqrt{6x - x^2} + \frac{9}{2} \int \frac{1}{\sqrt{6x - x^2}} \, dx \\
&= -\frac{1}{2}(3 - x)\sqrt{6x - x^2} - \frac{3}{4} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{36}}} \, dx, x, 6 - 2x \right) \\
&= -\frac{1}{2}(3 - x)\sqrt{6x - x^2} - \frac{9}{2} \sin^{-1} \left(1 - \frac{x}{3} \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.91

$$\frac{1}{2}(x - 3)\sqrt{-((x - 6)x)} - 9 \sin^{-1} \left(\sqrt{1 - \frac{x}{6}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[6*x - x^2], x]

[Out] ((-3 + x)*Sqrt[-((-6 + x)*x)])/2 - 9*ArcSin[Sqrt[1 - x/6]]

fricas [A] time = 0.73, size = 35, normalized size = 1.00

$$\frac{1}{2} \sqrt{-x^2 + 6x} (x - 3) - 9 \arctan \left(\frac{\sqrt{-x^2 + 6x}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+6*x)^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)

giac [A] time = 0.36, size = 25, normalized size = 0.71

$$\frac{1}{2} \sqrt{-x^2 + 6x} (x - 3) + \frac{9}{2} \arcsin \left(\frac{1}{3}x - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+6*x)^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)

maple [A] time = 0.05, size = 28, normalized size = 0.80

$$\frac{9 \arcsin\left(\frac{x}{3} - 1\right)}{2} - \frac{(-2x + 6) \sqrt{-x^2 + 6x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+6*x)^(1/2),x)

[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)

maxima [A] time = 2.97, size = 36, normalized size = 1.03

$$\frac{1}{2} \sqrt{-x^2 + 6x} x - \frac{3}{2} \sqrt{-x^2 + 6x} - \frac{9}{2} \arcsin\left(-\frac{1}{3}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{9 \operatorname{asin}\left(\frac{x}{3} - 1\right)}{2} + \left(\frac{x}{2} - \frac{3}{2}\right) \sqrt{6x - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x - x^2)^(1/2),x)

[Out] (9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + 6x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+6*x)**(1/2),x)

[Out] Integral(sqrt(-x**2 + 6*x), x)

3.11 $\int \sqrt{5x - 9x^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x) - \frac{25}{216}\sin^{-1}\left(1 - \frac{18x}{5}\right)$$

[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {612, 619, 216}

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x) - \frac{25}{216}\sin^{-1}\left(1 - \frac{18x}{5}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[5*x - 9*x^2], x]

[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/216

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{5x-9x^2} dx &= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} + \frac{25}{72} \int \frac{1}{\sqrt{5x-9x^2}} dx \\
&= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{5}{216} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{25}}} dx, x, 5-18x \right) \\
&= -\frac{1}{36}(5-18x)\sqrt{5x-9x^2} - \frac{25}{216} \sin^{-1} \left(1 - \frac{18x}{5} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.66

$$\frac{-3x(162x^2 - 135x + 25) - 25\sqrt{5-9x}\sqrt{x}\sin^{-1}\left(\sqrt{1-\frac{9x}{5}}\right)}{108\sqrt{-x(9x-5)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[5*x - 9*x^2], x]

[Out] (-3*x*(25 - 135*x + 162*x^2) - 25*Sqrt[5 - 9*x]*Sqrt[x]*ArcSin[Sqrt[1 - (9*x)/5]])/(108*Sqrt[-(x*(-5 + 9*x))])

fricas [A] time = 1.31, size = 38, normalized size = 1.09

$$\frac{1}{36} \sqrt{-9x^2 + 5x}(18x - 5) - \frac{25}{108} \arctan\left(\frac{\sqrt{-9x^2 + 5x}}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-9*x^2+5*x)^(1/2), x, algorithm="fricas")

[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^2 + 5*x)/x)

giac [A] time = 0.44, size = 27, normalized size = 0.77

$$\frac{1}{36} \sqrt{-9x^2 + 5x}(18x - 5) + \frac{25}{216} \arcsin\left(\frac{18}{5}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-9*x^2+5*x)^(1/2), x, algorithm="giac")

[Out] $1/36*\sqrt{-9*x^2 + 5*x}*(18*x - 5) + 25/216*\arcsin(18/5*x - 1)$

maple [A] time = 0.04, size = 28, normalized size = 0.80

$$\frac{25 \arcsin\left(\frac{18x}{5} - 1\right)}{216} - \frac{(-18x + 5) \sqrt{-9x^2 + 5x}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((-9*x^2+5*x)^{(1/2)}, x)$

[Out] $25/216*\arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^{(1/2)}$

maxima [A] time = 2.93, size = 36, normalized size = 1.03

$$\frac{1}{2} \sqrt{-9x^2 + 5x} x - \frac{5}{36} \sqrt{-9x^2 + 5x} - \frac{25}{216} \arcsin\left(-\frac{18}{5}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-9*x^2+5*x)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $1/2*\sqrt{-9*x^2 + 5*x}*x - 5/36*\sqrt{-9*x^2 + 5*x} - 25/216*\arcsin(-18/5*x + 1)$

mupad [B] time = 0.05, size = 26, normalized size = 0.74

$$\frac{25 \operatorname{asin}\left(\frac{18x}{5} - 1\right)}{216} + \left(\frac{x}{2} - \frac{5}{36}\right) \sqrt{5x - 9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((5*x - 9*x^2)^{(1/2)}, x)$

[Out] $(25*\operatorname{asin}((18*x)/5 - 1))/216 + (x/2 - 5/36)*(5*x - 9*x^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-9x^2 + 5x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-9*x**2+5*x)**(1/2), x)$

[Out] $\text{Integral}(\sqrt{-9*x**2 + 5*x}, x)$

3.12 $\int (x - x^2)^{3/2} dx$

Optimal. Leaf size=51

$$-\frac{1}{8}(1-2x)(x-x^2)^{3/2} - \frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{3}{128}\sin^{-1}(1-2x)$$

[Out] $-1/8*(1-2*x)*(-x^2+x)^{(3/2)}+3/128*\arcsin(-1+2*x)-3/64*(1-2*x)*(-x^2+x)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {612, 619, 216}

$$-\frac{1}{8}(1-2x)(x-x^2)^{3/2} - \frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{3}{128}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x - x^2)^{(3/2)}, x]$

[Out] $(-3*(1 - 2*x)*\text{Sqrt}[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^{(3/2)})/8 - (3*\text{ArcSin}[1 - 2*x])/128$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*x]/\text{Sqrt}[a]/\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 612

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \text{Int}[(a + b*x + c*x^2)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[4*p]$

Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rubi steps

$$\begin{aligned}
\int (x-x^2)^{3/2} dx &= -\frac{1}{8}(1-2x)(x-x^2)^{3/2} + \frac{3}{16} \int \sqrt{x-x^2} dx \\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)(x-x^2)^{3/2} + \frac{3}{128} \int \frac{1}{\sqrt{x-x^2}} dx \\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)(x-x^2)^{3/2} - \frac{3}{128} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right) \\
&= -\frac{3}{64}(1-2x)\sqrt{x-x^2} - \frac{1}{8}(1-2x)(x-x^2)^{3/2} - \frac{3}{128} \sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.86

$$\frac{1}{64} \left(-\sqrt{-(x-1)x} (16x^3 - 24x^2 + 2x + 3) - 3 \sin^{-1}(\sqrt{1-x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x - x^2)^(3/2), x]

[Out] $(-\operatorname{Sqrt}[-((-1+x)*x)]*(3+2*x-24*x^2+16*x^3))-3*\operatorname{ArcSin}[\operatorname{Sqrt}[1-x]]/64$

fricas [A] time = 0.86, size = 43, normalized size = 0.84

$$-\frac{1}{64} (16x^3 - 24x^2 + 2x + 3)\sqrt{-x^2 + x} - \frac{3}{64} \arctan\left(\frac{\sqrt{-x^2 + x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+x)^(3/2), x, algorithm="fricas")

[Out] $-1/64*(16*x^3-24*x^2+2*x+3)*\operatorname{sqrt}(-x^2+x)-3/64*\operatorname{arctan}(\operatorname{sqrt}(-x^2+x)/x)$

giac [A] time = 0.54, size = 35, normalized size = 0.69

$$-\frac{1}{64} (2(4(2x-3)x+1)x+3)\sqrt{-x^2+x} + \frac{3}{128} \arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+x)^(3/2), x, algorithm="giac")

[Out] $-1/64*(2*(4*(2*x-3)*x+1)*x+3)*\operatorname{sqrt}(-x^2+x)+3/128*\operatorname{arcsin}(2*x-1)$

maple [A] time = 0.04, size = 42, normalized size = 0.82

$$\frac{3 \arcsin(2x-1)}{128} - \frac{(-2x+1)(-x^2+x)^{\frac{3}{2}}}{8} - \frac{3(-2x+1)\sqrt{-x^2+x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+x)^(3/2),x)`

[Out] `-1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(2*x-1)-3/64*(1-2*x)*(-x^2+x)^(1/2)`

maxima [A] time = 2.89, size = 55, normalized size = 1.08

$$\frac{1}{4}(-x^2+x)^{\frac{3}{2}}x - \frac{1}{8}(-x^2+x)^{\frac{3}{2}} + \frac{3}{32}\sqrt{-x^2+x}x - \frac{3}{64}\sqrt{-x^2+x} + \frac{3}{128}\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+x)^(3/2),x, algorithm="maxima")`

[Out] `1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/64*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)`

mapad [B] time = 0.19, size = 39, normalized size = 0.76

$$\frac{3 \operatorname{asin}(2x-1)}{128} + \frac{3\sqrt{x-x^2}\left(\frac{x}{2}-\frac{1}{4}\right)}{16} + \frac{(x-x^2)^{3/2}\left(x-\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - x^2)^(3/2),x)`

[Out] `(3*asin(2*x - 1))/128 + (3*(x - x^2)^(1/2)*(x/2 - 1/4))/16 + ((x - x^2)^(3/2)*(x - 1/2))/4`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^2 + x)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+x)**(3/2),x)`

[Out] `Integral((-x**2 + x)**(3/2), x)`

3.13 $\int \sqrt{4x + x^2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}(x+2)\sqrt{x^2+4x} - 4 \tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

[Out] $-4*\operatorname{arctanh}(x/(x^2+4*x)^{(1/2}))+1/2*(2+x)*(x^2+4*x)^{(1/2}$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {612, 620, 206}

$$\frac{1}{2}(x+2)\sqrt{x^2+4x} - 4 \tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4*x + x^2], x]

[Out] $((2 + x)*\operatorname{Sqrt}[4*x + x^2])/2 - 4*\operatorname{ArcTanh}[x/\operatorname{Sqrt}[4*x + x^2]]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{4x + x^2} \, dx &= \frac{1}{2}(2 + x)\sqrt{4x + x^2} - 2 \int \frac{1}{\sqrt{4x + x^2}} \, dx \\
&= \frac{1}{2}(2 + x)\sqrt{4x + x^2} - 4 \operatorname{Subst} \left(\int \frac{1}{1 - x^2} \, dx, x, \frac{x}{\sqrt{4x + x^2}} \right) \\
&= \frac{1}{2}(2 + x)\sqrt{4x + x^2} - 4 \tanh^{-1} \left(\frac{x}{\sqrt{4x + x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.14

$$\frac{1}{2} \sqrt{x(x+4)} \left(x - \frac{8 \sinh^{-1} \left(\frac{\sqrt{x}}{2} \right)}{\sqrt{x+4} \sqrt{x}} + 2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4*x + x^2], x]

[Out] (Sqrt[x*(4 + x)]*(2 + x - (8*ArcSinh[Sqrt[x]/2])/(Sqrt[x]*Sqrt[4 + x])))/2

fricas [A] time = 0.84, size = 32, normalized size = 0.91

$$\frac{1}{2} \sqrt{x^2 + 4x} (x + 2) + 2 \log(-x + \sqrt{x^2 + 4x} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x)^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)

giac [A] time = 0.52, size = 33, normalized size = 0.94

$$\frac{1}{2} \sqrt{x^2 + 4x} (x + 2) + 2 \log \left(\left| -x + \sqrt{x^2 + 4x} - 2 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x)^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))

maple [A] time = 0.05, size = 33, normalized size = 0.94

$$-2 \ln \left(x + 2 + \sqrt{x^2 + 4x} \right) + \frac{(2x + 4) \sqrt{x^2 + 4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+4*x)^(1/2),x)`

[Out] `1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(x+2+(x^2+4*x)^(1/2))`

maxima [A] time = 1.42, size = 41, normalized size = 1.17

$$\frac{1}{2} \sqrt{x^2 + 4x} x + \sqrt{x^2 + 4x} - 2 \log(2x + 2\sqrt{x^2 + 4x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+4*x)^(1/2),x, algorithm="maxima")`

[Out] `1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) + 4)`

mupad [B] time = 0.20, size = 29, normalized size = 0.83

$$\sqrt{x^2 + 4x} \left(\frac{x}{2} + 1 \right) - 2 \ln(x + \sqrt{x(x+4)} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + x^2)^(1/2),x)`

[Out] `(4*x + x^2)^(1/2)*(x/2 + 1) - 2*log(x + (x*(x + 4))^(1/2) + 2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 + 4x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+4*x)**(1/2),x)`

[Out] `Integral(sqrt(x**2 + 4*x), x)`

3.14 $\int \sqrt{-8x + x^2} dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{x^2 - 8x}(4 - x) - 16 \tanh^{-1}\left(\frac{x}{\sqrt{x^2 - 8x}}\right)$$

[Out] -16*arctanh(x/(x^2-8*x)^(1/2))-1/2*(4-x)*(x^2-8*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {612, 620, 206}

$$-\frac{1}{2}\sqrt{x^2 - 8x}(4 - x) - 16 \tanh^{-1}\left(\frac{x}{\sqrt{x^2 - 8x}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-8*x + x^2], x]

[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned}\int \sqrt{-8x + x^2} dx &= -\frac{1}{2}(4-x)\sqrt{-8x + x^2} - 8 \int \frac{1}{\sqrt{-8x + x^2}} dx \\ &= -\frac{1}{2}(4-x)\sqrt{-8x + x^2} - 16 \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-8x + x^2}}\right) \\ &= -\frac{1}{2}(4-x)\sqrt{-8x + x^2} - 16 \tanh^{-1}\left(\frac{x}{\sqrt{-8x + x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 1.30

$$\frac{x(x^2 - 12x + 32) + 32\sqrt{-(x-8)x} \sin^{-1}\left(\sqrt{1 - \frac{x}{8}}\right)}{2\sqrt{(x-8)x}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[-8*x + x^2], x]

[Out] (x*(32 - 12*x + x^2) + 32*sqrt[-((-8 + x)*x)]*ArcSin[Sqrt[1 - x/8]])/(2*sqrt[(-8 + x)*x])

fricas [A] time = 0.71, size = 32, normalized size = 0.86

$$\frac{1}{2} \sqrt{x^2 - 8x}(x - 4) + 8 \log(-x + \sqrt{x^2 - 8x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-8*x)^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)

giac [A] time = 0.52, size = 33, normalized size = 0.89

$$\frac{1}{2} \sqrt{x^2 - 8x}(x - 4) + 8 \log\left(\left| -x + \sqrt{x^2 - 8x} + 4 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-8*x)^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))

maple [A] time = 0.04, size = 33, normalized size = 0.89

$$-8 \ln\left(x - 4 + \sqrt{x^2 - 8x}\right) + \frac{(2x - 8)\sqrt{x^2 - 8x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-8*x)^(1/2),x)`

[Out] `1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(-4+x+(x^2-8*x)^(1/2))`

maxima [A] time = 1.31, size = 43, normalized size = 1.16

$$\frac{1}{2} \sqrt{x^2 - 8x} x - 2 \sqrt{x^2 - 8x} - 8 \log(2x + 2 \sqrt{x^2 - 8x} - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-8*x)^(1/2),x, algorithm="maxima")`

[Out] `1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x) - 8)`

mupad [B] time = 0.11, size = 29, normalized size = 0.78

$$\left(\frac{x}{2} - 2\right) \sqrt{x^2 - 8x} - 8 \ln(x + \sqrt{x(x-8)} - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 8*x)^(1/2),x)`

[Out] `(x/2 - 2)*(x^2 - 8*x)^(1/2) - 8*log(x + (x*(x - 8))^(1/2) - 4)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-8*x)**(1/2),x)`

[Out] `Integral(sqrt(x**2 - 8*x), x)`

3.15 $\int \sqrt{-x + x^2} dx$

Optimal. Leaf size=39

$$-\frac{1}{4}\sqrt{x^2-x}(1-2x) - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

[Out] $-1/4*\operatorname{arctanh}(x/(x^2-x)^{(1/2)})-1/4*(1-2*x)*(x^2-x)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {612, 620, 206}

$$-\frac{1}{4}\sqrt{x^2-x}(1-2x) - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[-x + x^2], x]$

[Out] $-((1 - 2*x)*\operatorname{Sqrt}[-x + x^2])/4 - \operatorname{ArcTanh}[x/\operatorname{Sqrt}[-x + x^2]]/4$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 612

$\operatorname{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{p_}, x_Symbol] \rightarrow \operatorname{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \operatorname{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \operatorname{Int}[(a + b*x + c*x^2)^{p-1}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[4*p]$

Rule 620

$\operatorname{Int}[1/\operatorname{Sqrt}[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow \operatorname{Dist}[2, \operatorname{Subst}[\operatorname{Int}[1/(1 - c*x^2), x], x, x/\operatorname{Sqrt}[b*x + c*x^2]], x] /;$ $\operatorname{FreeQ}\{b, c\}, x]$

Rubi steps

$$\begin{aligned}
\int \sqrt{-x+x^2} dx &= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{8} \int \frac{1}{\sqrt{-x+x^2}} dx \\
&= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-x+x^2}} \right) \\
&= -\frac{1}{4}(1-2x)\sqrt{-x+x^2} - \frac{1}{4} \tanh^{-1} \left(\frac{x}{\sqrt{-x+x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.18

$$\frac{2x^3 - 3x^2 + x + \sqrt{-((x-1)x)} \sin^{-1}(\sqrt{1-x})}{4\sqrt{(x-1)x}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[-x + x^2], x]

[Out] (x - 3*x^2 + 2*x^3 + Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(4*Sqrt[(-1 + x)*x])

fricas [A] time = 0.87, size = 36, normalized size = 0.92

$$\frac{1}{4} \sqrt{x^2-x}(2x-1) + \frac{1}{8} \log(-2x + 2\sqrt{x^2-x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x)^(1/2), x, algorithm="fricas")

[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)

giac [A] time = 0.42, size = 37, normalized size = 0.95

$$\frac{1}{4} \sqrt{x^2-x}(2x-1) + \frac{1}{8} \log\left(\left|-2x + 2\sqrt{x^2-x} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-x)^(1/2), x, algorithm="giac")

[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))

maple [A] time = 0.05, size = 33, normalized size = 0.85

$$-\frac{\ln\left(x - \frac{1}{2} + \sqrt{x^2-x}\right)}{8} + \frac{(2x-1)\sqrt{x^2-x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-x)^(1/2),x)`

[Out] `1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(-1/2+x+(x^2-x)^(1/2))`

maxima [A] time = 1.35, size = 43, normalized size = 1.10

$$\frac{1}{2} \sqrt{x^2 - x} x - \frac{1}{4} \sqrt{x^2 - x} - \frac{1}{8} \log\left(2x + 2\sqrt{x^2 - x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-x)^(1/2),x, algorithm="maxima")`

[Out] `1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) - 1)`

mupad [B] time = 0.20, size = 29, normalized size = 0.74

$$\sqrt{x^2 - x} \left(\frac{x}{2} - \frac{1}{4}\right) - \frac{\ln\left(x + \sqrt{x(x-1)} - \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - x)^(1/2),x)`

[Out] `(x^2 - x)^(1/2)*(x/2 - 1/4) - log(x + (x*(x - 1))^(1/2) - 1/2)/8`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 - x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-x)**(1/2),x)`

[Out] `Integral(sqrt(x**2 - x), x)`

$$3.16 \quad \int \frac{1}{(bx+cx^2)^{7/2}} dx$$

Optimal. Leaf size=83

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}} + \frac{32c(b+2cx)}{15b^4(bx+cx^2)^{3/2}} - \frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

[Out] $-2/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/2)+32/15*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(3/2)-256/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/2)$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {614, 613}

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}} + \frac{32c(b+2cx)}{15b^4(bx+cx^2)^{3/2}} - \frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + c*x^2)^{-7/2}, x]$

[Out] $(-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))/(15*b^4*(b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*\text{Sqrt}[b*x + c*x^2])$

Rule 613

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-3/2}, x_Symbol] \rightarrow \text{Simp}[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]), x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 614

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^{(p + 1)} / ((p + 1)*(b^2 - 4*a*c)), x] - \text{Dist}[(2*c*(2*p + 3)) / ((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[p, -3/2] \&\& \text{IntegerQ}[4*p]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{7/2}} dx &= -\frac{2(b + 2cx)}{5b^2 (bx + cx^2)^{5/2}} - \frac{(16c) \int \frac{1}{(bx+cx^2)^{5/2}} dx}{5b^2} \\
&= -\frac{2(b + 2cx)}{5b^2 (bx + cx^2)^{5/2}} + \frac{32c(b + 2cx)}{15b^4 (bx + cx^2)^{3/2}} + \frac{(128c^2) \int \frac{1}{(bx+cx^2)^{3/2}} dx}{15b^4} \\
&= -\frac{2(b + 2cx)}{5b^2 (bx + cx^2)^{5/2}} + \frac{32c(b + 2cx)}{15b^4 (bx + cx^2)^{3/2}} - \frac{256c^2(b + 2cx)}{15b^6 \sqrt{bx + cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.84

$$\frac{2(3b^5 - 10b^4cx + 80b^3c^2x^2 + 480b^2c^3x^3 + 640bc^4x^4 + 256c^5x^5)}{15b^6(x(b + cx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-7/2), x]

[Out] (-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4 + 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))

fricas [A] time = 0.80, size = 105, normalized size = 1.27

$$\frac{2(256c^5x^5 + 640bc^4x^4 + 480b^2c^3x^3 + 80b^3c^2x^2 - 10b^4cx + 3b^5)\sqrt{cx^2 + bx}}{15(b^6c^3x^6 + 3b^7c^2x^5 + 3b^8cx^4 + b^9x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(7/2), x, algorithm="fricas")

[Out] -2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10*b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x^4 + b^9*x^3)

giac [A] time = 0.49, size = 74, normalized size = 0.89

$$\frac{2\left(8\left(2\left(4x\left(\frac{2c^5x}{b^6} + \frac{5c^4}{b^5}\right) + \frac{15c^3}{b^4}\right)x + \frac{5c^2}{b^3}\right)x - \frac{5c}{b^2}\right)x + \frac{3}{b}}{15(cx^2 + bx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")

[Out] $-2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)*x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)$

maple [A] time = 0.05, size = 75, normalized size = 0.90

$$\frac{2(cx + b)(256c^5x^5 + 640c^4x^4b + 480c^3x^3b^2 + 80c^2x^2b^3 - 10cx b^4 + 3b^5)x}{15(cx^2 + bx)^{\frac{7}{2}}b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(7/2),x)

[Out] $-2/15*(c*x+b)*x*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-10*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)$

maxima [A] time = 1.34, size = 111, normalized size = 1.34

$$-\frac{4cx}{5(cx^2 + bx)^{\frac{5}{2}}b^2} + \frac{64c^2x}{15(cx^2 + bx)^{\frac{3}{2}}b^4} - \frac{512c^3x}{15\sqrt{cx^2 + bx}b^6} - \frac{2}{5(cx^2 + bx)^{\frac{5}{2}}b} + \frac{32c}{15(cx^2 + bx)^{\frac{3}{2}}b^3} - \frac{256c^2}{15\sqrt{cx^2 + bx}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")

[Out] $-4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4) - 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32/15*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)$

mupad [B] time = 0.27, size = 96, normalized size = 1.16

$$\frac{6b^5 + 256bc^2(cx^2 + bx)^2 + 512c^3x(cx^2 + bx)^2 - 32b^3c(cx^2 + bx) + 12b^4cx - 64b^2c^2x(cx^2 + bx)}{15b^6(cx^2 + bx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(7/2),x)

[Out] $-(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c*(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c*x^2)^(5/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(7/2), x)

[Out] Integral((b*x + c*x**2)**(-7/2), x)

$$3.17 \quad \int \frac{1}{\sqrt{3ix+4x^2}} dx$$

Optimal. Leaf size=16

$$\frac{1}{2}i \sin^{-1}\left(1 - \frac{8ix}{3}\right)$$

[Out] -1/2*I*arcsin(-1+8/3*I*x)

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {619, 215}

$$\frac{1}{2}i \sin^{-1}\left(1 - \frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(3*I)*x + 4*x^2],x]

[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3ix+4x^2}} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{9}}} dx, x, 3i+8x \right) \\ &= \frac{1}{2}i \sin^{-1}\left(1 - \frac{8ix}{3}\right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 53, normalized size = 3.31

$$\frac{(-1)^{3/4} \sqrt{3-4ix} \sqrt{x} \sin^{-1}\left((1+i)\sqrt{\frac{2}{3}}\sqrt{x}\right)}{\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(3*I)*x + 4*x^2], x]

[Out] -(((-1)^(3/4)*Sqrt[3 - (4*I)*x]*Sqrt[x]*ArcSin[(1 + I)*Sqrt[2/3]*Sqrt[x]])/Sqrt[x*(3*I + 4*x)])

fricas [B] time = 0.94, size = 19, normalized size = 1.19

$$-\frac{1}{2} \log\left(-2x + \sqrt{4x^2 + 3ix} - \frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(1/2), x, algorithm="fricas")

[Out] -1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.

maple [A] time = 0.10, size = 10, normalized size = 0.62

$$\frac{\operatorname{arcsinh}\left(\frac{8x}{3} + i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2+3*I*x)^(1/2), x)

[Out] $\frac{1}{2} \operatorname{arcsinh}(8/3*x+I)$

maxima [B] time = 2.93, size = 21, normalized size = 1.31

$$\frac{1}{2} \log\left(8x + 4\sqrt{4x^2 + 3ix} + 3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2} \log(8*x + 4*\sqrt{4*x^2 + 3*I*x}) + 3*I$

mupad [B] time = 0.28, size = 19, normalized size = 1.19

$$\frac{\ln\left(x + \frac{\sqrt{x(4x+3i)}}{2} + \frac{3i}{8}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*3i + 4*x^2)^(1/2),x)`

[Out] $\log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8)/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x^2 + 3ix}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*I*x+4*x**2)**(1/2),x)`

[Out] `Integral(1/sqrt(4*x**2 + 3*I*x), x)`

$$3.18 \quad \int \frac{1}{(3ix+4x^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\frac{2(8x + 3i)}{9\sqrt{4x^2 + 3ix}}$$

[Out] 2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {613}

$$\frac{2(8x + 3i)}{9\sqrt{4x^2 + 3ix}}$$

Antiderivative was successfully verified.

[In] Int[((3*I)*x + 4*x^2)^(-3/2), x]

[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{(3ix + 4x^2)^{3/2}} dx = \frac{2(3i + 8x)}{9\sqrt{3ix + 4x^2}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.92

$$\frac{2(8x + 3i)}{9\sqrt{x(4x + 3i)}}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(-3/2), x]

[Out] (2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])

fricas [B] time = 0.81, size = 39, normalized size = 1.50

$$\frac{32x^2 + \sqrt{4x^2 + 3ix}(16x + 6i) + 24ix}{9(4x^2 + 3ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/9*(32*x^2 + sqrt(4*x^2 + 3*I*x)*(16*x + 6*I) + 24*I*x)/(4*x^2 + 3*I*x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Warning, replacing 0 by `u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
titution variable should perhaps be purged.

maple [A] time = 0.10, size = 21, normalized size = 0.81

$$\frac{\frac{16x}{9} + \frac{2i}{3}}{\sqrt{4x^2 + 3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2+3*I*x)^(3/2),x)

[Out] 2/9*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)

maxima [A] time = 1.37, size = 28, normalized size = 1.08

$$\frac{16x}{9\sqrt{4x^2 + 3ix}} + \frac{2i}{3\sqrt{4x^2 + 3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")

[Out] 16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)

mupad [B] time = 0.05, size = 20, normalized size = 0.77

$$\frac{16x + 6i}{9\sqrt{4x^2 + x3i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*3i + 4*x^2)^(3/2), x)

[Out] (16*x + 6i)/(9*(x*3i + 4*x^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4x^2 + 3ix)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x**2)**(3/2), x)

[Out] Integral((4*x**2 + 3*I*x)**(-3/2), x)

$$3.19 \quad \int \frac{1}{(3ix+4x^2)^{5/2}} dx$$

Optimal. Leaf size=53

$$\frac{64(8x + 3i)}{243\sqrt{4x^2 + 3ix}} + \frac{2(8x + 3i)}{27(4x^2 + 3ix)^{3/2}}$$

[Out] 2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {614, 613}

$$\frac{64(8x + 3i)}{243\sqrt{4x^2 + 3ix}} + \frac{2(8x + 3i)}{27(4x^2 + 3ix)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((3*I)*x + 4*x^2)^(-5/2), x]

[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(3*I)*x + 4*x^2])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\int \frac{1}{(3ix + 4x^2)^{5/2}} dx = \frac{2(3i + 8x)}{27(3ix + 4x^2)^{3/2}} + \frac{32}{27} \int \frac{1}{(3ix + 4x^2)^{3/2}} dx$$

$$= \frac{2(3i + 8x)}{27(3ix + 4x^2)^{3/2}} + \frac{64(3i + 8x)}{243\sqrt{3ix + 4x^2}}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.68

$$\frac{2048x^3 + 2304ix^2 - 432x + 54i}{243(x(4x + 3i))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(-5/2), x]

[Out] (54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))

fricas [A] time = 1.03, size = 62, normalized size = 1.17

$$\frac{4096x^4 + 6144ix^3 - 2304x^2 + (2048x^3 + 2304ix^2 - 432x + 54i)\sqrt{4x^2 + 3ix}}{3888x^4 + 5832ix^3 - 2187x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(5/2), x, algorithm="fricas")

[Out] (4096*x^4 + 6144*I*x^3 - 2304*x^2 + (2048*x^3 + 2304*I*x^2 - 432*x + 54*I)*sqrt(4*x^2 + 3*I*x))/(3888*x^4 + 5832*I*x^3 - 2187*x^2)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);;OUTPUT:Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.

maple [A] time = 0.10, size = 42, normalized size = 0.79

$$\frac{\frac{16x}{27} + \frac{2i}{9}}{(4x^2 + 3ix)^{\frac{3}{2}}} + \frac{\frac{512x}{243} + \frac{64i}{81}}{\sqrt{4x^2 + 3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2+3*I*x)^(5/2), x)

[Out] 2/27*(8*x+3*I)/(4*x^2+3*I*x)^(3/2)+64/243*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)

maxima [A] time = 1.37, size = 55, normalized size = 1.04

$$\frac{512x}{243\sqrt{4x^2 + 3ix}} + \frac{64i}{81\sqrt{4x^2 + 3ix}} + \frac{16x}{27(4x^2 + 3ix)^{\frac{3}{2}}} + \frac{2i}{9(4x^2 + 3ix)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(5/2), x, algorithm="maxima")

[Out] 512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x^2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)

mupad [B] time = 0.12, size = 31, normalized size = 0.58

$$\frac{(16x + 6i)(128x^2 + x96i + 9)}{243(4x^2 + x3i)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*3i + 4*x^2)^(5/2), x)

[Out] ((16*x + 6i)*(x*96i + 128*x^2 + 9))/(243*(x*3i + 4*x^2)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4x^2 + 3ix)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x**2)**(5/2), x)

[Out] Integral((4*x**2 + 3*I*x)**(-5/2), x)

$$3.20 \quad \int \frac{1}{(3ix+4x^2)^{7/2}} dx$$

Optimal. Leaf size=79

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}} + \frac{128(8x+3i)}{1215(4x^2+3ix)^{3/2}} + \frac{2(8x+3i)}{45(4x^2+3ix)^{5/2}}$$

[Out] $2/45*(3*I+8*x)/(3*I*x+4*x^2)^{(5/2)}+128/1215*(3*I+8*x)/(3*I*x+4*x^2)^{(3/2)}+4096/10935*(3*I+8*x)/(3*I*x+4*x^2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {614, 613}

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}} + \frac{128(8x+3i)}{1215(4x^2+3ix)^{3/2}} + \frac{2(8x+3i)}{45(4x^2+3ix)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((3*I)*x + 4*x^2)^(-7/2), x]

[Out] $(2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^{(5/2)}) + (128*(3*I + 8*x))/(1215*((3*I)*x + 4*x^2)^{(3/2)}) + (4096*(3*I + 8*x))/(10935*\text{Sqrt}[(3*I)*x + 4*x^2])$

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3ix + 4x^2)^{7/2}} dx &= \frac{2(3i + 8x)}{45(3ix + 4x^2)^{5/2}} + \frac{64}{45} \int \frac{1}{(3ix + 4x^2)^{5/2}} dx \\
&= \frac{2(3i + 8x)}{45(3ix + 4x^2)^{5/2}} + \frac{128(3i + 8x)}{1215(3ix + 4x^2)^{3/2}} + \frac{2048 \int \frac{1}{(3ix+4x^2)^{3/2}} dx}{1215} \\
&= \frac{2(3i + 8x)}{45(3ix + 4x^2)^{5/2}} + \frac{128(3i + 8x)}{1215(3ix + 4x^2)^{3/2}} + \frac{4096(3i + 8x)}{10935\sqrt{3ix + 4x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 0.61

$$\frac{524288x^5 + 983040ix^4 - 552960x^3 - 69120ix^2 - 6480x + 1458i}{10935(x(4x + 3i))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((3*I)*x + 4*x^2)^(-7/2), x]

[Out] (1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^5)/(10935*(x*(3*I + 4*x))^(5/2))

fricas [A] time = 0.81, size = 82, normalized size = 1.04

$$\frac{1048576x^6 + 2359296ix^5 - 1769472x^4 - 442368ix^3 + (524288x^5 + 983040ix^4 - 552960x^3 - 69120ix^2 - 6480x + 1458i)\sqrt{4x^2 + 3ix}}{699840x^6 + 1574640ix^5 - 1180980x^4 - 295245ix^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(7/2), x, algorithm="fricas")

[Out] (1048576*x^6 + 2359296*I*x^5 - 1769472*x^4 - 442368*I*x^3 + (524288*x^5 + 983040*I*x^4 - 552960*x^3 - 69120*I*x^2 - 6480*x + 1458*I)*sqrt(4*x^2 + 3*I*x))/(699840*x^6 + 1574640*I*x^5 - 1180980*x^4 - 295245*I*x^3)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(7/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x)::OUTPUT:Warning, replacing 0 by `u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by `u`, a substitu
 tion variable should perhaps be purged.Warning, replacing 0 by `u`, a subs
 titution variable should perhaps be purged.

maple [A] time = 0.10, size = 62, normalized size = 0.78

$$\frac{\frac{16x}{45} + \frac{2i}{15}}{(4x^2 + 3ix)^{\frac{5}{2}}} + \frac{\frac{1024x}{1215} + \frac{128i}{405}}{(4x^2 + 3ix)^{\frac{3}{2}}} + \frac{\frac{32768x}{10935} + \frac{4096i}{3645}}{\sqrt{4x^2 + 3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2+3*I*x)^(7/2),x)

[Out] 2/45*(8*x+3*I)/(4*x^2+3*I*x)^(5/2)+128/1215*(8*x+3*I)/(4*x^2+3*I*x)^(3/2)+4
 096/10935*(8*x+3*I)/(4*x^2+3*I*x)^(1/2)

maxima [A] time = 1.37, size = 82, normalized size = 1.04

$$\frac{32768x}{10935\sqrt{4x^2+3ix}} + \frac{4096i}{3645\sqrt{4x^2+3ix}} + \frac{1024x}{1215(4x^2+3ix)^{\frac{3}{2}}} + \frac{128i}{405(4x^2+3ix)^{\frac{3}{2}}} + \frac{16x}{45(4x^2+3ix)^{\frac{5}{2}}} + \frac{2i}{15(4x^2+3ix)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")

[Out] 32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024/
 1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/(4
 *x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)

mupad [B] time = 0.29, size = 40, normalized size = 0.51

$$-\frac{-524288x^5 - x^4 983040i + 552960x^3 + x^2 69120i + 6480x - 1458i}{10935(x(4x+3i))^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*3i + 4*x^2)^(7/2),x)

[Out] -(6480*x + x^2*69120i + 552960*x^3 - x^4*983040i - 524288*x^5 - 1458i)/(109
 35*(x*(4*x + 3i))^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4x^2 + 3ix)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-7/2), x)
```


$$3.21 \quad \int \frac{1}{\sqrt{3x-4x^2}} dx$$

Optimal. Leaf size=12

$$-\frac{1}{2} \sin^{-1} \left(1 - \frac{8x}{3} \right)$$

[Out] 1/2*arcsin(-1+8/3*x)

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {619, 216}

$$-\frac{1}{2} \sin^{-1} \left(1 - \frac{8x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3*x - 4*x^2], x]

[Out] -ArcSin[1 - (8*x)/3]/2

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3x-4x^2}} dx &= - \left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}} dx, x, 3-8x \right) \right) \\ &= -\frac{1}{2} \sin^{-1} \left(1 - \frac{8x}{3} \right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 40, normalized size = 3.33

$$-\frac{\sqrt{-x(4x-3)} \sin^{-1}\left(\sqrt{1-\frac{4x}{3}}\right)}{\sqrt{3-4x}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[3*x - 4*x^2], x]

[Out] -((Sqrt[-(x*(-3 + 4*x))]*ArcSin[Sqrt[1 - (4*x)/3]])/(Sqrt[3 - 4*x]*Sqrt[x]))

fricas [B] time = 0.94, size = 19, normalized size = 1.58

$$-\arctan\left(\frac{\sqrt{-4x^2 + 3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(1/2), x, algorithm="fricas")

[Out] -arctan(1/2*sqrt(-4*x^2 + 3*x)/x)

giac [A] time = 0.50, size = 8, normalized size = 0.67

$$\frac{1}{2} \arcsin\left(\frac{8}{3}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(1/2), x, algorithm="giac")

[Out] 1/2*arcsin(8/3*x - 1)

maple [A] time = 0.04, size = 9, normalized size = 0.75

$$\frac{\arcsin\left(\frac{8x}{3} - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2+3*x)^(1/2), x)

[Out] 1/2*arcsin(8/3*x-1)

maxima [A] time = 2.95, size = 8, normalized size = 0.67

$$-\frac{1}{2} \arcsin\left(-\frac{8}{3}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")

[Out] -1/2*arcsin(-8/3*x + 1)

mupad [B] time = 0.11, size = 8, normalized size = 0.67

$$\frac{\operatorname{asin}\left(\frac{8x}{3} - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 4*x^2)^(1/2),x)

[Out] asin((8*x)/3 - 1)/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2 + 3x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x**2+3*x)**(1/2),x)

[Out] Integral(1/sqrt(-4*x**2 + 3*x), x)

$$3.22 \quad \int \frac{1}{(3x-4x^2)^{3/2}} dx$$

Optimal. Leaf size=22

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

[Out] $-2/9*(3-8*x)/(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {613}

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(-3/2), x]

[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{(3x-4x^2)^{3/2}} dx = -\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{2(8x-3)}{9\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(-3/2), x]

[Out] (2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 0.96, size = 29, normalized size = 1.32

$$\frac{2\sqrt{-4x^2 + 3x}(8x - 3)}{9(4x^2 - 3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")

[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)

giac [A] time = 0.51, size = 29, normalized size = 1.32

$$\frac{2\sqrt{-4x^2 + 3x}(8x - 3)}{9(4x^2 - 3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")

[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)

maple [A] time = 0.05, size = 25, normalized size = 1.14

$$\frac{2(4x - 3)(8x - 3)x}{9(-4x^2 + 3x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2+3*x)^(3/2),x)

[Out] -2/9*x*(-3+4*x)*(-3+8*x)/(-4*x^2+3*x)^(3/2)

maxima [A] time = 1.34, size = 28, normalized size = 1.27

$$\frac{16x}{9\sqrt{-4x^2 + 3x}} - \frac{2}{3\sqrt{-4x^2 + 3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")

[Out] 16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)

mupad [B] time = 0.14, size = 18, normalized size = 0.82

$$\frac{16x - 6}{9\sqrt{3x - 4x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x - 4*x^2)^(3/2), x)
```

```
[Out] (16*x - 6)/(9*(3*x - 4*x^2)^(1/2))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(-4x^2 + 3x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(3/2), x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-3/2), x)
```

$$3.23 \quad \int \frac{1}{(3x-4x^2)^{5/2}} dx$$

Optimal. Leaf size=45

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}} - \frac{2(3-8x)}{27(3x-4x^2)^{3/2}}$$

[Out] $-2/27*(3-8*x)/(-4*x^2+3*x)^(3/2)-64/243*(3-8*x)/(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {614, 613}

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}} - \frac{2(3-8x)}{27(3x-4x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(-5/2), x]

[Out] $(-2*(3-8*x))/(27*(3*x-4*x^2)^(3/2)) - (64*(3-8*x))/(243*\text{Sqrt}[3*x-4*x^2])$

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\int \frac{1}{(3x - 4x^2)^{5/2}} dx = -\frac{2(3 - 8x)}{27(3x - 4x^2)^{3/2}} + \frac{32}{27} \int \frac{1}{(3x - 4x^2)^{3/2}} dx$$

$$= -\frac{2(3 - 8x)}{27(3x - 4x^2)^{3/2}} - \frac{64(3 - 8x)}{243\sqrt{3x - 4x^2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.69

$$-\frac{2048x^3 - 2304x^2 + 432x + 54}{243(-x(4x - 3))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(-5/2), x]

[Out] -1/243*(54 + 432*x - 2304*x^2 + 2048*x^3)/(-(x*(-3 + 4*x)))^(3/2)

fricas [A] time = 0.97, size = 46, normalized size = 1.02

$$-\frac{2(1024x^3 - 1152x^2 + 216x + 27)\sqrt{-4x^2 + 3x}}{243(16x^4 - 24x^3 + 9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(5/2), x, algorithm="fricas")

[Out] -2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*x^3 + 9*x^2)

giac [A] time = 0.41, size = 39, normalized size = 0.87

$$-\frac{2(8(16(8x - 9)x + 27)x + 27)\sqrt{-4x^2 + 3x}}{243(4x^2 - 3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(5/2), x, algorithm="giac")

[Out] -2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2

maple [A] time = 0.04, size = 35, normalized size = 0.78

$$\frac{2(4x - 3)(1024x^3 - 1152x^2 + 216x + 27)x}{243(-4x^2 + 3x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-4*x^2+3*x)^(5/2),x)`

[Out] $2/243*x*(4*x-3)*(1024*x^3-1152*x^2+216*x+27)/(-4*x^2+3*x)^(5/2)$

maxima [A] time = 1.36, size = 55, normalized size = 1.22

$$\frac{512x}{243\sqrt{-4x^2+3x}} - \frac{64}{81\sqrt{-4x^2+3x}} + \frac{16x}{27(-4x^2+3x)^{\frac{3}{2}}} - \frac{2}{9(-4x^2+3x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")`

[Out] $512/243*x/\text{sqrt}(-4*x^2 + 3*x) - 64/81/\text{sqrt}(-4*x^2 + 3*x) + 16/27*x/(-4*x^2 + 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)$

mupad [B] time = 0.03, size = 28, normalized size = 0.62

$$\frac{(16x - 6)(-128x^2 + 96x + 9)}{243(3x - 4x^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x - 4*x^2)^(5/2),x)`

[Out] $((16*x - 6)*(96*x - 128*x^2 + 9))/(243*(3*x - 4*x^2)^(3/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-4x^2 + 3x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4*x**2+3*x)**(5/2),x)`

[Out] `Integral((-4*x**2 + 3*x)**(-5/2), x)`

$$3.24 \quad \int \frac{1}{(3x-4x^2)^{7/2}} dx$$

Optimal. Leaf size=67

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}} - \frac{128(3-8x)}{1215(3x-4x^2)^{3/2}} - \frac{2(3-8x)}{45(3x-4x^2)^{5/2}}$$

[Out] $-2/45*(3-8*x)/(-4*x^2+3*x)^(5/2)-128/1215*(3-8*x)/(-4*x^2+3*x)^(3/2)-4096/10935*(3-8*x)/(-4*x^2+3*x)^(1/2)$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {614, 613}

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}} - \frac{128(3-8x)}{1215(3x-4x^2)^{3/2}} - \frac{2(3-8x)}{45(3x-4x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2)^(-7/2), x]

[Out] $(-2*(3-8*x))/(45*(3*x-4*x^2)^(5/2)) - (128*(3-8*x))/(1215*(3*x-4*x^2)^(3/2)) - (4096*(3-8*x))/(10935*\text{Sqrt}[3*x-4*x^2])$

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3x-4x^2)^{7/2}} dx &= -\frac{2(3-8x)}{45(3x-4x^2)^{5/2}} + \frac{64}{45} \int \frac{1}{(3x-4x^2)^{5/2}} dx \\
&= -\frac{2(3-8x)}{45(3x-4x^2)^{5/2}} - \frac{128(3-8x)}{1215(3x-4x^2)^{3/2}} + \frac{2048 \int \frac{1}{(3x-4x^2)^{3/2}} dx}{1215} \\
&= -\frac{2(3-8x)}{45(3x-4x^2)^{5/2}} - \frac{128(3-8x)}{1215(3x-4x^2)^{3/2}} - \frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.76

$$\frac{2(262144x^5 - 491520x^4 + 276480x^3 - 34560x^2 - 3240x - 729)}{10935(3-4x)^2x^2\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2)^(-7/2), x]

[Out] (2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(10935*(3 - 4*x)^2*x^2*Sqrt[-(x*(-3 + 4*x))])

fricas [A] time = 0.92, size = 61, normalized size = 0.91

$$\frac{2(262144x^5 - 491520x^4 + 276480x^3 - 34560x^2 - 3240x - 729)\sqrt{-4x^2 + 3x}}{10935(64x^6 - 144x^5 + 108x^4 - 27x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(7/2), x, algorithm="fricas")

[Out] -2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)*sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)

giac [A] time = 0.50, size = 49, normalized size = 0.73

$$\frac{2(8(32(8(16(8x-15)x+135)x-135)x-405)x-729)\sqrt{-4x^2+3x}}{10935(4x^2-3x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")

[Out] $-2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*\sqrt{-4*x^2 + 3*x}/(4*x^2 - 3*x)^3$

maple [A] time = 0.05, size = 45, normalized size = 0.67

$$\frac{2(4x - 3)(262144x^5 - 491520x^4 + 276480x^3 - 34560x^2 - 3240x - 729)x}{10935(-4x^2 + 3x)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2+3*x)^(7/2),x)

[Out] $-2/10935*x*(4*x-3)*(262144*x^5-491520*x^4+276480*x^3-34560*x^2-3240*x-729)/(-4*x^2+3*x)^{(7/2)}$

maxima [A] time = 1.37, size = 82, normalized size = 1.22

$$\frac{32768x}{10935\sqrt{-4x^2+3x}} - \frac{4096}{3645\sqrt{-4x^2+3x}} + \frac{1024x}{1215(-4x^2+3x)^{\frac{3}{2}}} - \frac{128}{405(-4x^2+3x)^{\frac{3}{2}}} + \frac{16x}{45(-4x^2+3x)^{\frac{5}{2}}} - \frac{2}{15(-4x^2+3x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")

[Out] $32768/10935*x/\sqrt{-4*x^2 + 3*x} - 4096/3645/\sqrt{-4*x^2 + 3*x} + 1024/1215*x/(-4*x^2 + 3*x)^{(3/2)} - 128/405/(-4*x^2 + 3*x)^{(3/2)} + 16/45*x/(-4*x^2 + 3*x)^{(5/2)} - 2/15/(-4*x^2 + 3*x)^{(5/2)}$

mupad [B] time = 0.20, size = 73, normalized size = 1.09

$$\frac{6480x - 9216x(3x - 4x^2) - 32768x(3x - 4x^2)^2 + 12288(3x - 4x^2)^2 - 13824x^2 + 1458}{(3x - 4x^2)^{3/2}(32805x - 43740x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x - 4*x^2)^(7/2),x)

[Out] $-(6480*x - 9216*x*(3*x - 4*x^2) - 32768*x*(3*x - 4*x^2)^2 + 12288*(3*x - 4*x^2)^2 - 13824*x^2 + 1458)/((3*x - 4*x^2)^{(3/2)}*(32805*x - 43740*x^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-4x^2 + 3x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-7/2), x)
```

$$3.25 \quad \int \frac{1}{\sqrt{bx - b^2x^2}} dx$$

Optimal. Leaf size=12

$$-\frac{\sin^{-1}(1 - 2bx)}{b}$$

[Out] arcsin(2*b*x-1)/b

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {619, 216}

$$-\frac{\sin^{-1}(1 - 2bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[b*x - b^2*x^2], x]

[Out] -(ArcSin[1 - 2*b*x]/b)

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{bx - b^2x^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{b^2}}} dx, x, b - 2b^2x \right)}{b^2} = -\frac{\sin^{-1}(1 - 2bx)}{b}$$

Mathematica [B] time = 0.01, size = 47, normalized size = 3.92

$$\frac{2\sqrt{x}\sqrt{1-bx}\sin^{-1}(\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{-bx(bx-1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x - b^2*x^2],x]

[Out] (2*Sqrt[x]*Sqrt[1 - b*x]*ArcSin[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[-(b*x*(-1 + b*x))])

fricas [B] time = 0.94, size = 27, normalized size = 2.25

$$-\frac{2 \arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")

[Out] -2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b

giac [A] time = 0.52, size = 15, normalized size = 1.25

$$-\frac{\arcsin(-2bx+1)\operatorname{sgn}(b)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")

[Out] -arcsin(-2*b*x + 1)*sgn(b)/abs(b)

maple [B] time = 0.07, size = 35, normalized size = 2.92

$$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b^2*x^2+b*x)^(1/2),x)

[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x)^(1/2))

maxima [A] time = 2.92, size = 21, normalized size = 1.75

$$-\frac{\arcsin\left(-\frac{2b^2x-b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-(2*b^2*x - b)/b)/b

mupad [B] time = 0.31, size = 42, normalized size = 3.50

$$\frac{\ln\left(\frac{\frac{b}{2}-b^2x}{\sqrt{-b^2}} + \sqrt{bx - b^2x^2}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x - b^2*x^2)^(1/2),x)

[Out] log((b/2 - b^2*x)/(-b^2)^(1/2) + (b*x - b^2*x^2)^(1/2))/(-b^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b^2x^2 + bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b**2*x**2+b*x)**(1/2),x)

[Out] Integral(1/sqrt(-b**2*x**2 + b*x), x)

$$3.26 \quad \int \frac{1}{\sqrt{bx+b^2x^2}} dx$$

Optimal. Leaf size=24

$$\frac{2 \tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

[Out] $2*\operatorname{arctanh}(b*x/(b^2*x^2+b*x)^{(1/2)})/b$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/\operatorname{Sqrt}[b*x + b^2*x^2], x]$

[Out] $(2*\operatorname{ArcTanh}[(b*x)/\operatorname{Sqrt}[b*x + b^2*x^2]])/b$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 620

$\operatorname{Int}[1/\operatorname{Sqrt}[(b_)*(x_) + (c_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Dist}[2, \operatorname{Subst}[\operatorname{Int}[1/(1 - c*x^2), x], x, x/\operatorname{Sqrt}[b*x + c*x^2]], x] /;$ $\operatorname{FreeQ}\{b, c\}, x$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{bx+b^2x^2}} dx &= 2 \operatorname{Subst}\left(\int \frac{1}{1-b^2x^2} dx, x, \frac{x}{\sqrt{bx+b^2x^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.88

$$\frac{2\sqrt{x}\sqrt{bx+1}\sinh^{-1}(\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{bx(bx+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[b*x + b^2*x^2], x]

[Out] (2*Sqrt[x]*Sqrt[1 + b*x]*ArcSinh[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[b*x*(1 + b*x)])

fricas [A] time = 0.81, size = 27, normalized size = 1.12

$$-\frac{\log\left(-2bx + 2\sqrt{b^2x^2 + bx} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^2+b*x)^(1/2), x, algorithm="fricas")

[Out] -log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b

giac [A] time = 0.55, size = 36, normalized size = 1.50

$$-\frac{\log\left(\left|-2\left(x|b| - \sqrt{b^2x^2 + bx}\right)|b| - b\right|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^2+b*x)^(1/2), x, algorithm="giac")

[Out] -log(abs(-2*(x*abs(b) - sqrt(b^2*x^2 + b*x))*abs(b) - b))/abs(b)

maple [A] time = 0.05, size = 37, normalized size = 1.54

$$\frac{\ln\left(\frac{b^2x + \frac{1}{2}b}{\sqrt{b^2}} + \sqrt{b^2x^2 + bx}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^2+b*x)^(1/2), x)

[Out] ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)

maxima [A] time = 1.37, size = 29, normalized size = 1.21

$$\frac{\log\left(2b^2x + 2\sqrt{b^2x^2 + bx}b + b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")

[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x)*b + b)/b

mupad [B] time = 0.23, size = 36, normalized size = 1.50

$$\frac{\ln\left(\frac{x\frac{b^2+\frac{b}{2}}{\sqrt{b^2}} + \sqrt{b^2x^2 + bx}}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + b^2*x^2)^(1/2),x)

[Out] log((b/2 + b^2*x)/(b^2)^(1/2) + (b*x + b^2*x^2)^(1/2))/(b^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x^2 + bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**2*x**2+b*x)**(1/2),x)

[Out] Integral(1/sqrt(b**2*x**2 + b*x), x)

$$3.27 \quad \int \frac{1}{\sqrt{6x-x^2}} dx$$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1 - \frac{x}{3}\right)$$

[Out] arcsin(-1+1/3*x)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {619, 216}

$$-\sin^{-1}\left(1 - \frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[6*x - x^2], x]

[Out] -ArcSin[1 - x/3]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{6x-x^2}} dx &= -\left(\frac{1}{6} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}} dx, x, 6-2x\right)\right) \\ &= -\sin^{-1}\left(1 - \frac{x}{3}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-2 \sin^{-1}\left(\sqrt{1 - \frac{x}{6}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[6*x - x^2],x]

[Out] -2*ArcSin[Sqrt[1 - x/6]]

fricas [B] time = 0.60, size = 18, normalized size = 1.80

$$-2 \arctan\left(\frac{\sqrt{-x^2 + 6x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")

[Out] -2*arctan(sqrt(-x^2 + 6*x)/x)

giac [A] time = 0.39, size = 6, normalized size = 0.60

$$\arcsin\left(\frac{1}{3}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")

[Out] arcsin(1/3*x - 1)

maple [A] time = 0.05, size = 7, normalized size = 0.70

$$\arcsin\left(\frac{x}{3} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+6*x)^(1/2),x)

[Out] arcsin(1/3*x-1)

maxima [A] time = 3.03, size = 8, normalized size = 0.80

$$-\arcsin\left(-\frac{1}{3}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")

[Out] $-\arcsin(-1/3*x + 1)$

mupad [B] time = 0.11, size = 6, normalized size = 0.60

$$\arcsin\left(\frac{x}{3} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(6*x - x^2)^(1/2), x)`

[Out] $\arcsin(x/3 - 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + 6x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**2+6*x)**(1/2), x)`

[Out] `Integral(1/sqrt(-x**2 + 6*x), x)`

$$3.28 \quad \int \frac{1}{\sqrt{4x+x^2}} dx$$

Optimal. Leaf size=16

$$2 \tanh^{-1} \left(\frac{x}{\sqrt{x^2 + 4x}} \right)$$

[Out] 2*arctanh(x/(x^2+4*x)^(1/2))

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {620, 206}

$$2 \tanh^{-1} \left(\frac{x}{\sqrt{x^2 + 4x}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4*x + x^2], x]

[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{4x+x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{4x+x^2}} \right) \\ &= 2 \tanh^{-1} \left(\frac{x}{\sqrt{4x+x^2}} \right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2\sqrt{x}\sqrt{x+4}\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x(x+4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4*x + x^2], x]

[Out] (2*Sqrt[x]*Sqrt[4 + x]*ArcSinh[Sqrt[x]/2])/Sqrt[x*(4 + x)]

fricas [A] time = 0.91, size = 17, normalized size = 1.06

$$-\log\left(-x + \sqrt{x^2 + 4x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x)^(1/2), x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 4*x) - 2)

giac [A] time = 0.54, size = 18, normalized size = 1.12

$$-\log\left(\left|-x + \sqrt{x^2 + 4x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x)^(1/2), x, algorithm="giac")

[Out] -log(abs(-x + sqrt(x^2 + 4*x) - 2))

maple [A] time = 0.04, size = 14, normalized size = 0.88

$$\ln\left(x + 2 + \sqrt{x^2 + 4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+4*x)^(1/2), x)

[Out] ln(x+2+(x^2+4*x)^(1/2))

maxima [A] time = 1.31, size = 17, normalized size = 1.06

$$\log\left(2x + 2\sqrt{x^2 + 4x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x)^(1/2), x, algorithm="maxima")

[Out] log(2*x + 2*sqrt(x^2 + 4*x) + 4)

mupad [B] time = 0.51, size = 11, normalized size = 0.69

$$\ln(x + \sqrt{x(x+4)} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x + x^2)^(1/2), x)`

[Out] `log(x + (x*(x + 4))^(1/2) + 2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 + 4x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+4*x)**(1/2), x)`

[Out] `Integral(1/sqrt(x**2 + 4*x), x)`

$$3.29 \quad \int \frac{1}{\sqrt{-2x+x^2}} dx$$

Optimal. Leaf size=16

$$2 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

[Out] 2*arctanh(x/(x^2-2*x)^(1/2))

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {620, 206}

$$2 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2*x + x^2], x]

[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2x+x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-2x+x^2}} \right) \\ &= 2 \tanh^{-1} \left(\frac{x}{\sqrt{-2x+x^2}} \right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{2\sqrt{(x-2)x} \sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)}{\sqrt{-((x-2)x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2*x + x^2],x]

[Out] (2*sqrt[(-2 + x)*x]*ArcSin[Sqrt[1 - x/2]])/sqrt[-((-2 + x)*x)]

fricas [A] time = 0.82, size = 17, normalized size = 1.06

$$-\log\left(-x + \sqrt{x^2 - 2x + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 - 2*x) + 1)

giac [A] time = 0.54, size = 18, normalized size = 1.12

$$-\log\left(\left|-x + \sqrt{x^2 - 2x + 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")

[Out] -log(abs(-x + sqrt(x^2 - 2*x) + 1))

maple [A] time = 0.05, size = 14, normalized size = 0.88

$$\ln\left(x - 1 + \sqrt{x^2 - 2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2-2*x)^(1/2),x)

[Out] ln(x-1+(x^2-2*x)^(1/2))

maxima [A] time = 1.33, size = 17, normalized size = 1.06

$$\log\left(2x + 2\sqrt{x^2 - 2x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")

[Out] log(2*x + 2*sqrt(x^2 - 2*x) - 2)

mupad [B] time = 0.53, size = 11, normalized size = 0.69

$$\ln(x + \sqrt{x(x-2)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 - 2*x)^(1/2), x)`

[Out] `log(x + (x*(x - 2))^(1/2) - 1)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 - 2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-2*x)**(1/2), x)`

[Out] `Integral(1/sqrt(x**2 - 2*x), x)`

3.30 $\int (bx + cx^2)^{4/3} dx$

Optimal. Leaf size=448

$$\frac{3 \left(-\frac{cx(b+cx)}{b^2} \right)^{4/3} (b+2cx)(bx+cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{4/3}} + \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b+2cx)(bx+cx^2)^{4/3}}{55c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{4/3}} + \frac{\sqrt[3]{2} 3^{3/4} \sqrt{2-\sqrt{3}} b^2 (bx+cx^2)^{4/3}}{\dots}$$

[Out] $3/55 * (-c*x*(c*x+b)/b^2)^{(1/3)} * (2*c*x+b) * (c*x^2+b*x)^{(4/3)} / c / (-c*(c*x^2+b*x)/b^2)^{(4/3)} + 3/22 * (-c*x*(c*x+b)/b^2)^{(4/3)} * (2*c*x+b) * (c*x^2+b*x)^{(4/3)} / c / (-c*(c*x^2+b*x)/b^2)^{(4/3)} + 1/55 * 2^{(1/3)} * 3^{(3/4)} * b^2 * (c*x^2+b*x)^{(4/3)} * (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) * \text{EllipticF}((1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 3^{(1/2)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)}), 2*I-I*3^{(1/2)}) * (1/2 * 6^{(1/2)} - 1/2 * 2^{(1/2)}) * ((1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 2*2^{(1/3)} * (-c*x*(c*x+b)/b^2)^{(2/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)} / c / (2*c*x+b) / (-c*(c*x^2+b*x)/b^2)^{(4/3)} / ((-1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)}))^2)^{(1/2)}$

Rubi [A] time = 0.72, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {622, 619, 195, 236, 219}

$$\frac{3 \left(-\frac{cx(b+cx)}{b^2} \right)^{4/3} (b+2cx)(bx+cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{4/3}} + \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b+2cx)(bx+cx^2)^{4/3}}{55c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{4/3}} + \frac{\sqrt[3]{2} 3^{3/4} \sqrt{2-\sqrt{3}} b^2 (bx+cx^2)^{4/3}}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(4/3), x]

[Out] $(3 * (-((c*x*(b+c*x))/b^2))^{(1/3)} * (b+2*c*x) * (b*x+c*x^2)^{(4/3)}) / (55*c * (-((c*(b*x+c*x^2))/b^2))^{(4/3)}) + (3 * (-((c*x*(b+c*x))/b^2))^{(4/3)} * (b+2*c*x) * (b*x+c*x^2)^{(4/3)}) / (22*c * (-((c*(b*x+c*x^2))/b^2))^{(4/3)}) + (2^{(1/3)} * 3^{(3/4)} * \text{Sqrt}[2-\text{Sqrt}[3]] * b^2 * (b*x+c*x^2)^{(4/3)} * (1-2^{(2/3)} * (-((c*x*(b+c*x))/b^2))^{(1/3)}) * \text{Sqrt}[(1+2^{(2/3)} * (-((c*x*(b+c*x))/b^2))^{(1/3)} + 2*$

$$2^{(1/3)} * (-((c*x*(b + c*x))/b^2))^{(2/3)} / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})^2 * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]] / (55*c*(b + 2*c*x) * (-((c*(b*x + c*x^2))/b^2))^{(4/3)} * \text{Sqrt}[-((1 - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})^2)])$$

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^(p), Int[(-(c*x)/b) - (c^2*x^2)/b^2)^(p), x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
\int (bx + cx^2)^{4/3} dx &= \frac{(bx + cx^2)^{4/3} \int \left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{4/3} dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} \\
&= -\frac{(b^2 (bx + cx^2)^{4/3}) \text{Subst}\left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{4/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{8 \cdot 2^{2/3} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (b + 2cx) (bx + cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} - \frac{(b^2 (bx + cx^2)^{4/3}) \text{Subst}\left(\int \sqrt[3]{1 - \frac{b^2x^2}{c^2}} dx, x, -\frac{c}{b}\right)}{11 \cdot 2^{2/3} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) (bx + cx^2)^{4/3}}{55c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (b + 2cx) (bx + cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} - \frac{\left(\sqrt[3]{2} b^2 (bx + cx^2)^{4/3}\right)}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) (bx + cx^2)^{4/3}}{55c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (b + 2cx) (bx + cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{\left(3 (bx + cx^2)^{4/3}\right)}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) (bx + cx^2)^{4/3}}{55c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (b + 2cx) (bx + cx^2)^{4/3}}{22c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}} + \frac{\left(\sqrt[3]{2} 3^{3/4} \sqrt[3]{bx + cx^2}\right)}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.11

$$\frac{3bx^2 \sqrt[3]{x(b+cx)} {}_2F_1\left(-\frac{4}{3}, \frac{7}{3}; \frac{10}{3}; -\frac{cx}{b}\right)}{7 \sqrt[3]{\frac{cx}{b}} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(4/3),x]

[Out] (3*b*x^2*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-4/3, 7/3, 10/3, -((c*x)/b)])/(7*(1 + (c*x)/b)^(1/3))

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(4/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(4/3), x)

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(4/3),x)

[Out] int((c*x^2+b*x)^(4/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(4/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(4/3), x)

mupad [B] time = 0.23, size = 36, normalized size = 0.08

$$\frac{3x(c x^2 + b x)^{4/3} {}_2F_1\left(-\frac{4}{3}, \frac{7}{3}; \frac{10}{3}; -\frac{c x}{b}\right)}{7\left(\frac{c x}{b} + 1\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(4/3), x)

[Out] (3*x*(b*x + c*x^2)^(4/3)*hypergeom([-4/3, 7/3], 10/3, -(c*x)/b))/(7*((c*x)/b + 1)^(4/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(4/3), x)

[Out] Integral((b*x + c*x**2)**(4/3), x)

3.31 $\int \sqrt[3]{bx + cx^2} dx$

Optimal. Leaf size=387

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}{5\cdot 2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

[Out] $3/10*(-c*x*(c*x+b)/b^2)^{(1/3)}*(2*c*x+b)*(c*x^2+b*x)^{(1/3)}/c/(-c*(c*x^2+b*x)/b^2)^{(1/3)}+1/10*3^{(3/4)}*b^2*(c*x^2+b*x)^{(1/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*\text{EllipticF}((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}*2^{(1/3)}/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^{(1/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.45, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {622, 619, 195, 236, 219}

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}{5\cdot 2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(1/3), x]

[Out] $(3*(-((c*x*(b+c*x))/b^2))^{(1/3)}*(b+2*c*x)*(b*x+c*x^2)^{(1/3)})/(10*c*(-((c*(b*x+c*x^2))/b^2))^{(1/3)})+(3^{(3/4)}*\text{Sqrt}[2-\text{Sqrt}[3]]*b^2*(b*x+c*x^2)^{(1/3)}*(1-2^{(2/3)}*(-((c*x*(b+c*x))/b^2))^{(1/3)})*\text{Sqrt}[(1+2^{(2/3)}*(-((c*x*(b+c*x))/b^2))^{(1/3)}+2*2^{(1/3)}*(-((c*x*(b+c*x))/b^2))^{(2/3)})/(1-\text{Sqrt}[3]-2^{(2/3)}*(-((c*x*(b+c*x))/b^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3]-2^{(2/3)}*(-((c*x*(b+c*x))/b^2))^{(1/3)})/(1-\text{Sqrt}[3]-2^{(2/3)}*(-((c*x*(b+c*x))/b^2))^{(1/3)})]$

$$\frac{(-((c*x*(b + c*x))/b^2))^{(1/3)}], -7 + 4*\text{Sqrt}[3]]/(5*2^{(2/3)}*c*(b + 2*c*x) * (-((c*(b*x + c*x^2))/b^2))^{(1/3)}*\text{Sqrt}[-((1 - 2^{(2/3)}*(-((c*x*(b + c*x))/b^2))^{(1/3)})^2)]/(1 - \text{Sqrt}[3] - 2^{(2/3)}*(-((c*x*(b + c*x))/b^2))^{(1/3)})^2]]$$

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2)^(p), Int[-((c*x)/b) - (c^2*x^2)/b^2)^(p), x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
\int \sqrt[3]{bx + cx^2} dx &= \frac{\sqrt[3]{bx + cx^2} \int \sqrt[3]{-\frac{cx}{b} - \frac{c^2x^2}{b^2}} dx}{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} \\
&= \frac{\left(b^2 \sqrt[3]{bx + cx^2}\right) \text{Subst}\left(\int \sqrt[3]{1 - \frac{b^2x^2}{c^2}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{2^{2/3} c^2 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) \sqrt[3]{bx + cx^2}}{10c \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} - \frac{\left(b^2 \sqrt[3]{bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{2/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{5^{2/3} c^2 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) \sqrt[3]{bx + cx^2}}{10c \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{\left(3 \sqrt[3]{bx + cx^2} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}} dx, x, 2\left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right)\right)}{10^{2/3} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} \\
&= \frac{3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (b + 2cx) \sqrt[3]{bx + cx^2}}{10c \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4} \sqrt{2 - \sqrt{3}} b^2 \sqrt[3]{bx + cx^2} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \left(1 - 2^{2/3} \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\right)}{5^{2/3} c (b + 2cx) \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.12

$$\frac{3x \sqrt[3]{x(b + cx)} {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{cx}{b}\right)}{4 \sqrt[3]{\frac{cx}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(1/3), x]

[Out] (3*x*(x*(b + c*x))^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -(c*x)/b])/(4*(1 + (c*x)/b)^(1/3))

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(1/3), x)

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(1/3),x)

[Out] int((c*x^2+b*x)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(1/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(1/3), x)

mupad [B] time = 0.17, size = 36, normalized size = 0.09

$$\frac{3x(cx^2 + bx)^{1/3} {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{cx}{b}\right)}{4\left(\frac{cx}{b} + 1\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^(1/3),x)
```

```
[Out] (3*x*(b*x + c*x^2)^(1/3)*hypergeom([-1/3, 4/3], 7/3, -(c*x)/b))/(4*((c*x)/b + 1)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)**(1/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(1/3), x)
```

$$3.32 \quad \int \frac{1}{(bx+cx^2)^{2/3}} dx$$

Optimal. Leaf size=322

$$\frac{\sqrt[3]{2} 3^{3/4} \sqrt{2-\sqrt{3}} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \left(1-2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right) \sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}} F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}\right)}{\right)}{c(b+2cx)(bx+cx^2)^{2/3} \sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

[Out] $2^{(1/3)} * 3^{(3/4)} * b^2 * (-c * (c * x^2 + b * x) / b^2)^{(2/3)} * (1 - 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)}) * \text{EllipticF}((1 - 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)} + 3^{(1/2)}) / (1 - 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)} - 3^{(1/2)}), 2 * I - I * 3^{(1/2)}) * (1/2 * 6^{(1/2)} - 1/2 * 2^{(1/2)}) * ((1 + 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)} + 2 * 2^{(1/3)} * (-c * x * (c * x + b) / b^2)^{(2/3)}) / (1 - 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)} / c / (2 * c * x + b) / (c * x^2 + b * x)^{(2/3)} / ((-1 + 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)}) / (1 - 2^{(2/3)} * (-c * x * (c * x + b) / b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.39, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {622, 619, 236, 219}

$$\frac{\sqrt[3]{2} 3^{3/4} \sqrt{2-\sqrt{3}} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \left(1-2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right) \sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}} F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}\right)}{\right)}{c(b+2cx)(bx+cx^2)^{2/3} \sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-2/3), x]

[Out] $(2^{(1/3)} * 3^{(3/4)} * \text{Sqrt}[2 - \text{Sqrt}[3]] * b^2 * (-((c * (b * x + c * x^2)) / b^2))^{(2/3)} * (1 - 2^{(2/3)} * (-((c * x * (b + c * x)) / b^2))^{(1/3)}) * \text{Sqrt}[(1 + 2^{(2/3)} * (-((c * x * (b + c * x)) / b^2))^{(1/3)} + 2 * 2^{(1/3)} * (-((c * x * (b + c * x)) / b^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c * x * (b + c * x)) / b^2))^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)} * (-((c * x * (b + c * x)) / b^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c * x * (b + c * x)) / b^2))^{(1/3)})], -7 + 4 * \text{Sqrt}[3]]) / (c * (b + 2 * c * x) * (b * x + c * x^2)^{(2/3)} * S$

$\text{qrt}[-((1 - 2^{(2/3)})*(-((c*x*(b + c*x))/b^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*(-((c*x*(b + c*x))/b^2))^{(1/3)})^2]]$

Rule 219

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^3], x_Symbol] \text{ :> With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 - \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 - \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*s + r*x]/((1 - \text{Sqrt}[3])*s + r*x)], -7 + 4*\text{Sqrt}[3]])/(3^{(1/4)}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[-((s*(s + r*x))/((1 - \text{Sqrt}[3])*s + r*x)^2)]), x]] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a]$

Rule 236

$\text{Int}[(a_) + (b_)*(x_)^2]^{-2/3}, x_Symbol] \text{ :> Dist}[(3*\text{Sqrt}[b*x^2])/(2*b*x), \text{Subst}[\text{Int}[1/\text{Sqrt}[-a + x^3], x], x, (a + b*x^2)^{(1/3)}], x] \text{ /; FreeQ}\{a, b\}, x]$

Rule 619

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_)}, x_Symbol] \text{ :> Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p)}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]]^{(p)}, x], x, b + 2*c*x], x] \text{ /; FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rule 622

$\text{Int}[(b_.)*(x_) + (c_.)*(x_)^2]^{(p_)}, x_Symbol] \text{ :> Dist}[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2)^p, \text{Int}[(-(c*x)/b) - (c^2*x^2)/b^2]^{(p)}, x], x] \text{ /; FreeQ}\{b, c\}, x] \ \&\& \ \text{RationalQ}[p] \ \&\& \ 3 \leq \text{Denominator}[p] \leq 4$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{2/3}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \int \frac{1}{\left(\frac{-cx}{b} - \frac{c^2x^2}{b^2}\right)^{2/3}} dx}{(bx + cx^2)^{2/3}} \\
&= \frac{\left(\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{2/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{2/3}} \\
&= \frac{\left(3 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{-1+x^3}} dx, x, 2^{2/3} \sqrt[3]{-\frac{cx(1+\frac{cx}{b})}{b}} \right)}{2^{2/3} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{2/3}} \\
&= \frac{\sqrt[3]{2} 3^{3/4} \sqrt{2 - \sqrt{3}} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \left(1 - 2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right) \sqrt{\frac{1+2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{(1-\sqrt{3}-2)}}}{c(b + 2cx) (bx + cx^2)^{2/3} \sqrt{-1 - \frac{4cx(b+cx)}{b^2}} \sqrt{\frac{1+2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{(1-\sqrt{3}-2)}}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.13

$$\frac{3x \left(\frac{cx}{b} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{cx}{b}\right)}{(x(b + cx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-2/3), x]

[Out] (3*x*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*x)/b)])/(x*(b + c*x))^(2/3)

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{1}{(cx^2 + bx)^{2/3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-2/3), x)

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(2/3),x)

[Out] int(1/(c*x^2+b*x)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(2/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-2/3), x)

mupad [B] time = 0.22, size = 36, normalized size = 0.11

$$\frac{3x \left(\frac{cx}{b} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{cx}{b}\right)}{(cx^2 + bx)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x + c*x^2)^(2/3),x)`

[Out] `(3*x*((c*x)/b + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(c*x)/b))/(b*x + c*x^2)^(2/3)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x)**(2/3),x)`

[Out] `Integral((b*x + c*x**2)**(-2/3), x)`

$$3.33 \quad \int \frac{1}{(bx+cx^2)^{5/3}} dx$$

Optimal. Leaf size=384

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\right)}}}{c(b+2cx)(bx+cx^2)^{5/3}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\right)}}$$

[Out] $3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^(5/3)/c/(-c*x*(c*x+b)/b^2)^(2/3)/(c*x^2+b*x)^(5/3)+2^(1/3)*3^(3/4)*b^2*(-c*(c*x^2+b*x)/b^2)^(5/3)*(1-2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)*\text{EllipticF}((1-2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3))*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2^(1/2)/c/(2*c*x+b)/(c*x^2+b*x)^(5/3)/((-1+2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3))*(-c*x*(c*x+b)/b^2)^(1/3)-3^(1/2))^2^(1/2)$

Rubi [A] time = 0.46, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {622, 619, 199, 236, 219}

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\right)}}}{c(b+2cx)(bx+cx^2)^{5/3}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + c*x^2)^{-5/3}, x]$

[Out] $(3*(b+2*c*x)*(-((c*(b*x+c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b+c*x))/b^2))^(2/3)*(b*x+c*x^2)^(5/3))+(2^(1/3)*3^(3/4)*\text{Sqrt}[2-\text{Sqrt}[3]]*b^2*(-((c*(b*x+c*x^2))/b^2))^(5/3)*(1-2^(2/3))*(-((c*x*(b+c*x))/b^2))^(1/3))*\text{Sqrt}[(1+2^(2/3))*(-((c*x*(b+c*x))/b^2))^(1/3)+2*2^(1/3))*(-((c*x*(b+c*x))/b^2))^(2/3)]/(1-\text{Sqrt}[3]-2^(2/3))*(-((c*x*(b+c*x))/b^2))^(1/3))^2]$

```
*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)], -7 + 4*Sqrt[3]]/(c*(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[-((c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{5/3}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3} \int \frac{1}{\left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{5/3}} dx}{(bx + cx^2)^{5/3}} \\
&= \frac{\left(4\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{5/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{5/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{5/3}} - \frac{\left(\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{2/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{5/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{5/3}} + \frac{\left(3 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{-1+x^3}} dx, \right)}{2^{2/3} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{5/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{5/3}} + \frac{\sqrt[3]{2} 3^{3/4} \sqrt{2 - \sqrt{3}} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \left(1 - \right)}{c(b + 2cx) (bx -}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.12

$$\frac{3 \left(\frac{cx}{b} + 1\right)^{2/3} {}_2F_1\left(-\frac{2}{3}, \frac{5}{3}, \frac{1}{3}, -\frac{cx}{b}\right)}{2b(x(b + cx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-5/3), x]

[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 1/3, -((c*x)/b)])/(2*b*(x*(b + c*x))^(2/3))

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^2 + bx)^{\frac{1}{3}}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(1/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-5/3), x)

maple [F] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(5/3),x)

[Out] int(1/(c*x^2+b*x)^(5/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-5/3), x)

mupad [B] time = 0.25, size = 36, normalized size = 0.09

$$-\frac{3x\left(\frac{cx}{b} + 1\right)^{5/3} {}_2F_1\left(-\frac{2}{3}, \frac{5}{3}; \frac{1}{3}; -\frac{cx}{b}\right)}{2\left(cx^2 + bx\right)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(5/3), x)

[Out] -(3*x*((c*x)/b + 1)^(5/3)*hypergeom([-2/3, 5/3], 1/3, -(c*x)/b))/(2*(b*x + c*x^2)^(5/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{5/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(5/3), x)

[Out] Integral((b*x + c*x**2)**(-5/3), x)

$$3.34 \quad \int \frac{1}{(bx+cx^2)^{8/3}} dx$$

Optimal. Leaf size=448

$$\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}} + \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}{5c(b+\sqrt[3]{-\frac{2c^2(b+cx)}{b^2}})}$$

[Out] $3/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^{(8/3)}/c/(-c*x*(c*x+b)/b^2)^{(5/3)}/(c*x^2+b*x)^{(8/3)}+21/5*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^{(8/3)}/c/(-c*x*(c*x+b)/b^2)^{(2/3)}/(c*x^2+b*x)^{(8/3)}+14/5*2^{(1/3)}*3^{(3/4)}*b^2*(-c*(c*x^2+b*x)/b^2)^{(8/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*EllipticF((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}/c/(2*c*x+b)/(c*x^2+b*x)^{(8/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.51, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {622, 619, 199, 236, 219}

$$\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}} + \frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}} + \frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}{5c(b+\sqrt[3]{-\frac{2c^2(b+cx)}{b^2}})}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-8/3), x]

[Out] $(3*(b+2*c*x)*(-((c*(b*x+c*x^2))/b^2))^{(8/3)})/(5*c*(-((c*x*(b+c*x))/b^2))^{(5/3)}*(b*x+c*x^2)^{(8/3)})+(21*(b+2*c*x)*(-((c*(b*x+c*x^2))/b^2))^{(8/3)})/(5*c*(-((c*x*(b+c*x))/b^2))^{(2/3)}*(b*x+c*x^2)^{(8/3)})+(14*2^{(1/3)}*3^{(3/4)}*sqrt[2-Sqrt[3]]*b^2*(-((c*(b*x+c*x^2))/b^2))^{(8/3)}*(1-2^{(2/3)}*sqrt[3]{-\frac{cx(b+cx)}{b^2}}))$

$$\frac{2}{3} * (-((c*x*(b + c*x))/b^2))^{(1/3)} * \text{Sqrt}[(1 + 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)} + 2 * 2^{(1/3)} * (-((c*x*(b + c*x))/b^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})], -7 + 4 * \text{Sqrt}[3]] / (5 * c * (b + 2 * c * x) * (b * x + c * x^2)^{(8/3)} * \text{Sqrt}[-((1 - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - 2^{(2/3)} * (-((c*x*(b + c*x))/b^2))^{(1/3)})^2]])$$

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/
(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]},
Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[(-(c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{8/3}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3} \int \frac{1}{\left(\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{8/3}} dx}{(bx + cx^2)^{8/3}} \\
&= \frac{\left(16\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{8/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{8/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (bx + cx^2)^{8/3}} - \frac{\left(56\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{5/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{5c^2 (bx + cx^2)^{8/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (bx + cx^2)^{8/3}} + \frac{21(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{8/3}} - \frac{\left(14\sqrt[3]{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right)}{5c^2} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (bx + cx^2)^{8/3}} + \frac{21(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{8/3}} + \frac{\left(21\sqrt[3]{2} \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}\right) \sqrt{2 - \sqrt{3}}}{5c^2} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (bx + cx^2)^{8/3}} + \frac{21(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{8/3}} + \frac{14\sqrt[3]{2} 3^{3/4} \sqrt{2 - \sqrt{3}} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c^2}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.11

$$-\frac{3\left(\frac{cx}{b} + 1\right)^{2/3} {}_2F_1\left(-\frac{5}{3}, \frac{8}{3}; -\frac{2}{3}; -\frac{cx}{b}\right)}{5b^2x(x(b + cx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-8/3), x]

[Out] (-3*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[-5/3, 8/3, -2/3, -((c*x)/b)])/(5*b^2*x*(x*(b + c*x))^(2/3))

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^2 + bx)^{\frac{1}{3}}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(8/3), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(1/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(8/3), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-8/3), x)

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(8/3), x)

[Out] int(1/(c*x^2+b*x)^(8/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(8/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-8/3), x)

mupad [B] time = 0.27, size = 36, normalized size = 0.08

$$\frac{3x\left(\frac{cx}{b} + 1\right)^{8/3} {}_2F_1\left(-\frac{5}{3}, \frac{8}{3}; -\frac{2}{3}; -\frac{cx}{b}\right)}{5\left(cx^2 + bx\right)^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(8/3),x)

[Out] -(3*x*((c*x)/b + 1)^(8/3)*hypergeom([-5/3, 8/3], -2/3, -(c*x)/b))/(5*(b*x + c*x^2)^(8/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{8/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(8/3),x)

[Out] Integral((b*x + c*x**2)**(-8/3), x)

3.35 $\int (bx + cx^2)^{5/3} dx$

Optimal. Leaf size=842

$$15\sqrt[4]{3}\sqrt{2+\sqrt{3}}(cx^2+bx)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}}\right)\right)$$

$$364\sqrt[3]{2}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}$$

[Out] $15/364*(-c*x*(c*x+b)/b^2)^(2/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c/(-c*(c*x^2+b*x)/b^2)^(5/3)+3/26*(-c*x*(c*x+b)/b^2)^(5/3)*(2*c*x+b)*(c*x^2+b*x)^(5/3)/c/(-c*(c*x^2+b*x)/b^2)^(5/3)-15/364*(2*c*x+b)*(c*x^2+b*x)^(5/3)*2^(2/3)/c/(-c*(c*x^2+b*x)/b^2)^(5/3)/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2)+5/182*3^(3/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticF((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2))^2)^(1/2)*2^(1/6)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2))^2)^(1/2)-15/728*3^(1/4)*b^2*(c*x^2+b*x)^(5/3)*(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))*EllipticE((1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3)+2*2^(1/3)*(-c*x*(c*x+b)/b^2)^(2/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*2^(2/3)/c/(2*c*x+b)/(-c*(c*x^2+b*x)/b^2)^(5/3)/((-1+2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))/(1-2^(2/3)*(-c*x*(c*x+b)/b^2)^(1/3))-3^(1/2))^2)^(1/2)$

Rubi [A] time = 1.06, antiderivative size = 842, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {622, 619, 195, 235, 304, 219, 1879}

$$15\sqrt[4]{3}\sqrt{2+\sqrt{3}}(cx^2+bx)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}}\right)\right)$$

$$364\sqrt[3]{2}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(5/3),x]

[Out]
$$\frac{15 \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{2/3} (b + 2 c x) (b x + c x^2)^{5/3} \right)}{364 c \left(-\left(\frac{c (b x + c x^2)}{b^2} \right)^{5/3} \right) + 3 \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{5/3} (b + 2 c x) (b x + c x^2)^{5/3} \right)}{(26 c \left(-\left(\frac{c (b x + c x^2)}{b^2} \right)^{5/3} \right) - (15 (b + 2 c x) (b x + c x^2)^{5/3}) / (182 2^{1/3} c \left(-\left(\frac{c (b x + c x^2)}{b^2} \right)^{5/3} \right) * (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right)) - (15 3^{1/4} \sqrt{2 + \sqrt{3}}) b^2 (b x + c x^2)^{5/3} (1 - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) \sqrt{(1 + 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) + 2 2^{1/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{2/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))^{1/3})^2} \right) * \text{EllipticE}[\text{ArcSin}[(1 + \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))] / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))], -7 + 4 \sqrt{3}} \right) / (364 2^{1/3} c (b + 2 c x) \left(-\left(\frac{c (b x + c x^2)}{b^2} \right)^{5/3} \sqrt{-(1 - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))^{1/3})^2} \right) + (5 3^{3/4} b^2 (b x + c x^2)^{5/3} (1 - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) \sqrt{(1 + 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) + 2 2^{1/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{2/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))^{1/3})^2} \right) * \text{EllipticF}[\text{ArcSin}[(1 + \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))] / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))], -7 + 4 \sqrt{3}} \right) / (91 2^{5/6} c (b + 2 c x) \left(-\left(\frac{c (b x + c x^2)}{b^2} \right)^{5/3} \sqrt{-(1 - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\left(\frac{c x (b + c x)}{b^2} \right)^{1/3} \right))^{1/3})^2} \right)$$

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 219

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*sqrt[2 - sqrt[3]]*(s + r*x)*sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + sqrt[3])*s + r*x)/((1 - sqrt[3])*s + r*x)]], -7 + 4*sqrt[3]])/(3^(1/4)*r*sqrt[a + b*x^3]*sqrt[-((s*(s + r*x))/((1 - sqrt[3])*s + r*x)^2)]), x] /; FreeQ[{a, b}, x] && NegQ[a]

Rule 235

Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] := Dist[(3*sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-
(c*(b*x + c*x^2))/b^2)^(p), Int[-((c*x)/b) - (c^2*x^2)/b^2)^(p), x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int (bx + cx^2)^{5/3} dx &= \frac{(bx + cx^2)^{5/3} \int \left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{5/3} dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{\left(b^2 (bx + cx^2)^{5/3}\right) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{5/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{16\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b + 2cx) (bx + cx^2)^{5/3}}{26c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} - \frac{\left(5b^2 (bx + cx^2)^{5/3}\right) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{2/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{104\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{15 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{5/3}}{364c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b + 2cx) (bx + cx^2)^{5/3}}{26c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} - \frac{\left(5b^2 (bx + cx^2)^{5/3}\right) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{1/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{104\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{15 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{5/3}}{364c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b + 2cx) (bx + cx^2)^{5/3}}{26c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{\left(15 (bx + cx^2)^{5/3}\right) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{1/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{104\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{15 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{5/3}}{364c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b + 2cx) (bx + cx^2)^{5/3}}{26c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} - \frac{\left(15 (bx + cx^2)^{5/3}\right) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{1/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{104\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} \\
&= \frac{15 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{5/3}}{364c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{5/3} (b + 2cx) (bx + cx^2)^{5/3}}{26c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}} + \frac{15 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (bx + cx^2)^{5/3}}{182\sqrt[3]{2} c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.06

$$\frac{3bx^2(x(b+cx))^{2/3} {}_2F_1\left(-\frac{5}{3}, \frac{8}{3}; \frac{11}{3}; -\frac{cx}{b}\right)}{8\left(\frac{cx}{b} + 1\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(5/3), x]

[Out] (3*b*x^2*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-5/3, 8/3, 11/3, -((c*x)/b)])/(8*(1 + (c*x)/b)^(2/3))

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/3), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(5/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/3), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(5/3), x)

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(5/3), x)

[Out] int((c*x^2+b*x)^(5/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(5/3), x)

mupad [B] time = 0.18, size = 36, normalized size = 0.04

$$\frac{3x(c x^2 + b x)^{5/3} {}_2F_1\left(-\frac{5}{3}, \frac{8}{3}; \frac{11}{3}; -\frac{c x}{b}\right)}{8\left(\frac{c x}{b} + 1\right)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(5/3),x)

[Out] (3*x*(b*x + c*x^2)^(5/3)*hypergeom([-5/3, 8/3], 11/3, -(c*x)/b))/(8*((c*x)/b + 1)^(5/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(5/3),x)

[Out] Integral((b*x + c*x**2)**(5/3), x)

3.36 $\int (bx + cx^2)^{2/3} dx$

Optimal. Leaf size=781

$$\frac{3 \left(-\frac{cx(b+cx)}{b^2} \right)^{2/3} (b+2cx) (bx+cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{2/3}} - \frac{3(b+2cx) (bx+cx^2)^{2/3}}{7\sqrt[3]{2}c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{2/3} \left(-2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}} - \sqrt{3} + 1 \right)} + \frac{\sqrt[6]{2} 3^{3/4} b^2 (bx+cx^2)^2}{}$$

[Out] $\frac{3}{14} * (-c*x*(c*x+b)/b^2)^{(2/3)} * (2*c*x+b) * (c*x^2+b*x)^{(2/3)} / c / (-c*(c*x^2+b*x)/b^2)^{(2/3)} - \frac{3}{14} * (2*c*x+b) * (c*x^2+b*x)^{(2/3)} * 2^{(2/3)} / c / (-c*(c*x^2+b*x)/b^2)^{(2/3)} / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)}) + \frac{1}{7} * 2^{(1/6)} * 3^{(3/4)} * b^2 * (c*x^2+b*x)^{(2/3)} * (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) * \text{EllipticF}((1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 3^{(1/2)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)}), 2 * I - I * 3^{(1/2)}) * ((1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 2 * 2^{(1/3)} * (-c*x*(c*x+b)/b^2)^{(2/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)} / c / (2*c*x+b) / (-c*(c*x^2+b*x)/b^2)^{(2/3)} / ((-1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)} - \frac{3}{28} * 3^{(1/4)} * b^2 * (c*x^2+b*x)^{(2/3)} * (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) * \text{EllipticE}((1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 3^{(1/2)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)}), 2 * I - I * 3^{(1/2)}) * ((1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} + 2 * 2^{(1/3)} * (-c*x*(c*x+b)/b^2)^{(2/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)} * (1/2 * 6^{(1/2)} + 1/2 * 2^{(1/2)}) * 2^{(2/3)} / c / (2*c*x+b) / (-c*(c*x^2+b*x)/b^2)^{(2/3)} / ((-1+2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)}) / (1-2^{(2/3)} * (-c*x*(c*x+b)/b^2)^{(1/3)} - 3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.95, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {622, 619, 195, 235, 304, 219, 1879}

$$\frac{3 \left(-\frac{cx(b+cx)}{b^2} \right)^{2/3} (b+2cx) (bx+cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{2/3}} - \frac{3(b+2cx) (bx+cx^2)^{2/3}}{7\sqrt[3]{2}c \left(-\frac{c(bx+cx^2)}{b^2} \right)^{2/3} \left(-2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}} - \sqrt{3} + 1 \right)} + \frac{\sqrt[6]{2} 3^{3/4} b^2 (bx+cx^2)^2}{}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(2/3), x]

[Out]
$$\begin{aligned} & (3*((c*x*(b + c*x))/b^2))^{(2/3)}*(b + 2*c*x)*(b*x + c*x^2)^{(2/3)}/(14*c*((c*(b*x + c*x^2))/b^2))^{(2/3)} - (3*(b + 2*c*x)*(b*x + c*x^2)^{(2/3)})/(7*2^{(1/3)}*c*((c*(b*x + c*x^2))/b^2))^{(2/3)}*(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)}) - (3*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*b^2*(b*x + c*x^2)^{(2/3)}*(1 - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})*\text{Sqrt}[(1 + 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)} + 2*2^{(1/3)}*((c*x*(b + c*x))/b^2))^{(2/3)}]/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})^2*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)}], -7 + 4*\text{Sqrt}[3]]/(14*2^{(1/3)}*c*(b + 2*c*x)*((c*(b*x + c*x^2))/b^2))^{(2/3)}*\text{Sqrt}[-((1 - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})^2] + (2^{(1/6)}*3^{(3/4)}*b^2*(b*x + c*x^2)^{(2/3)}*(1 - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})*\text{Sqrt}[(1 + 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)} + 2*2^{(1/3)}*((c*x*(b + c*x))/b^2))^{(2/3)}]/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})^2*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)}], -7 + 4*\text{Sqrt}[3]]/(7*c*(b + 2*c*x)*((c*(b*x + c*x^2))/b^2))^{(2/3)}*\text{Sqrt}[-((1 - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})/(1 - \text{Sqrt}[3] - 2^{(2/3)}*((c*x*(b + c*x))/b^2))^{(1/3)})^2]) \end{aligned}$$

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 219

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)]], -7 + 4*Sqrt[3])]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]

Rule 235

Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[-((c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int (bx + cx^2)^{2/3} dx &= \frac{(bx + cx^2)^{2/3} \int \left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{2/3} dx}{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} \\
&= \frac{(b^2 (bx + cx^2)^{2/3}) \text{Subst} \left(\int \left(1 - \frac{b^2x^2}{c^2}\right)^{2/3} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{4\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} - \frac{(b^2 (bx + cx^2)^{2/3}) \text{Subst} \left(\int \frac{1}{\sqrt[3]{1 - \frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{7\sqrt[3]{2} c^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} + \frac{\left(3 (bx + cx^2)^{2/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{14\sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} - \frac{\left(3 (bx + cx^2)^{2/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{14\sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} \\
&= \frac{3 \left(-\frac{cx(b+cx)}{b^2}\right)^{2/3} (b + 2cx) (bx + cx^2)^{2/3}}{14c \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} + \frac{3b^2 (bx + cx^2)^{2/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}}{7\sqrt[3]{2} c(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3} \left(1 - \sqrt{3} - 2^{2/3} \sqrt[3]{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right)}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.06

$$\frac{3x(x(b + cx))^{2/3} {}_2F_1\left(-\frac{2}{3}, \frac{5}{3}, \frac{8}{3}, -\frac{cx}{b}\right)}{5\left(\frac{cx}{b} + 1\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(2/3),x]

[Out] (3*x*(x*(b + c*x))^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((c*x)/b)])/(5*(1 + (c*x)/b)^(2/3))

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(2/3), x)

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(2/3),x)

[Out] int((c*x^2+b*x)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(2/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(2/3), x)

mupad [B] time = 0.16, size = 36, normalized size = 0.05

$$\frac{3x(c x^2 + b x)^{2/3} {}_2F_1\left(-\frac{2}{3}, \frac{5}{3}; \frac{8}{3}; -\frac{c x}{b}\right)}{5\left(\frac{c x}{b} + 1\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(2/3), x)

[Out] (3*x*(b*x + c*x^2)^(2/3)*hypergeom([-2/3, 5/3], 8/3, -(c*x)/b))/(5*((c*x)/b + 1)^(2/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(2/3), x)

[Out] Integral((b*x + c*x**2)**(2/3), x)

$$3.37 \quad \int \frac{1}{\sqrt[3]{bx+cx^2}} dx$$

Optimal. Leaf size=715

$$\frac{3(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)} + \frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}$$

[Out] $-3/2*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^{(1/3)}*2^{(2/3)}/c/(c*x^2+b*x)^{(1/3)}/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})+2^{(1/6)}*3^{(3/4)}*b^2*(-c*(c*x^2+b*x)/b^2)^{(1/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*EllipticF((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}/c/(2*c*x+b)/(c*x^2+b*x)^{(1/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}-3/4*3^{(1/4)}*b^2*(-c*(c*x^2+b*x)/b^2)^{(1/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*EllipticE((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*2^{(2/3)}/c/(2*c*x+b)/(c*x^2+b*x)^{(1/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.86, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {622, 619, 235, 304, 219, 1879}

$$\frac{3(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)} + \frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-1/3), x]

```
[Out] (-3*(b + 2*c*x)*(-(c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x + c*x^2)^(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-(c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-(c*x*(b + c*x))/b^2))^(2/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)], -7 + 4*Sqrt[3]]]/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2]) + (2^(1/6)*3^(3/4)*b^2*(-(c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)*Sqrt[(1 + 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-(c*x*(b + c*x))/b^2))^(2/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)], -7 + 4*Sqrt[3]]]/(c*(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-(c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{bx+cx^2}} dx &= \frac{\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}} \int \frac{1}{\sqrt[3]{-\frac{cx}{b} - \frac{c^2x^2}{b^2}}} dx}{\sqrt[3]{bx+cx^2}} \\
&= \frac{\left(b^2 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\right) \text{Subst} \left(\int \frac{1}{\sqrt[3]{1-\frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{\sqrt[3]{2} c^2 \sqrt[3]{bx+cx^2}} \\
&= \frac{\left(3 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{x}{\sqrt{-1+x^3}} dx, x, 2^{2/3} \sqrt[3]{-\frac{cx(1+\frac{cx}{b})}{b}} \right)}{2 \sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) \sqrt[3]{bx+cx^2}} \\
&= \frac{\left(3 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx, x, 2^{2/3} \sqrt[3]{-\frac{cx(1+\frac{cx}{b})}{b}} \right)}{2 \sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) \sqrt[3]{bx+cx^2}} + \frac{\left(3 \sqrt{2+\sqrt{3}}\right)}{\sqrt[3]{bx+cx^2}} \\
&= \frac{3b^2 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \sqrt{-1 - \frac{4cx(b+cx)}{b^2}}}{\sqrt[3]{2} c(b+2cx) \sqrt[3]{bx+cx^2} \left(1 - \sqrt{3} - 2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} - \frac{3^4 \sqrt{2+\sqrt{3}} b^2 \sqrt[3]{-\frac{c(bx+cx^2)}{b^2}} \sqrt{-1 - \frac{4cx(b+cx)}{b^2}}}{\sqrt[3]{bx+cx^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.06

$$\frac{3x \sqrt[3]{\frac{cx}{b} + 1} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{cx}{b}\right)}{2 \sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-1/3), x]

[Out] (3*x*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*x)/b)])/(2*(x*(b + c*x))^(1/3))

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-1/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-1/3), x)

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(1/3),x)

[Out] int(1/(c*x^2+b*x)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-1/3), x)

mupad [B] time = 0.20, size = 36, normalized size = 0.05

$$\frac{3x\left(\frac{cx}{b} + 1\right)^{1/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{cx}{b}\right)}{2\left(cx^2 + bx\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(1/3),x)

[Out] (3*x*((c*x)/b + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -(c*x)/b))/(2*(b*x + c*x^2)^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(1/3),x)

[Out] Integral((b*x + c*x**2)**(-1/3), x)

$$3.38 \quad \int \frac{1}{(bx+cx^2)^{4/3}} dx$$

Optimal. Leaf size=773

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(bx+cx^2)^{4/3}} + \frac{3 \cdot 2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c(bx+cx^2)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}} - \sqrt{3} + 1\right)} - \frac{2\sqrt[6]{2} \cdot 3^{3/4} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \left(1 - 2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}{c(bx+cx^2)^{4/3}}$$

[Out] $3*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^{(4/3)}/c/(-c*x*(c*x+b)/b^2)^{(1/3)}/(c*x^2+b*x)^{(4/3)}+3*2^{(2/3)}*(2*c*x+b)*(-c*(c*x^2+b*x)/b^2)^{(4/3)}/c/(c*x^2+b*x)^{(4/3)}/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})-2*2^{(1/6)}*3^{(3/4)}*b^2*(-c*(c*x^2+b*x)/b^2)^{(4/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*EllipticF((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}/c/(2*c*x+b)/(c*x^2+b*x)^{(4/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}+3/2*3^{(1/4)}*b^2*(-c*(c*x^2+b*x)/b^2)^{(4/3)}*(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})*EllipticE((1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+3^{(1/2)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)}),2*I-I*3^{(1/2)})*((1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}+2*2^{(1/3)}*(-c*x*(c*x+b)/b^2)^{(2/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*2^{(2/3)}/c/(2*c*x+b)/(c*x^2+b*x)^{(4/3)}/((-1+2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)})/(1-2^{(2/3)}*(-c*x*(c*x+b)/b^2)^{(1/3)}-3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.95, antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {622, 619, 199, 235, 304, 219, 1879}

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(bx+cx^2)^{4/3}} + \frac{3 \cdot 2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c(bx+cx^2)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}} - \sqrt{3} + 1\right)} - \frac{2\sqrt[6]{2} \cdot 3^{3/4} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \left(1 - 2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)}{c(bx+cx^2)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-4/3), x]

[Out] (3*(b + 2*c*x)*(-(c*(b*x + c*x^2))/b^2))^(4/3)/(c*(-(c*x*(b + c*x))/b^2))^(1/3)*(b*x + c*x^2)^(4/3) + (3*2^(2/3)*(b + 2*c*x)*(-(c*(b*x + c*x^2))/b^2))^(4/3)/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-(c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)*Sqrt[(1 + 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-(c*x*(b + c*x))/b^2))^(2/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)], -7 + 4*Sqrt[3]]/(2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2]) - (2*2^(1/6)*3^(3/4)*b^2*(-(c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)*Sqrt[(1 + 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-(c*x*(b + c*x))/b^2))^(2/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)]/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3)], -7 + 4*Sqrt[3]]/(c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-(c*x*(b + c*x))/b^2))^(1/3))^2])]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 219

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[3])*s + r*x)]], -7 + 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]

Rule 235

Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] := Dist[(3*Sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}, x]

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[-((c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2]))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{4/3}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \int \frac{1}{\left(\frac{-cx}{b} - \frac{c^2x^2}{b^2}\right)^{4/3}} dx}{(bx + cx^2)^{4/3}} \\
&= \frac{\left(2^{2/3} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{4/3}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{4/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c^3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{4/3}} + \frac{\left(2^{2/3} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\right) \text{Subst} \left(\int \frac{1}{\sqrt[3]{1 - \frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2} \right)}{c^2 (bx + cx^2)^{4/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c^3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{4/3}} - \frac{\left(3 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{x}{\sqrt{-1+x^3}} dx, x, \sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{4/3} \right)}{\sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{4/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c^3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{4/3}} + \frac{\left(3 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}}\right) \text{Subst} \left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}} dx, x, \sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{4/3} \right)}{\sqrt[3]{2} \left(-\frac{c}{b} - \frac{2c^2x}{b^2}\right) (bx + cx^2)^{4/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c^3 \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{4/3}} - \frac{3 \cdot 2^{2/3} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3} \sqrt{-1 - \frac{4cx}{b} - \frac{4c^2x^2}{b^2}} \sqrt{-1 - \frac{4cx(b+cx)}{b^2}}}{c(b + 2cx) (bx + cx^2)^{4/3} \left(1 - \sqrt{3} - 2^{2/3} \sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)} +
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.06

$$\frac{3 \sqrt[3]{\frac{cx}{b}} + 1}{b \sqrt[3]{x(b+cx)}} {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{2}{3}; -\frac{cx}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-4/3), x]

[Out] $(-3*(1 + (c*x)/b)^{(1/3)}*Hypergeometric2F1[-1/3, 4/3, 2/3, -((c*x)/b)])/(b*(x*(b + c*x))^{(1/3)})$

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^2 + bx)^{\frac{2}{3}}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(4/3), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(2/3)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(4/3), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-4/3), x)

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(4/3), x)

[Out] int(1/(c*x^2+b*x)^(4/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(4/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-4/3), x)

mupad [B] time = 0.23, size = 36, normalized size = 0.05

$$\frac{3x\left(\frac{cx}{b} + 1\right)^{4/3} {}_2F_1\left(-\frac{1}{3}, \frac{4}{3}; \frac{2}{3}; -\frac{cx}{b}\right)}{(cx^2 + bx)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(4/3),x)

[Out] -(3*x*((c*x)/b + 1)^(4/3)*hypergeom([-1/3, 4/3], 2/3, -(c*x)/b))/(b*x + c*x^2)^(4/3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(4/3),x)

[Out] Integral((b*x + c*x**2)**(-4/3), x)

$$3.39 \quad \int \frac{1}{(bx+cx^2)^{7/3}} dx$$

Optimal. Leaf size=838

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right)-7+4}{2\sqrt[3]{2}c(b+2cx)(cx^2+bx)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

[Out] $\frac{3}{4}(2cx+b)(-c(cx^2+bx)/b^2)^{7/3}/c/(-c(cx^2+bx)/b^2)^{4/3}/(cx^2+bx)^{7/3}+15/2(2cx+b)(-c(cx^2+bx)/b^2)^{7/3}/c/(-c(cx^2+bx)/b^2)^{1/3}/(cx^2+bx)^{7/3}+15/2(2cx+b)(-c(cx^2+bx)/b^2)^{7/3}2^{2/3}/c/(cx^2+bx)^{7/3}/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2})-5*2^{1/6}*3^{3/4}*b^2*(-c(cx^2+bx)/b^2)^{7/3}*(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}*E\text{llipticF}((1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}+3^{1/2})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2}),2*I-I*3^{1/2})*((1+2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}+2*2^{1/3})(-c(cx^2+bx)/b^2)^{2/3})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2})^2)^{1/2}/c/(2cx+b)/(cx^2+bx)^{7/3}/((-1+2^{2/3})(-c(cx^2+bx)/b^2)^{1/3})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2})^2)^{1/2}+15/4*3^{1/4}*b^2*(-c(cx^2+bx)/b^2)^{7/3}*(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}*E\text{llipticE}((1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}+3^{1/2})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2}),2*I-I*3^{1/2})*((1+2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}+2*2^{1/3})(-c(cx^2+bx)/b^2)^{2/3})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2})^2)^{1/2}*(1/2*6^{1/2}+1/2*2^{1/2})*2^{2/3}/c/(2cx+b)/(cx^2+bx)^{7/3}/((-1+2^{2/3})(-c(cx^2+bx)/b^2)^{1/3})/(1-2^{2/3})(-c(cx^2+bx)/b^2)^{1/3}-3^{1/2})^2)^{1/2}$

Rubi [A] time = 1.04, antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {622, 619, 199, 235, 304, 219, 1879}

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right)-7+4}{2\sqrt[3]{2}c(b+2cx)(cx^2+bx)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-7/3), x]

[Out]
$$\frac{3(b + 2cx) \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3}}{4c \left(-\frac{c(bx + cx^2)}{b^2} \right)^{4/3} (bx + cx^2)^{7/3}} + \frac{15(b + 2cx) \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3}}{(2c \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} (bx + cx^2)^{7/3})} + \frac{15(b + 2cx) \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3}}{(2^{1/3} c (bx + cx^2)^{7/3} (1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}))} + \frac{15 \cdot 3^{1/4} \sqrt{2 + \sqrt{3}} b^2 \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3} (1 - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}) \sqrt{(1 + 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} + 2 \cdot 2^{1/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{2/3})}}{(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3})^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}\right], -7 + 4\sqrt{3}\right] \right. \\ \left. - \frac{5 \cdot 2^{1/6} \cdot 3^{3/4} b^2 \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3} (1 - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}) \sqrt{(1 + 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} + 2 \cdot 2^{1/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{2/3})}}{(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3})^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}\right], -7 + 4\sqrt{3}\right] \right. \\ \left. \right) / (c(b + 2cx)(bx + cx^2)^{7/3} \sqrt{\left(1 - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3})^2}} - \frac{(5 \cdot 2^{1/6} \cdot 3^{3/4} b^2 \left(-\frac{c(bx + cx^2)}{b^2} \right)^{7/3} (1 - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}) \sqrt{(1 + 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} + 2 \cdot 2^{1/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{2/3})}}{(1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3})^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}{1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3}}\right], -7 + 4\sqrt{3}\right] \right) / (c(b + 2cx)(bx + cx^2)^{7/3} \sqrt{\left(1 - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3} \right) / (1 - \sqrt{3} - 2^{2/3} \left(-\frac{c(bx + cx^2)}{b^2} \right)^{1/3})^2}} \right)$$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 219

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*sqrt[2 - sqrt[3]]*(s + r*x)*sqrt[(s^2 - r*s*x + r^2*x^2)/((1 - sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + sqrt[3])*s + r*x)/((1 - sqrt[3])*s + r*x)]], -7 + 4*sqrt[3]])/(3^(1/4)*r*sqrt[a + b*x^3]*sqrt[-((s*(s + r*x))/((1 - sqrt[3])*s + r*x)^2))], x] /; FreeQ[{a, b}, x] && NegQ[a]

Rule 235

Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] := Dist[(3*sqrt[b*x^2])/(2*b*x), Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}

, x]

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-
(c*(b*x + c*x^2))/b^2)^(p, Int[(-(c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2))], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx + cx^2)^{7/3}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3} \int \frac{1}{\left(\frac{-cx}{b} - \frac{cx^2}{b^2}\right)^{7/3}} dx}{(bx + cx^2)^{7/3}} \\
&= \frac{\left(8 \cdot 2^{2/3} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2 x^2}{c^2}\right)^{7/3}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2} \right)}{c^2 (bx + cx^2)^{7/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (bx + cx^2)^{7/3}} - \frac{\left(5 \cdot 2^{2/3} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2 x^2}{c^2}\right)^{4/3}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2} \right)}{c^2 (bx + cx^2)^{7/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (bx + cx^2)^{7/3}} + \frac{15(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{7/3}} + \frac{\left(5b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}\right) \text{Subst} \left(\int \frac{1}{\left(1 - \frac{b^2 x^2}{c^2}\right)^{3/3}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2} \right)}{\sqrt[3]{2} c^2 (bx + cx^2)^{7/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (bx + cx^2)^{7/3}} + \frac{15(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{7/3}} - \frac{\left(15 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3} \sqrt{-1 - \frac{4c^2 x}{b}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{b^2 x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2} \right)}{2 \sqrt[3]{2} c^2 (bx + cx^2)^{7/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (bx + cx^2)^{7/3}} + \frac{15(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{7/3}} + \frac{\left(15 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3} \sqrt{-1 - \frac{4c^2 x}{b}}\right) \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{b^2 x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2} \right)}{2 \sqrt[3]{2} c^2 (bx + cx^2)^{7/3}} \\
&= \frac{3(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{4c \left(-\frac{cx(b+cx)}{b^2}\right)^{4/3} (bx + cx^2)^{7/3}} + \frac{15(b + 2cx) \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3}}{2c \sqrt[3]{-\frac{cx(b+cx)}{b^2}} (bx + cx^2)^{7/3}} - \frac{15b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{7/3} \sqrt{-1 - \frac{4c^2 x}{b}}}{\sqrt[3]{2} c (b + 2cx) (bx + cx^2)^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.06

$$\frac{3\sqrt[3]{\frac{cx}{b} + 1} {}_2F_1\left(-\frac{4}{3}, \frac{7}{3}; -\frac{1}{3}; -\frac{cx}{b}\right)}{4b^2x\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-7/3), x]

[Out] (-3*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[-4/3, 7/3, -1/3, -((c*x)/b)])/(4*b^2*x*(x*(b + c*x))^(1/3))

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^2 + bx)^{\frac{2}{3}}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(7/3), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(2/3)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(7/3), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-7/3), x)

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(7/3), x)

[Out] `int(1/(c*x^2+b*x)^(7/3),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x)^(7/3),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x)^(-7/3), x)`

mupad [B] time = 0.25, size = 36, normalized size = 0.04

$$-\frac{3x\left(\frac{cx}{b} + 1\right)^{7/3} {}_2F_1\left(-\frac{4}{3}, \frac{7}{3}; -\frac{1}{3}; -\frac{cx}{b}\right)}{4(cx^2 + bx)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x + c*x^2)^(7/3),x)`

[Out] `-(3*x*((c*x)/b + 1)^(7/3)*hypergeom([-4/3, 7/3], -1/3, -(c*x)/b))/(4*(b*x + c*x^2)^(7/3))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x)**(7/3),x)`

[Out] `Integral((b*x + c*x**2)**(-7/3), x)`

3.40 $\int (bx + cx^2)^{5/4} dx$

Optimal. Leaf size=119

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}c^3(bx+cx^2)^{3/4}} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c}$$

[Out] $-5/84*b^2*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c^2+1/7*(2*c*x+b)*(c*x^2+b*x)^(5/4)/c+5/168*b^5*(-c*(c*x^2+b*x)/b^2)^(3/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticF(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))/c^3/(c*x^2+b*x)^(3/4)*2^(1/2)$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {612, 622, 619, 232}

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2} + \frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}c^3(bx+cx^2)^{3/4}} + \frac{(b+2cx)(bx+cx^2)^{5/4}}{7c}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(5/4), x]

[Out] $(-5*b^2*(b+2*c*x)*(b*x+c*x^2)^(1/4))/(84*c^2) + ((b+2*c*x)*(b*x+c*x^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x+c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1+(2*c*x)/b]/2, 2])/(84*sqrt[2]*c^3*(b*x+c*x^2)^(3/4))$

Rule 232

Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] := Simp[(2*EllipticF[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^(p, Int[(-(c*x)/b) - (c^2*x^2)/b^2)^(p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
 \int (bx + cx^2)^{5/4} dx &= \frac{(b + 2cx)(bx + cx^2)^{5/4}}{7c} - \frac{(5b^2) \int \sqrt[4]{bx + cx^2} dx}{28c} \\
 &= -\frac{5b^2(b + 2cx)\sqrt[4]{bx + cx^2}}{84c^2} + \frac{(b + 2cx)(bx + cx^2)^{5/4}}{7c} + \frac{(5b^4) \int \frac{1}{(bx+cx^2)^{3/4}} dx}{336c^2} \\
 &= -\frac{5b^2(b + 2cx)\sqrt[4]{bx + cx^2}}{84c^2} + \frac{(b + 2cx)(bx + cx^2)^{5/4}}{7c} + \frac{\left(5b^4 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right) \int \frac{1}{\left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{3/4}}}{336c^2 (bx + cx^2)^{3/4}} \\
 &= -\frac{5b^2(b + 2cx)\sqrt[4]{bx + cx^2}}{84c^2} + \frac{(b + 2cx)(bx + cx^2)^{5/4}}{7c} - \frac{\left(5b^6 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{b^2}{c}\right)}\right)}{168\sqrt{2}c^4 (bx + cx^2)^{3/4}} \\
 &= -\frac{5b^2(b + 2cx)\sqrt[4]{bx + cx^2}}{84c^2} + \frac{(b + 2cx)(bx + cx^2)^{5/4}}{7c} + \frac{5b^5 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2}{\dots}\right)\right)}{84\sqrt{2}c^3 (bx + cx^2)^{3/4}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.40

$$\frac{4bx^2 \sqrt[4]{x(b+cx)} {}_2F_1\left(-\frac{5}{4}, \frac{9}{4}; \frac{13}{4}; -\frac{cx}{b}\right)}{9 \sqrt[4]{\frac{cx}{b} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(5/4), x]

[Out] $(4*b*x^2*(x*(b + c*x))^{1/4}*Hypergeometric2F1[-5/4, 9/4, 13/4, -((c*x)/b)])/(9*(1 + (c*x)/b)^{1/4})$

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/4), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(5/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/4), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(5/4), x)

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(5/4), x)

[Out] int((c*x^2+b*x)^(5/4), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(5/4), x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(5/4), x)

mupad [B] time = 0.19, size = 36, normalized size = 0.30

$$\frac{4x(c x^2 + b x)^{5/4} {}_2F_1\left(-\frac{5}{4}, \frac{9}{4}; \frac{13}{4}; -\frac{c x}{b}\right)}{9\left(\frac{c x}{b} + 1\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(5/4), x)

[Out] (4*x*(b*x + c*x^2)^(5/4)*hypergeom([-5/4, 9/4], 13/4, -(c*x)/b))/(9*((c*x)/b + 1)^(5/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(5/4), x)

[Out] Integral((b*x + c*x**2)**(5/4), x)

3.41 $\int (bx + cx^2)^{3/4} dx$

Optimal. Leaf size=90

$$\frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} - \frac{3b^3 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right)\right) 2}{10\sqrt{2} c^2 \sqrt[4]{bx + cx^2}}$$

[Out] 1/5*(2*c*x+b)*(c*x^2+b*x)^(3/4)/c-3/20*b^3*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b))^2)^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(1/4)*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {612, 622, 619, 228}

$$\frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} - \frac{3b^3 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right)\right) 2}{10\sqrt{2} c^2 \sqrt[4]{bx + cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(3/4), x]

[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*sqrt[2]*c^2*(b*x + c*x^2)^(1/4))

Rule 228

Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] := Simp[(2*EllipticE[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b

+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 622

Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^(p), Int[(-(c*x)/b) - (c^2*x^2)/b^2)^(p), x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rubi steps

$$\begin{aligned}
 \int (bx + cx^2)^{3/4} dx &= \frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} - \frac{(3b^2) \int \frac{1}{\sqrt[4]{bx+cx^2}} dx}{20c} \\
 &= \frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} - \frac{\left(3b^2 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right) \int \frac{1}{\sqrt[4]{-\frac{cx}{b} - \frac{c^2x^2}{b^2}}} dx}{20c \sqrt[4]{bx + cx^2}} \\
 &= \frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} + \frac{\left(3b^4 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[4]{1 - \frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{20\sqrt{2}c^3 \sqrt[4]{bx + cx^2}} \\
 &= \frac{(b + 2cx)(bx + cx^2)^{3/4}}{5c} - \frac{3b^3 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right)\right) \Big|_2}{10\sqrt{2}c^2 \sqrt[4]{bx + cx^2}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.50

$$\frac{4x(x(b + cx))^{3/4} {}_2F_1\left(-\frac{3}{4}, \frac{7}{4}; \frac{11}{4}; -\frac{cx}{b}\right)}{7\left(\frac{cx}{b} + 1\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(3/4), x]

[Out] (4*x*(x*(b + c*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -(c*x)/b])/(7*(1 + (c*x)/b)^(3/4))

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(3/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(3/4), x)

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^(3/4),x)

[Out] int((c*x^2+b*x)^(3/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^(3/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(3/4), x)

mupad [B] time = 0.17, size = 36, normalized size = 0.40

$$\frac{4x (cx^2 + bx)^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{4}, \frac{7}{4}; \frac{11}{4}; -\frac{cx}{b}\right)}{7\left(\frac{cx}{b} + 1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(3/4),x)

[Out] $(4*x*(b*x + c*x^2)^{3/4}*\text{hypergeom}([-3/4, 7/4], 11/4, -(c*x)/b))/(7*((c*x)/b + 1)^{3/4})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(3/4),x)

[Out] Integral((b*x + c*x**2)**(3/4), x)

3.42 $\int \sqrt[4]{bx + cx^2} dx$

Optimal. Leaf size=90

$$\frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} - \frac{b^3 \left(-\frac{c(bx+cx^2)}{b^2} \right)^{3/4} F\left(\frac{1}{2} \sin^{-1} \left(\frac{2cx}{b} + 1 \right) \middle| 2 \right)}{3\sqrt{2} c^2 (bx + cx^2)^{3/4}}$$

[Out] $1/3*(2*c*x+b)*(c*x^2+b*x)^(1/4)/c-1/6*b^3*(-c*(c*x^2+b*x)/b^2)^(3/4)*(\cos(1/2*\arcsin(1+2*c*x/b))^2)^(1/2)/\cos(1/2*\arcsin(1+2*c*x/b))*\text{EllipticF}(\sin(1/2*\arcsin(1+2*c*x/b)),2^(1/2))/c^2/(c*x^2+b*x)^(3/4)*2^(1/2)$

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {612, 622, 619, 232}

$$\frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} - \frac{b^3 \left(-\frac{c(bx+cx^2)}{b^2} \right)^{3/4} F\left(\frac{1}{2} \sin^{-1} \left(\frac{2cx}{b} + 1 \right) \middle| 2 \right)}{3\sqrt{2} c^2 (bx + cx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(1/4), x]

[Out] $((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2))/b^2))^(3/4)*\text{EllipticF}[\text{ArcSin}[1 + (2*c*x)/b]/2, 2])/(3*\text{Sqrt}[2]*c^2*(b*x + c*x^2)^(3/4))$

Rule 232

Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] := Simp[(2*EllipticF[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^(p, Int[(-(c*x)/b) - (c^2*x^2)/b^2)^(p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
 \int \sqrt[4]{bx + cx^2} dx &= \frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} - \frac{b^2 \int \frac{1}{(bx+cx^2)^{3/4}} dx}{12c} \\
 &= \frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} - \frac{\left(b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right) \int \frac{1}{\left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{3/4}} dx}{12c (bx + cx^2)^{3/4}} \\
 &= \frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} + \frac{\left(b^4 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{3/4}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{6\sqrt{2} c^3 (bx + cx^2)^{3/4}} \\
 &= \frac{(b + 2cx)\sqrt[4]{bx + cx^2}}{3c} - \frac{b^3 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right) \middle| 2\right)}{3\sqrt{2} c^2 (bx + cx^2)^{3/4}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.50

$$\frac{4x \sqrt[4]{x(b + cx)} {}_2F_1\left(-\frac{1}{4}, \frac{5}{4}; \frac{9}{4}; -\frac{cx}{b}\right)}{5 \sqrt[4]{\frac{cx}{b} + 1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(1/4), x]
```

[Out] $(4*x*(x*(b + c*x))^{1/4}*\text{Hypergeometric2F1}[-1/4, 5/4, 9/4, -((c*x)/b)])/(5*(1 + (c*x)/b)^{1/4})$

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)^(1/4),x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x)^(1/4), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)^(1/4),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x)^(1/4), x)`

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x)^(1/4),x)`

[Out] `int((c*x^2+b*x)^(1/4),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)^(1/4),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x)^(1/4), x)`

mupad [B] time = 0.17, size = 36, normalized size = 0.40

$$\frac{4x(c x^2 + b x)^{1/4} {}_2F_1\left(-\frac{1}{4}, \frac{5}{4}; \frac{9}{4}; -\frac{c x}{b}\right)}{5\left(\frac{c x}{b} + 1\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^(1/4), x)

[Out] (4*x*(b*x + c*x^2)^(1/4)*hypergeom([-1/4, 5/4], 9/4, -(c*x)/b))/(5*((c*x)/b + 1)^(1/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**(1/4), x)

[Out] Integral((b*x + c*x**2)**(1/4), x)

$$3.43 \quad \int \frac{1}{\sqrt[4]{bx+cx^2}} dx$$

Optimal. Leaf size=58

$$\frac{\sqrt{2} b \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right) \middle| 2\right)}{c \sqrt[4]{bx+cx^2}}$$

[Out] $b*(-c*(c*x^2+b*x)/b^2)^{(1/4)}*(\cos(1/2*\arcsin(1+2*c*x/b)))^{(1/2)}/\cos(1/2*\arcsin(1+2*c*x/b))*\text{EllipticE}(\sin(1/2*\arcsin(1+2*c*x/b)), 2^{(1/2)})*2^{(1/2)}/c/(c*x^2+b*x)^{(1/4)}$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {622, 619, 228}

$$\frac{\sqrt{2} b \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right) \middle| 2\right)}{c \sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-1/4), x]

[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(c*(b*x + c*x^2)^(1/4))

Rule 228

Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] := Simp[(2*EllipticE[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 622

Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2)^p, Int[(-(c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{bx+cx^2}} dx &= \frac{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} \int \frac{1}{\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}} dx}{\sqrt[4]{bx+cx^2}} \\ &= \frac{\left(b^2 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{\sqrt{2} c^2 \sqrt[4]{bx+cx^2}} \\ &= \frac{\sqrt{2} b \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right) \middle| 2\right)}{c \sqrt[4]{bx+cx^2}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.78

$$\frac{4x \sqrt[4]{\frac{cx}{b} + 1} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{cx}{b}\right)}{3 \sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-1/4), x]

[Out] (4*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)])/(3*(x*(b + c*x))^(1/4))

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/4), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-1/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-1/4), x)

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(1/4),x)

[Out] int(1/(c*x^2+b*x)^(1/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-1/4), x)

mupad [B] time = 0.20, size = 36, normalized size = 0.62

$$\frac{4x\left(\frac{cx}{b} + 1\right)^{1/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{cx}{b}\right)}{3(cx^2 + bx)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(1/4),x)

[Out] (4*x*((c*x)/b + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(c*x)/b))/(3*(b*x + c*x^2)^(1/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x)**(1/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-1/4), x)
```

$$3.44 \quad \int \frac{1}{(bx+cx^2)^{3/4}} dx$$

Optimal. Leaf size=59

$$\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c(bx+cx^2)^{3/4}}$$

[Out] $2*b*(-c*(c*x^2+b*x)/b^2)^{(3/4)}*(\cos(1/2*\arcsin(1+2*c*x/b))^2)^{(1/2)}/\cos(1/2*\arcsin(1+2*c*x/b))*\text{EllipticF}(\sin(1/2*\arcsin(1+2*c*x/b)),2^{(1/2)})*2^{(1/2)}/c/(c*x^2+b*x)^{(3/4)}$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {622, 619, 232}

$$\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c(bx+cx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + c*x^2)^{-3/4}, x]$

[Out] $(2*\text{Sqrt}[2]*b*(-((c*(b*x + c*x^2))/b^2))^{(3/4)}*\text{EllipticF}[\text{ArcSin}[1 + (2*c*x)/b]/2, 2])/c*(b*x + c*x^2)^{(3/4)}$

Rule 232

$\text{Int}[(a_ + (b_)*(x_)^2)^{-3/4}, x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*\text{ArcSin}[\text{Rt}[-(b/a), 2]*x])/2, 2])/(a^{(3/4)}*\text{Rt}[-(b/a), 2]), x] /;$ $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b/a]$

Rule 619

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p)}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c, p, x\} \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rule 622

$\text{Int}[(b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2)^p, \text{Int}[(-(c*x)/b - (c^2*x^2)/b^2)^p, x], x] /;$ Fr

eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rubi steps

$$\begin{aligned} \int \frac{1}{(bx + cx^2)^{3/4}} dx &= \frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} \int \frac{1}{\left(-\frac{cx}{b} - \frac{c^2x^2}{b^2}\right)^{3/4}} dx}{(bx + cx^2)^{3/4}} \\ &= \frac{\left(\sqrt{2} b^2 \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\right) \text{Subst}\left(\int \frac{1}{\left(1 - \frac{b^2x^2}{c^2}\right)^{3/4}} dx, x, -\frac{c}{b} - \frac{2c^2x}{b^2}\right)}{c^2 (bx + cx^2)^{3/4}} \\ &= \frac{2\sqrt{2} b \left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right) \middle| 2\right)}{c (bx + cx^2)^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.73

$$\frac{4x \left(\frac{cx}{b} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{cx}{b}\right)}{(x(b + cx))^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-3/4), x]

[Out] (4*x*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -(c*x)/b])/(x*(b + c*x))^(3/4)

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(3/4), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-3/4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-3/4), x)

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(3/4),x)

[Out] int(1/(c*x^2+b*x)^(3/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-3/4), x)

mupad [B] time = 0.19, size = 36, normalized size = 0.61

$$\frac{4x \left(\frac{cx}{b} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{cx}{b}\right)}{(cx^2 + bx)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(3/4),x)

[Out] $(4*x*((c*x)/b + 1)^{3/4}*\text{hypergeom}([1/4, 3/4], 5/4, -(c*x)/b))/(b*x + c*x^2)^{3/4}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(3/4),x)

[Out] Integral((b*x + c*x**2)**(-3/4), x)

$$3.45 \quad \int \frac{1}{(bx+cx^2)^{5/4}} dx$$

Optimal. Leaf size=83

$$\frac{4\sqrt{2} \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right) \middle| 2\right)}{b\sqrt[4]{bx+cx^2}} - \frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

[Out] $-4*(2*c*x+b)/b^2/(c*x^2+b*x)^(1/4)+4*(-c*(c*x^2+b*x)/b^2)^(1/4)*(\cos(1/2*\arcsin(1+2*c*x/b)))^(1/2)/\cos(1/2*\arcsin(1+2*c*x/b))*\text{EllipticE}(\sin(1/2*\arcsin(1+2*c*x/b)), 2^(1/2))*2^(1/2)/b/(c*x^2+b*x)^(1/4)$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {614, 622, 619, 228}

$$\frac{4\sqrt{2} \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{2cx}{b} + 1\right) \middle| 2\right)}{b\sqrt[4]{bx+cx^2}} - \frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*x + c*x^2)^{-5/4}, x]$

[Out] $(-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*\text{Sqrt}[2]*(-((c*(b*x + c*x^2))/b^2))^(1/4)*\text{EllipticE}[\text{ArcSin}[1 + (2*c*x)/b]/2, 2])/(b*(b*x + c*x^2)^(1/4))$

Rule 228

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1/4}, x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*\text{ArcSin}[\text{Rt}[-(b/a), 2]*x])/2, 2])/(a^(1/4)*\text{Rt}[-(b/a), 2]), x] /;$ $\text{FreeQ}\{a, b\}, x$ && $\text{GtQ}[a, 0]$ && $\text{NegQ}[b/a]$

Rule 614

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^{(p+1)} / ((p+1)*(b^2 - 4*a*c)), x] - \text{Dist}[(2*c*(2*p+3)) / ((p+1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p+1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $\text{LtQ}[p, -1]$ && $\text{NeQ}[p, -3/2]$ && $\text{IntegerQ}[4*p]$

Rule 619


```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^(p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2)/b^2))^(p), Int[(-(c*x)/b) - (c^2*x^2)/b^2]^(p), x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(bx + cx^2)^{5/4}} dx &= -\frac{4(b + 2cx)}{b^2 \sqrt[4]{bx + cx^2}} + \frac{(4c) \int \frac{1}{\sqrt[4]{bx + cx^2}} dx}{b^2} \\
 &= -\frac{4(b + 2cx)}{b^2 \sqrt[4]{bx + cx^2}} + \frac{\left(4c \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}}\right) \int \frac{1}{\sqrt[4]{-\frac{cx}{b} - \frac{c^2 x^2}{b^2}}} dx}{b^2 \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{b^2 \sqrt[4]{bx + cx^2}} - \frac{\left(2\sqrt{2} \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[4]{1 - \frac{b^2 x^2}{c^2}}} dx, x, -\frac{c}{b} - \frac{2c^2 x}{b^2}\right)}{c \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{b^2 \sqrt[4]{bx + cx^2}} + \frac{4\sqrt{2} \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right)\right) \Big|_2}{b \sqrt[4]{bx + cx^2}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.54

$$\frac{4 \sqrt[4]{\frac{cx}{b} + 1} {}_2F_1\left(-\frac{1}{4}, \frac{5}{4}; \frac{3}{4}; -\frac{cx}{b}\right)}{b \sqrt[4]{x(b + cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)^(-5/4), x]
```

```
[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((c*x)/b)])/(b*(
x*(b + c*x))^(1/4))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^2 + bx)^{\frac{3}{4}}}{c^2x^4 + 2bcx^3 + b^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-5/4), x)

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(5/4),x)

[Out] int(1/(c*x^2+b*x)^(5/4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-5/4), x)

mupad [B] time = 0.22, size = 36, normalized size = 0.43

$$\frac{4x \left(\frac{cx}{b} + 1\right)^{5/4} {}_2F_1\left(-\frac{1}{4}, \frac{5}{4}; \frac{3}{4}; -\frac{cx}{b}\right)}{(cx^2 + bx)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(5/4), x)

[Out] $-(4*x*((c*x)/b + 1)^{(5/4)}*hypergeom([-1/4, 5/4], 3/4, -(c*x)/b))/(b*x + c*x^2)^{(5/4)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{5/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(5/4), x)

[Out] Integral((b*x + c*x**2)**(-5/4), x)

$$3.46 \quad \int \frac{1}{(bx+cx^2)^{9/4}} dx$$

Optimal. Leaf size=115

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} - \frac{48\sqrt{2}c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\right)\Big|_2}{5b^3\sqrt[4]{bx+cx^2}}$$

[Out] $-4/5*(2*c*x+b)/b^2/(c*x^2+b*x)^(5/4)+48/5*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(1/4)$
 $-48/5*c*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b)))^(1/2)/cos$
 $(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/$
 $2)/b^3/(c*x^2+b*x)^(1/4)$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {614, 622, 619, 228}

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}} - \frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}} - \frac{48\sqrt{2}c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\right)\Big|_2}{5b^3\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-9/4), x]

[Out] $(-4*(b+2*c*x))/(5*b^2*(b*x+c*x^2)^(5/4)) + (48*c*(b+2*c*x))/(5*b^4*(b*x+c*x^2)^(1/4)) - (48*sqrt[2]*c*(-((c*(b*x+c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1+(2*c*x)/b]/2, 2])/(5*b^3*(b*x+c*x^2)^(1/4))$

Rule 228

Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] := Simp[(2*EllipticE[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b+2*c*x)*(a+b*x+c*x^2)^(p+1))/((p+1)*(b^2-4*a*c)), x] - Dist[(2*c*(2*p+3))/((p+1)*(b^2-4*a*c)), Int[(a+b*x+c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 622

Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[(-(c*x)/b) - (c^2*x^2)/b^2]^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(bx + cx^2)^{9/4}} dx &= -\frac{4(b + 2cx)}{5b^2 (bx + cx^2)^{5/4}} - \frac{(12c) \int \frac{1}{(bx + cx^2)^{5/4}} dx}{5b^2} \\
 &= -\frac{4(b + 2cx)}{5b^2 (bx + cx^2)^{5/4}} + \frac{48c(b + 2cx)}{5b^4 \sqrt[4]{bx + cx^2}} - \frac{(48c^2) \int \frac{1}{\sqrt[4]{bx + cx^2}} dx}{5b^4} \\
 &= -\frac{4(b + 2cx)}{5b^2 (bx + cx^2)^{5/4}} + \frac{48c(b + 2cx)}{5b^4 \sqrt[4]{bx + cx^2}} - \frac{\left(48c^2 \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}}\right) \int \frac{1}{\sqrt[4]{-\frac{cx}{b} - \frac{c^2x^2}{b^2}}} dx}{5b^4 \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{5b^2 (bx + cx^2)^{5/4}} + \frac{48c(b + 2cx)}{5b^4 \sqrt[4]{bx + cx^2}} + \frac{\left(24\sqrt{2} \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt[4]{1 - \frac{b^2x^2}{c^2}}} dx, x, -\frac{c}{b}\right)}{5b^2 \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{5b^2 (bx + cx^2)^{5/4}} + \frac{48c(b + 2cx)}{5b^4 \sqrt[4]{bx + cx^2}} - \frac{48\sqrt{2} c \sqrt[4]{-\frac{c(bx + cx^2)}{b^2}} E\left(\frac{1}{2} \sin^{-1}\left(1 + \frac{2cx}{b}\right) \middle| 2\right)}{5b^3 \sqrt[4]{bx + cx^2}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.43

$$-\frac{4\sqrt[4]{\frac{cx}{b} + 1} {}_2F_1\left(-\frac{5}{4}, \frac{9}{4}; -\frac{1}{4}; -\frac{cx}{b}\right)}{5b^2 x \sqrt[4]{x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-9/4), x]

[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, -((c*x)/b)])/(5*b^2*x*(x*(b + c*x))^(1/4))

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^2 + bx)^{\frac{3}{4}}}{c^3x^6 + 3bc^2x^5 + 3b^2cx^4 + b^3x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(9/4), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(3/4)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(9/4), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-9/4), x)

maple [F] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(9/4), x)

[Out] int(1/(c*x^2+b*x)^(9/4), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-9/4), x)

mupad [B] time = 0.26, size = 36, normalized size = 0.31

$$\frac{4x\left(\frac{cx}{b} + 1\right)^{9/4} {}_2F_1\left(-\frac{5}{4}, \frac{9}{4}; -\frac{1}{4}; -\frac{cx}{b}\right)}{5\left(cx^2 + bx\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + c*x^2)^(9/4),x)

[Out] -(4*x*((c*x)/b + 1)^(9/4)*hypergeom([-5/4, 9/4], -1/4, -(c*x)/b))/(5*(b*x + c*x^2)^(9/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{9/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+b*x)**(9/4),x)

[Out] Integral((b*x + c*x**2)**(-9/4), x)

$$3.47 \quad \int \frac{1}{(bx+cx^2)^{13/4}} dx$$

Optimal. Leaf size=146

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}} + \frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\right)\Big|_2}{15b^5\sqrt[4]{bx+cx^2}}$$

[Out] $-4/9*(2*c*x+b)/b^2/(c*x^2+b*x)^(9/4)+112/45*c*(2*c*x+b)/b^4/(c*x^2+b*x)^(5/4)-448/15*c^2*(2*c*x+b)/b^6/(c*x^2+b*x)^(1/4)+448/15*c^2*(-c*(c*x^2+b*x)/b^2)^(1/4)*(cos(1/2*arcsin(1+2*c*x/b)))^(1/2)/cos(1/2*arcsin(1+2*c*x/b))*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/b^5/(c*x^2+b*x)^(1/4)$

Rubi [A] time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {614, 622, 619, 228}

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}} + \frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\right)\Big|_2}{15b^5\sqrt[4]{bx+cx^2}} + \frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}} - \frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^(-13/4), x]

[Out] $(-4*(b+2*c*x))/(9*b^2*(b*x+c*x^2)^(9/4))+(112*c*(b+2*c*x))/(45*b^4*(b*x+c*x^2)^(5/4))-(448*c^2*(b+2*c*x))/(15*b^6*(b*x+c*x^2)^(1/4))+(448*sqrt[2]*c^2*(-((c*(b*x+c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1+(2*c*x)/b]/2,2])/(15*b^5*(b*x+c*x^2)^(1/4))$

Rule 228

Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] := Simp[(2*EllipticE[(1*ArcSin[Rt[-(b/a), 2]*x])/2, 2])/(a^(1/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b/a]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b+2*c*x)*(a+b*x+c*x^2)^(p+1))/((p+1)*(b^2-4*a*c)), x] - Dist[(2*c*(2*p+3))/((p+1)*(b^2-4*a*c)), Int[(a+b*x+c*x^2)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2-4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int

egerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 622

Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(b*x + c*x^2)^p/(-(c*(b*x + c*x^2))/b^2))^p, Int[(-(c*x)/b - (c^2*x^2)/b^2)^p, x], x] /; FreeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(bx + cx^2)^{13/4}} dx &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} - \frac{(28c) \int \frac{1}{(bx+cx^2)^{9/4}} dx}{9b^2} \\
 &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} + \frac{112c(b + 2cx)}{45b^4 (bx + cx^2)^{5/4}} + \frac{(112c^2) \int \frac{1}{(bx+cx^2)^{5/4}} dx}{15b^4} \\
 &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} + \frac{112c(b + 2cx)}{45b^4 (bx + cx^2)^{5/4}} - \frac{448c^2(b + 2cx)}{15b^6 \sqrt[4]{bx + cx^2}} + \frac{(448c^3) \int \frac{1}{\sqrt[4]{bx+cx^2}} dx}{15b^6} \\
 &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} + \frac{112c(b + 2cx)}{45b^4 (bx + cx^2)^{5/4}} - \frac{448c^2(b + 2cx)}{15b^6 \sqrt[4]{bx + cx^2}} + \frac{\left(448c^3 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right) \int \frac{1}{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}} dx}{15b^6 \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} + \frac{112c(b + 2cx)}{45b^4 (bx + cx^2)^{5/4}} - \frac{448c^2(b + 2cx)}{15b^6 \sqrt[4]{bx + cx^2}} - \frac{\left(224\sqrt{2} c^4 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right) \text{Subst}\left[\int \frac{1}{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}} dx, x, \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\right]}{15b^4 \sqrt[4]{bx + cx^2}} \\
 &= -\frac{4(b + 2cx)}{9b^2 (bx + cx^2)^{9/4}} + \frac{112c(b + 2cx)}{45b^4 (bx + cx^2)^{5/4}} - \frac{448c^2(b + 2cx)}{15b^6 \sqrt[4]{bx + cx^2}} + \frac{448\sqrt{2} c^2 \sqrt[4]{-\frac{c(bx+cx^2)}{b^2}} E\left(\frac{1}{2}\right)}{15b^5 \sqrt[4]{bx + cx^2}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.34

$$\frac{4\sqrt[4]{\frac{cx}{b} + 1} {}_2F_1\left(-\frac{9}{4}, \frac{13}{4}; -\frac{5}{4}; -\frac{cx}{b}\right)}{9b^3x^2\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^(-13/4), x]

[Out] (-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-9/4, 13/4, -5/4, -((c*x)/b)])/(9*b^3*x^2*(x*(b + c*x))^(1/4))

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^2 + bx)^{\frac{3}{4}}}{c^4x^8 + 4bc^3x^7 + 6b^2c^2x^6 + 4b^3cx^5 + b^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(13/4), x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(3/4)/(c^4*x^8 + 4*b*c^3*x^7 + 6*b^2*c^2*x^6 + 4*b^3*c*x^5 + b^4*x^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{13}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+b*x)^(13/4), x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-13/4), x)

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{13}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+b*x)^(13/4), x)

[Out] `int(1/(c*x^2+b*x)^(13/4),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2 + bx)^{\frac{13}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x)^(-13/4), x)`

mupad [B] time = 0.29, size = 36, normalized size = 0.25

$$-\frac{4x\left(\frac{cx}{b} + 1\right)^{13/4} {}_2F_1\left(-\frac{9}{4}, \frac{13}{4}; -\frac{5}{4}; -\frac{cx}{b}\right)}{9(cx^2 + bx)^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x + c*x^2)^(13/4),x)`

[Out] `-(4*x*((c*x)/b + 1)^(13/4)*hypergeom([-9/4, 13/4], -5/4, -(c*x)/b))/(9*(b*x + c*x^2)^(13/4))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx + cx^2)^{\frac{13}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+b*x)**(13/4),x)`

[Out] `Integral((b*x + c*x**2)**(-13/4), x)`

3.48 $\int (bx + cx^2)^p dx$

Optimal. Leaf size=55

$$\frac{\left(-\frac{cx}{b}\right)^{-p-1} (bx + cx^2)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{b+cx}{b}\right)}{b(p+1)}$$

[Out] $-(-c*x/b)^{-1-p}*(c*x^2+b*x)^{(1+p)}*\text{hypergeom}([-p, 1+p], [2+p], (c*x+b)/b)/b/(1+p)$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {624}

$$\frac{\left(-\frac{cx}{b}\right)^{-p-1} (bx + cx^2)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{b+cx}{b}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)^p, x]

[Out] $-(((-(c*x)/b))^{-1-p}*(b*x + c*x^2)^{(1+p)}*\text{Hypergeometric2F1}[-p, 1+p, 2+p, (b+c*x)/b])/(b*(1+p))$

Rule 624

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p+1)*Hypergeometric2F1[-p, p+1, p+2, (b+q+2*c*x)/(2*q)])/(q*(p+1)*((q-b-2*c*x)/(2*q))^(p+1)), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]

Rubi steps

$$\int (bx + cx^2)^p dx = -\frac{\left(-\frac{cx}{b}\right)^{-1-p} (bx + cx^2)^{1+p} {}_2F_1\left(-p, 1+p; 2+p; \frac{b+cx}{b}\right)}{b(1+p)}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.82

$$\frac{x(x(b+cx))^p \left(\frac{cx}{b} + 1\right)^{-p} {}_2F_1\left(-p, p+1; p+2; -\frac{cx}{b}\right)}{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)^p,x]

[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 + p)*(1 + (c*x)/b)^p)

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^p,x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^p,x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^p, x)

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)^p,x)

[Out] int((c*x^2+b*x)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)^p,x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^p, x)

mupad [B] time = 0.32, size = 48, normalized size = 0.87

$$\frac{x (c x^2 + b x)^p {}_2F_1\left(-p, p + 1; p + 2; -\frac{c x}{b}\right)}{\left(\frac{c x}{b} + 1\right)^p (p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^p,x)

[Out] (x*(b*x + c*x^2)^p*hypergeom([-p, p + 1], p + 2, -(c*x)/b))/(((c*x)/b + 1)^p*(p + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)**p,x)

[Out] Integral((b*x + c*x**2)**p, x)

3.49 $\int (a + cx^2)^4 dx$

Optimal. Leaf size=51

$$a^4x + \frac{4}{3}a^3cx^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9}$$

[Out] $a^4x + 4/3a^3c*x^3 + 6/5a^2c^2*x^5 + 4/7a*c^3*x^7 + 1/9*c^4*x^9$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {194}

$$\frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^4, x]

[Out] $a^4x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + cx^2)^4 dx &= \int (a^4 + 4a^3cx^2 + 6a^2c^2x^4 + 4ac^3x^6 + c^4x^8) dx \\ &= a^4x + \frac{4}{3}a^3cx^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 51, normalized size = 1.00

$$a^4x + \frac{4}{3}a^3cx^3 + \frac{6}{5}a^2c^2x^5 + \frac{4}{7}ac^3x^7 + \frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^4, x]

[Out] $a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (c^4*x^9)/9$

fricas [A] time = 0.73, size = 43, normalized size = 0.84

$$\frac{1}{9}x^9c^4 + \frac{4}{7}x^7c^3a + \frac{6}{5}x^5c^2a^2 + \frac{4}{3}x^3ca^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^4,x, algorithm="fricas")`

[Out] $1/9*x^9*c^4 + 4/7*x^7*c^3*a + 6/5*x^5*c^2*a^2 + 4/3*x^3*c*a^3 + x*a^4$

giac [A] time = 0.39, size = 43, normalized size = 0.84

$$\frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^4,x, algorithm="giac")`

[Out] $1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x$

maple [A] time = 0.04, size = 44, normalized size = 0.86

$$\frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^4,x)`

[Out] $a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9$

maxima [A] time = 1.31, size = 43, normalized size = 0.84

$$\frac{1}{9}c^4x^9 + \frac{4}{7}ac^3x^7 + \frac{6}{5}a^2c^2x^5 + \frac{4}{3}a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^4,x, algorithm="maxima")`

[Out] $1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a^4*x$

mupad [B] time = 0.03, size = 43, normalized size = 0.84

$$a^4x + \frac{4a^3cx^3}{3} + \frac{6a^2c^2x^5}{5} + \frac{4ac^3x^7}{7} + \frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^4,x)`

[Out] $a^4x + (c^4x^9)/9 + (4a^3cx^3)/3 + (4ac^3x^7)/7 + (6a^2c^2x^5)/5$

sympy [A] time = 0.07, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3cx^3}{3} + \frac{6a^2c^2x^5}{5} + \frac{4ac^3x^7}{7} + \frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**4,x)`

[Out] $a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**9/9$

3.50 $\int (a + cx^2)^3 dx$

Optimal. Leaf size=35

$$a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

[Out] $a^3x + a^2cx^3 + 3/5ac^2x^5 + 1/7c^3x^7$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {194}

$$a^2cx^3 + a^3x + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^3, x]

[Out] $a^3x + a^2cx^3 + (3ac^2x^5)/5 + (c^3x^7)/7$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + cx^2)^3 dx &= \int (a^3 + 3a^2cx^2 + 3ac^2x^4 + c^3x^6) dx \\ &= a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x + a^2cx^3 + \frac{3}{5}ac^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^3, x]

[Out] $a^3x + a^2cx^3 + (3ac^2x^5)/5 + (c^3x^7)/7$

fricas [A] time = 0.75, size = 31, normalized size = 0.89

$$\frac{1}{7}x^7c^3 + \frac{3}{5}x^5c^2a + x^3ca^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3,x, algorithm="fricas")

[Out] 1/7*x^7*c^3 + 3/5*x^5*c^2*a + x^3*c*a^2 + x*a^3

giac [A] time = 0.53, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3,x, algorithm="giac")

[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x

maple [A] time = 0.04, size = 32, normalized size = 0.91

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^3,x)

[Out] a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7

maxima [A] time = 1.35, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3,x, algorithm="maxima")

[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3x + a^2cx^3 + \frac{3ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^3,x)`

[Out] $a^3x + (c^3x^7)/7 + a^2cx^3 + (3ac^2x^5)/5$

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^3x + a^2cx^3 + \frac{3ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**3,x)`

[Out] $a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7$

3.51 $\int (a + cx^2)^2 dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

[Out] $a^2x + 2/3*a*c*x^3 + 1/5*c^2*x^5$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {194}

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^2, x]

[Out] $a^2x + (2*a*c*x^3)/3 + (c^2*x^5)/5$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + cx^2)^2 dx &= \int (a^2 + 2acx^2 + c^2x^4) dx \\ &= a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^2, x]

[Out] $a^2x + (2*a*c*x^3)/3 + (c^2*x^5)/5$

fricas [A] time = 0.73, size = 21, normalized size = 0.84

$$\frac{1}{5}x^5c^2 + \frac{2}{3}x^3ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^2,x, algorithm="fricas")

[Out] 1/5*x^5*c^2 + 2/3*x^3*c*a + x*a^2

giac [A] time = 0.43, size = 21, normalized size = 0.84

$$\frac{1}{5}c^2x^5 + \frac{2}{3}acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^2,x, algorithm="giac")

[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x

maple [A] time = 0.04, size = 22, normalized size = 0.88

$$\frac{1}{5}c^2x^5 + \frac{2}{3}acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^2,x)

[Out] a^2*x+2/3*a*c*x^3+1/5*c^2*x^5

maxima [A] time = 1.36, size = 21, normalized size = 0.84

$$\frac{1}{5}c^2x^5 + \frac{2}{3}acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^2,x, algorithm="maxima")

[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^2,x)`

[Out] `a^2*x + (c^2*x^5)/5 + (2*a*c*x^3)/3`

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**2,x)`

[Out] `a**2*x + 2*a*c*x**3/3 + c**2*x**5/5`

3.52 $\int (a + cx^2) dx$

Optimal. Leaf size=12

$$ax + \frac{cx^3}{3}$$

[Out] a*x+1/3*c*x^3

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[a + c*x^2,x]

[Out] a*x + (c*x^3)/3

Rubi steps

$$\int (a + cx^2) dx = ax + \frac{cx^3}{3}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[a + c*x^2,x]

[Out] a*x + (c*x^3)/3

fricas [A] time = 0.71, size = 10, normalized size = 0.83

$$\frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c*x^2+a,x, algorithm="fricas")

[Out] $\frac{1}{3}x^3c + xa$

giac [A] time = 0.35, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2+a,x, algorithm="giac")`

[Out] $\frac{1}{3}c*x^3 + a*x$

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(c*x^2+a,x)`

[Out] $a*x + \frac{1}{3}c*x^3$

maxima [A] time = 1.29, size = 10, normalized size = 0.83

$$\frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^2+a,x, algorithm="maxima")`

[Out] $\frac{1}{3}c*x^3 + a*x$

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{cx^3}{3} + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + c*x^2,x)`

[Out] $a*x + (c*x^3)/3$

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**2+a,x)
```

```
[Out] a*x + c*x**3/3
```

$$3.53 \quad \int \frac{1}{a+cx^2} dx$$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

[Out] arctan(x*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{a+cx^2} dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])

fricas [A] time = 0.81, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x - a}{cx^2 + a}\right)}{2ac}, \frac{\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right)}{ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))/(a*c), sqrt(a*c)*arctan(sqrt(a*c)*x/a)/(a*c)]

giac [A] time = 0.40, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a),x, algorithm="giac")

[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)

maple [A] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a),x)

[Out] 1/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))

maxima [A] time = 2.90, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a),x, algorithm="maxima")

[Out] $\arctan(c*x/\sqrt{a*c})/\sqrt{a*c}$

mupad [B] time = 0.07, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(1/(a + c*x^2), x)$

[Out] $\operatorname{atan}((c^{1/2}*x)/a^{1/2})/(a^{1/2}*c^{1/2})$

sympy [B] time = 0.14, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ac}} \log\left(-a\sqrt{-\frac{1}{ac}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ac}} \log\left(a\sqrt{-\frac{1}{ac}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/(c*x**2+a), x)$

[Out] $-\sqrt{-1/(a*c)}*\log(-a*\sqrt{-1/(a*c)} + x)/2 + \sqrt{-1/(a*c)}*\log(a*\sqrt{-1/(a*c)} + x)/2$

$$3.54 \quad \int \frac{1}{(a+cx^2)^2} dx$$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{x}{2a(a+cx^2)}$$

[Out] $1/2*x/a/(c*x^2+a)+1/2*\arctan(x*c^{(1/2)}/a^{(1/2)})/a^{(3/2)}/c^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{x}{2a(a+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-2), x]

[Out] $x/(2*a*(a + c*x^2)) + \text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a]]/(2*a^{(3/2)}*\text{Sqrt}[c])$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{(a + cx^2)^2} dx = \frac{x}{2a(a + cx^2)} + \frac{\int \frac{1}{a+cx^2} dx}{2a}$$

$$= \frac{x}{2a(a + cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{x}{2a(a + cx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-2),x]

[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])

fricas [A] time = 0.83, size = 120, normalized size = 2.67

$$\left[\frac{2acx - (cx^2 + a)\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x - a}{cx^2 + a}\right)}{4(a^2c^2x^2 + a^3c)}, \frac{acx + (cx^2 + a)\sqrt{ac} \arctan\left(\frac{\sqrt{ac}x}{a}\right)}{2(a^2c^2x^2 + a^3c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*c*x - (c*x^2 + a)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^2*c^2*x^2 + a^3*c), 1/2*(a*c*x + (c*x^2 + a)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^2*c^2*x^2 + a^3*c)]

giac [A] time = 0.42, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{x}{2(cx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^2,x, algorithm="giac")

[Out] $1/2*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a) + 1/2*x/((c*x^2 + a)*a)$

maple [A] time = 0.05, size = 36, normalized size = 0.80

$$\frac{x}{2(c x^2 + a) a} + \frac{\arctan\left(\frac{c x}{\sqrt{a c}}\right)}{2 \sqrt{a c} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+a)^2,x)`

[Out] $1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^{(1/2)*\arctan(1/(a*c)^{(1/2)*c*x)}$

maxima [A] time = 3.11, size = 35, normalized size = 0.78

$$\frac{x}{2(a c x^2 + a^2)} + \frac{\arctan\left(\frac{c x}{\sqrt{a c}}\right)}{2 \sqrt{a c} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+a)^2,x, algorithm="maxima")`

[Out] $1/2*x/(a*c*x^2 + a^2) + 1/2*\arctan(c*x/\sqrt{a*c})/(\sqrt{a*c}*a)$

mupad [B] time = 0.14, size = 33, normalized size = 0.73

$$\frac{x}{2 a (c x^2 + a)} + \frac{\operatorname{atan}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2 a^{3/2} \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x^2)^2,x)`

[Out] $x/(2*a*(a + c*x^2)) + \operatorname{atan}((c^{(1/2)*x})/a^{(1/2)})/(2*a^{(3/2)*c^{(1/2)})}$

sympy [B] time = 0.22, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2acx^2} - \frac{\sqrt{-\frac{1}{a^3c}} \log\left(-a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c}} \log\left(a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+a)**2,x)`

[Out] $x/(2*a**2 + 2*a*c*x**2) - \sqrt{-1/(a**3*c)}*\log(-a**2*\sqrt{-1/(a**3*c)}) + x)/4 + \sqrt{-1/(a**3*c)}*\log(a**2*\sqrt{-1/(a**3*c)}) + x)/4$

$$3.55 \quad \int \frac{1}{(a+cx^2)^3} dx$$

Optimal. Leaf size=62

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} + \frac{3x}{8a^2(a+cx^2)} + \frac{x}{4a(a+cx^2)^2}$$

[Out] 1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {199, 205}

$$\frac{3x}{8a^2(a+cx^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} + \frac{x}{4a(a+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-3), x]

[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^2)^3} dx &= \frac{x}{4a(a+cx^2)^2} + \frac{3 \int \frac{1}{(a+cx^2)^2} dx}{4a} \\
&= \frac{x}{4a(a+cx^2)^2} + \frac{3x}{8a^2(a+cx^2)} + \frac{3 \int \frac{1}{a+cx^2} dx}{8a^2} \\
&= \frac{x}{4a(a+cx^2)^2} + \frac{3x}{8a^2(a+cx^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} + \frac{5ax + 3cx^3}{8a^2(a+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-3), x]

[Out] (5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])

fricas [A] time = 0.87, size = 188, normalized size = 3.03

$$\left[\frac{6ac^2x^3 + 10a^2cx - 3(c^2x^4 + 2acx^2 + a^2)\sqrt{-ac} \log\left(\frac{cx^2 - 2\sqrt{-ac}x - a}{cx^2 + a}\right)}{16(a^3c^3x^4 + 2a^4c^2x^2 + a^5c)}, \frac{3ac^2x^3 + 5a^2cx + 3(c^2x^4 + 2acx^2 + a^2)\sqrt{a}}{8(a^3c^3x^4 + 2a^4c^2x^2 + a^5c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^3,x, algorithm="fricas")

[Out] [1/16*(6*a*c^2*x^3 + 10*a^2*c*x - 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c), 1/8*(3*a*c^2*x^3 + 5*a^2*c*x + 3*(c^2*x^4 + 2*a*c*x^2 + a^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a^3*c^3*x^4 + 2*a^4*c^2*x^2 + a^5*c)]

giac [A] time = 0.34, size = 45, normalized size = 0.73

$$\frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2} + \frac{3cx^3 + 5ax}{8(cx^2 + a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^3,x, algorithm="giac")

[Out] $\frac{3}{8} \arctan\left(\frac{cx}{\sqrt{ac}}\right) / (\sqrt{ac} a^2) + \frac{1}{8} (3cx^3 + 5ax) / ((cx^2 + a)^2 a^2)$

maple [A] time = 0.05, size = 51, normalized size = 0.82

$$\frac{x}{4(c x^2 + a)^2 a} + \frac{3x}{8(c x^2 + a) a^2} + \frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^3,x)

[Out] $\frac{1}{4} x/a / (c x^2 + a)^2 + \frac{3}{8} x/a^2 / (c x^2 + a) + \frac{3}{8} / a^2 / (a c)^{(1/2)} \arctan(1/(a c)^{(1/2)} c x)$

maxima [A] time = 2.99, size = 58, normalized size = 0.94

$$\frac{3cx^3 + 5ax}{8(a^2c^2x^4 + 2a^3cx^2 + a^4)} + \frac{3 \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} (3cx^3 + 5ax) / (a^2c^2x^4 + 2a^3cx^2 + a^4) + \frac{3}{8} \arctan(cx/\sqrt{ac}) / (\sqrt{ac} a^2)$

mupad [B] time = 0.16, size = 55, normalized size = 0.89

$$\frac{\frac{5x}{8a} + \frac{3cx^3}{8a^2}}{a^2 + 2acx^2 + c^2x^4} + \frac{3 \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^2)^3,x)

[Out] $\left(\frac{5x}{8a} + \frac{3cx^3}{8a^2}\right) / (a^2 + c^2x^4 + 2acx^2) + \frac{3 \operatorname{atan}\left(\frac{c^{1/2}x}{a^{1/2}}\right)}{8a^{5/2}c^{1/2}}$

sympy [A] time = 0.34, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5c}} \log\left(-a^3\sqrt{-\frac{1}{a^5c}} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5c}} \log\left(a^3\sqrt{-\frac{1}{a^5c}} + x\right)}{16} + \frac{5ax + 3cx^3}{8a^4 + 16a^3cx^2 + 8a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**3,x)

[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt(-1/(a**5*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x**3)/(8*a**4 + 16*a**3*c*x**2 + 8*a**2*c**2*x**4)

3.56 $\int (a + cx^2)^{5/2} dx$

Optimal. Leaf size=84

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}} + \frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5}{24}ax(a+cx^2)^{3/2} + \frac{1}{6}x(a+cx^2)^{5/2}$$

[Out] $5/24*a*x*(c*x^2+a)^{(3/2)}+1/6*x*(c*x^2+a)^{(5/2)}+5/16*a^3*\operatorname{arctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})/c^{(1/2)}+5/16*a^2*x*(c*x^2+a)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {195, 217, 206}

$$\frac{5}{16}a^2x\sqrt{a+cx^2} + \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}} + \frac{5}{24}ax(a+cx^2)^{3/2} + \frac{1}{6}x(a+cx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + c*x^2)^{(5/2)}, x]$

[Out] $(5*a^2*x*\operatorname{Sqrt}[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^{(3/2)})/24 + (x*(a + c*x^2)^{(5/2)})/6 + (5*a^3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*x)/\operatorname{Sqrt}[a + c*x^2]])/(16*\operatorname{Sqrt}[c])$

Rule 195

$\operatorname{Int}[(a_ + (b_)*(x_)^n)^p, x_Symbol] \rightarrow \operatorname{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \operatorname{Dist}[(a*n*p)/(n*p + 1), \operatorname{Int}[(a + b*x^n)^{p-1}, x], x] /;$ Free Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(1 - b*x^2), x], x, x/\operatorname{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int (a + cx^2)^{5/2} dx &= \frac{1}{6}x(a + cx^2)^{5/2} + \frac{1}{6}(5a) \int (a + cx^2)^{3/2} dx \\
&= \frac{5}{24}ax(a + cx^2)^{3/2} + \frac{1}{6}x(a + cx^2)^{5/2} + \frac{1}{8}(5a^2) \int \sqrt{a + cx^2} dx \\
&= \frac{5}{16}a^2x\sqrt{a + cx^2} + \frac{5}{24}ax(a + cx^2)^{3/2} + \frac{1}{6}x(a + cx^2)^{5/2} + \frac{1}{16}(5a^3) \int \frac{1}{\sqrt{a + cx^2}} dx \\
&= \frac{5}{16}a^2x\sqrt{a + cx^2} + \frac{5}{24}ax(a + cx^2)^{3/2} + \frac{1}{6}x(a + cx^2)^{5/2} + \frac{1}{16}(5a^3) \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{\sqrt{cx^2 + a}}{\sqrt{c}}\right) \\
&= \frac{5}{16}a^2x\sqrt{a + cx^2} + \frac{5}{24}ax(a + cx^2)^{3/2} + \frac{1}{6}x(a + cx^2)^{5/2} + \frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}}\right)}{16\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 76, normalized size = 0.90

$$\frac{1}{48}\sqrt{a + cx^2} \left(\frac{15a^{5/2} \sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a} + 1}} + 33a^2x + 26acx^3 + 8c^2x^5 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(5/2), x]

[Out] (Sqrt[a + c*x^2]*(33*a^2*x + 26*a*c*x^3 + 8*c^2*x^5 + (15*a^(5/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]))/48

fricas [A] time = 0.82, size = 146, normalized size = 1.74

$$\left[\frac{15a^3\sqrt{c} \log\left(-2cx^2 - 2\sqrt{cx^2 + a}\sqrt{c}x - a\right) + 2(8c^3x^5 + 26ac^2x^3 + 33a^2cx)\sqrt{cx^2 + a}}{96c}, -\frac{15a^3\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2 + a}}\right)}{16\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(5/2), x, algorithm="fricas")

[Out] [1/96*(15*a^3*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c, -1/48*(15*a^3*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (8*c^3*x^5 + 26*a*c^2*x^3 + 33*a^2*c*x)*sqrt(c*x^2 + a))/c]

giac [A] time = 0.36, size = 63, normalized size = 0.75

$$-\frac{5a^3 \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{16\sqrt{c}} + \frac{1}{48} \left(2(4c^2x^2 + 13ac)x^2 + 33a^2\right)\sqrt{cx^2 + a}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(5/2),x, algorithm="giac")

[Out] -5/16*a^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2*(4*c^2*x^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x

maple [A] time = 0.04, size = 66, normalized size = 0.79

$$\frac{5a^3 \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{16\sqrt{c}} + \frac{5\sqrt{cx^2 + a}a^2x}{16} + \frac{5(cx^2 + a)^{\frac{3}{2}}ax}{24} + \frac{(cx^2 + a)^{\frac{5}{2}}x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(5/2),x)

[Out] 1/6*x*(c*x^2+a)^(5/2)+5/24*a*x*(c*x^2+a)^(3/2)+5/16*a^2*x*(c*x^2+a)^(1/2)+5/16*a^3/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))

maxima [A] time = 1.38, size = 58, normalized size = 0.69

$$\frac{1}{6}(cx^2 + a)^{\frac{5}{2}}x + \frac{5}{24}(cx^2 + a)^{\frac{3}{2}}ax + \frac{5}{16}\sqrt{cx^2 + a}a^2x + \frac{5a^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 1/6*(c*x^2 + a)^(5/2)*x + 5/24*(c*x^2 + a)^(3/2)*a*x + 5/16*sqrt(c*x^2 + a)*a^2*x + 5/16*a^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)

mupad [B] time = 0.20, size = 37, normalized size = 0.44

$$\frac{x(c x^2 + a)^{5/2} {}_2F_1\left(-\frac{5}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{c x^2}{a}\right)}{\left(\frac{c x^2}{a} + 1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(5/2), x)`

[Out] `(x*(a + c*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a + 1)^(5/2)`

sympy [A] time = 4.43, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16} + \frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24} + \frac{\sqrt{a}c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6} + \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(5/2), x)`

[Out] `11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 + c*x**2/a)/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c))`

3.57 $\int (a + cx^2)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}} + \frac{3}{8}ax\sqrt{a+cx^2} + \frac{1}{4}x(a+cx^2)^{3/2}$$

[Out] $1/4*x*(c*x^2+a)^{(3/2)}+3/8*a^2*\arctanh(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})/c^{(1/2)}+3/8*a*x*(c*x^2+a)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {195, 217, 206}

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}} + \frac{3}{8}ax\sqrt{a+cx^2} + \frac{1}{4}x(a+cx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(3/2), x]

[Out] $(3*a*x*\text{Sqrt}[a + c*x^2])/8 + (x*(a + c*x^2)^{(3/2)})/4 + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(8*\text{Sqrt}[c])$

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int (a + cx^2)^{3/2} dx &= \frac{1}{4}x(a + cx^2)^{3/2} + \frac{1}{4}(3a) \int \sqrt{a + cx^2} dx \\
&= \frac{3}{8}ax\sqrt{a + cx^2} + \frac{1}{4}x(a + cx^2)^{3/2} + \frac{1}{8}(3a^2) \int \frac{1}{\sqrt{a + cx^2}} dx \\
&= \frac{3}{8}ax\sqrt{a + cx^2} + \frac{1}{4}x(a + cx^2)^{3/2} + \frac{1}{8}(3a^2) \text{Subst} \left(\int \frac{1}{1 - cx^2} dx, x, \frac{x}{\sqrt{a + cx^2}} \right) \\
&= \frac{3}{8}ax\sqrt{a + cx^2} + \frac{1}{4}x(a + cx^2)^{3/2} + \frac{3a^2 \tanh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{a + cx^2}} \right)}{8\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.00

$$\frac{1}{8}\sqrt{a + cx^2} \left(\frac{3a^{3/2} \sinh^{-1} \left(\frac{\sqrt{c}x}{\sqrt{a}} \right)}{\sqrt{c} \sqrt{\frac{cx^2}{a} + 1}} + 5ax + 2cx^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(3/2), x]

[Out] (Sqrt[a + c*x^2]*(5*a*x + 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]))/8

fricas [A] time = 0.96, size = 124, normalized size = 1.91

$$\left[\frac{3a^2\sqrt{c} \log(-2cx^2 - 2\sqrt{cx^2 + a}\sqrt{c}x - a) + 2(2c^2x^3 + 5acx)\sqrt{cx^2 + a}}{16c}, -\frac{3a^2\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2 + a}}\right) - (2c^2x^3 + 5acx)\sqrt{cx^2 + a}}{8c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2), x, algorithm="fricas")

[Out] [1/16*(3*a^2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c, -1/8*(3*a^2*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (2*c^2*x^3 + 5*a*c*x)*sqrt(c*x^2 + a))/c]

giac [A] time = 0.43, size = 49, normalized size = 0.75

$$\frac{1}{8} (2cx^2 + 5a)\sqrt{cx^2 + a}x - \frac{3a^2 \log\left(|-\sqrt{c}x + \sqrt{cx^2 + a}|\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2),x, algorithm="giac")

[Out] 1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c)

maple [A] time = 0.04, size = 51, normalized size = 0.78

$$\frac{3a^2 \ln\left(\sqrt{c} x + \sqrt{cx^2 + a}\right)}{8\sqrt{c}} + \frac{3\sqrt{cx^2 + a} ax}{8} + \frac{(cx^2 + a)^{\frac{3}{2}} x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^(3/2),x)

[Out] 1/4*x*(c*x^2+a)^(3/2)+3/8*a*x*(c*x^2+a)^(1/2)+3/8*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))

maxima [A] time = 1.30, size = 43, normalized size = 0.66

$$\frac{1}{4} (cx^2 + a)^{\frac{3}{2}} x + \frac{3}{8} \sqrt{cx^2 + a} ax + \frac{3a^2 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2),x, algorithm="maxima")

[Out] 1/4*(c*x^2 + a)^(3/2)*x + 3/8*sqrt(c*x^2 + a)*a*x + 3/8*a^2*arcsinh(c*x/sqrt(a*c))/sqrt(c)

mupad [B] time = 0.16, size = 37, normalized size = 0.57

$$\frac{x (cx^2 + a)^{3/2} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{cx^2}{a}\right)}{\left(\frac{cx^2}{a} + 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^2)^(3/2),x)

[Out] (x*(a + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(c*x^2)/a))/((c*x^2)/a + 1)^(3/2)

sympy [A] time = 3.01, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{cx^2}{a}}}{8} + \frac{\sqrt{a}cx^3\sqrt{1+\frac{cx^2}{a}}}{4} + \frac{3a^2 \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**(3/2),x)

[Out] 5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**2/a)/4 + 3*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))

3.58 $\int \sqrt{a + cx^2} dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a + cx^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

[Out] $1/2*a*\operatorname{arctanh}(x*c^{(1/2)}/(c*x^2+a)^{(1/2)})/c^{(1/2)}+1/2*x*(c*x^2+a)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {195, 217, 206}

$$\frac{1}{2}x\sqrt{a + cx^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*x^2], x]

[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{a+cx^2} dx &= \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a \int \frac{1}{\sqrt{a+cx^2}} dx \\
&= \frac{1}{2}x\sqrt{a+cx^2} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right) \\
&= \frac{1}{2}x\sqrt{a+cx^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+cx^2} + \frac{a \log\left(\sqrt{c}\sqrt{a+cx^2} + cx\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c*x^2], x]

[Out] (x*Sqrt[a + c*x^2])/2 + (a*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2*Sqrt[c])

fricas [A] time = 0.93, size = 94, normalized size = 2.04

$$\left[\frac{2\sqrt{cx^2+acx} + a\sqrt{c} \log\left(-2cx^2 - 2\sqrt{cx^2+a}\sqrt{cx} - a\right)}{4c}, \frac{\sqrt{cx^2+acx} - a\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/4*(2*sqrt(c*x^2 + a)*c*x + a*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/c, 1/2*(sqrt(c*x^2 + a)*c*x - a*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/c]

giac [A] time = 0.40, size = 37, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2+ax} - \frac{a \log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(1/2), x, algorithm="giac")

[Out] $\frac{1}{2}\sqrt{cx^2 + a}x - \frac{1}{2}a\log(\text{abs}(-\sqrt{c}x + \sqrt{cx^2 + a}))/\sqrt{c}$
)

maple [A] time = 0.04, size = 36, normalized size = 0.78

$$\frac{a \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{2\sqrt{c}} + \frac{\sqrt{cx^2 + a}x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(1/2),x)`

[Out] $\frac{1}{2}x\sqrt{cx^2+a} + \frac{1}{2}a/c^{1/2} \ln(c^{1/2}x + \sqrt{cx^2+a})$

maxima [A] time = 1.36, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{cx^2 + a}x + \frac{a \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2}\sqrt{cx^2 + a}x + \frac{1}{2}a\operatorname{arcsinh}(cx/\sqrt{ac})/\sqrt{c}$

mupad [B] time = 0.13, size = 35, normalized size = 0.76

$$\frac{x\sqrt{cx^2 + a}}{2} + \frac{a \ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)^(1/2),x)`

[Out] $\frac{x\sqrt{a + cx^2}}{2} + \frac{a\log(c^{1/2}x + \sqrt{a + cx^2})}{2c^{1/2}}$
)

sympy [A] time = 1.91, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1 + \frac{cx^2}{a}}}{2} + \frac{a \operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(1/2),x)`

[Out] $\sqrt{a}x\sqrt{1 + cx^2/a}/2 + a\operatorname{asinh}(\sqrt{c}x/\sqrt{a})/(2\sqrt{c})$

$$3.59 \quad \int \frac{1}{\sqrt{a+cx^2}} dx$$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + c*x^2], x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+cx^2}} dx &= \text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{a+cx^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + c*x^2],x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]

fricas [A] time = 0.99, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2cx^2 - 2\sqrt{cx^2+a}\sqrt{c}x - a\right)}{2\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)/sqrt(c), -sqrt(-c)*arc tan(sqrt(-c)*x/sqrt(c*x^2 + a))/c]

giac [A] time = 0.41, size = 23, normalized size = 0.92

$$\frac{\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(1/2),x, algorithm="giac")

[Out] -log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c)

maple [A] time = 0.04, size = 21, normalized size = 0.84

$$\frac{\ln\left(\sqrt{c}x + \sqrt{cx^2+a}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(1/2),x)

[Out] $\ln(c^{(1/2)}*x+(c*x^2+a)^{(1/2)})/c^{(1/2)}$

maxima [A] time = 1.39, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^2+a)^(1/2),x, algorithm="maxima")`

[Out] `arcsinh(c*x/sqrt(a*c))/sqrt(c)`

mupad [B] time = 0.19, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{c}x + \sqrt{cx^2 + a}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x^2)^(1/2),x)`

[Out] `log(c^(1/2)*x + (a + c*x^2)^(1/2))/c^(1/2)`

sympy [A] time = 1.05, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**2+a)**(1/2),x)`

[Out] `asinh(sqrt(c)*x/sqrt(a))/sqrt(c)`

$$3.60 \quad \int \frac{1}{(a+cx^2)^{3/2}} dx$$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+cx^2}}$$

[Out] x/a/(c*x^2+a)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {191}

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-3/2), x]

[Out] x/(a*Sqrt[a + c*x^2])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rubi steps

$$\int \frac{1}{(a+cx^2)^{3/2}} dx = \frac{x}{a\sqrt{a+cx^2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-3/2), x]

[Out] x/(a*Sqrt[a + c*x^2])

fricas [A] time = 0.74, size = 23, normalized size = 1.44

$$\frac{\sqrt{cx^2 + a} x}{acx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="fricas")

[Out] sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)

giac [A] time = 0.48, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2 + a} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="giac")

[Out] x/(sqrt(c*x^2 + a)*a)

maple [A] time = 0.04, size = 15, normalized size = 0.94

$$\frac{x}{\sqrt{c x^2 + a} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(3/2),x)

[Out] x/a/(c*x^2+a)^(1/2)

maxima [A] time = 1.25, size = 14, normalized size = 0.88

$$\frac{x}{\sqrt{cx^2 + a} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(3/2),x, algorithm="maxima")

[Out] x/(sqrt(c*x^2 + a)*a)

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\frac{x}{a \sqrt{c x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^2)^(3/2),x)
```

```
[Out] x/(a*(a + c*x^2)^(1/2))
```

sympy [A] time = 0.65, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1 + \frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

$$3.61 \quad \int \frac{1}{(a+cx^2)^{5/2}} dx$$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+cx^2}} + \frac{x}{3a(a+cx^2)^{3/2}}$$

[Out] $1/3*x/a/(c*x^2+a)^{(3/2)}+2/3*x/a^2/(c*x^2+a)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{2x}{3a^2\sqrt{a+cx^2}} + \frac{x}{3a(a+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-5/2), x]

[Out] $x/(3*a*(a + c*x^2)^{(3/2)}) + (2*x)/(3*a^2*\text{Sqrt}[a + c*x^2])$

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+cx^2)^{5/2}} dx &= \frac{x}{3a(a+cx^2)^{3/2}} + \frac{2 \int \frac{1}{(a+cx^2)^{3/2}} dx}{3a} \\ &= \frac{x}{3a(a+cx^2)^{3/2}} + \frac{2x}{3a^2\sqrt{a+cx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.74

$$\frac{x(3a + 2cx^2)}{3a^2(a + cx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-5/2), x]

[Out] (x*(3*a + 2*c*x^2))/(3*a^2*(a + c*x^2)^(3/2))

fricas [A] time = 0.71, size = 47, normalized size = 1.21

$$\frac{(2cx^3 + 3ax)\sqrt{cx^2 + a}}{3(a^2c^2x^4 + 2a^3cx^2 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(5/2), x, algorithm="fricas")

[Out] 1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)

giac [A] time = 0.51, size = 27, normalized size = 0.69

$$\frac{x\left(\frac{2cx^2}{a^2} + \frac{3}{a}\right)}{3(cx^2 + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(5/2), x, algorithm="giac")

[Out] 1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)

maple [A] time = 0.04, size = 26, normalized size = 0.67

$$\frac{(2cx^2 + 3a)x}{3(cx^2 + a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(5/2), x)

[Out] 1/3*x*(2*c*x^2+3*a)/(c*x^2+a)^(3/2)/a^2

maxima [A] time = 1.02, size = 31, normalized size = 0.79

$$\frac{2x}{3\sqrt{cx^2 + a}a^2} + \frac{x}{3(cx^2 + a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)

mupad [B] time = 0.19, size = 28, normalized size = 0.72

$$\frac{2x(cx^2 + a) + ax}{3a^2(cx^2 + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^2)^(5/2),x)

[Out] (2*x*(a + c*x^2) + a*x)/(3*a^2*(a + c*x^2)^(3/2))

sympy [B] time = 0.86, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1 + \frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1 + \frac{cx^2}{a}}} + \frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1 + \frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1 + \frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**(5/2),x)

[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2/a))

$$3.62 \quad \int \frac{1}{(a+cx^2)^{7/2}} dx$$

Optimal. Leaf size=58

$$\frac{8x}{15a^3\sqrt{a+cx^2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{x}{5a(a+cx^2)^{5/2}}$$

[Out] 1/5*x/a/(c*x^2+a)^(5/2)+4/15*x/a^2/(c*x^2+a)^(3/2)+8/15*x/a^3/(c*x^2+a)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{8x}{15a^3\sqrt{a+cx^2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{x}{5a(a+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-7/2), x]

[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8*x)/(15*a^3*Sqrt[a + c*x^2])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^2)^{7/2}} dx &= \frac{x}{5a(a+cx^2)^{5/2}} + \frac{4 \int \frac{1}{(a+cx^2)^{5/2}} dx}{5a} \\
&= \frac{x}{5a(a+cx^2)^{5/2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{8 \int \frac{1}{(a+cx^2)^{3/2}} dx}{15a^2} \\
&= \frac{x}{5a(a+cx^2)^{5/2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{8x}{15a^3\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{x(15a^2 + 20acx^2 + 8c^2x^4)}{15a^3(a+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-7/2), x]

[Out] (x*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4))/(15*a^3*(a + c*x^2)^(5/2))

fricas [A] time = 1.08, size = 69, normalized size = 1.19

$$\frac{(8c^2x^5 + 20acx^3 + 15a^2x)\sqrt{cx^2 + a}}{15(a^3c^3x^6 + 3a^4c^2x^4 + 3a^5cx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(7/2), x, algorithm="fricas")

[Out] 1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3*x^6 + 3*a^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)

giac [A] time = 0.46, size = 41, normalized size = 0.71

$$\frac{\left(4x^2\left(\frac{2c^2x^2}{a^3} + \frac{5c}{a^2}\right) + \frac{15}{a}\right)x}{15(cx^2 + a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="giac")

[Out] 1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)

maple [A] time = 0.04, size = 37, normalized size = 0.64

$$\frac{(8c^2x^4 + 20acx^2 + 15a^2)x}{15(cx^2 + a)^{\frac{5}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(7/2),x)

[Out] 1/15*x*(8*c^2*x^4+20*a*c*x^2+15*a^2)/(c*x^2+a)^(5/2)/a^3

maxima [A] time = 1.31, size = 46, normalized size = 0.79

$$\frac{8x}{15\sqrt{cx^2+a}a^3} + \frac{4x}{15(cx^2+a)^{\frac{3}{2}}a^2} + \frac{x}{5(cx^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(7/2),x, algorithm="maxima")

[Out] 8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*x/((c*x^2 + a)^(5/2)*a)

mupad [B] time = 0.19, size = 44, normalized size = 0.76

$$\frac{8x(cx^2+a)^2 + 3a^2x + 4ax(cx^2+a)}{15a^3(cx^2+a)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^2)^(7/2),x)

[Out] (8*x*(a + c*x^2)^2 + 3*a^2*x + 4*a*x*(a + c*x^2))/(15*a^3*(a + c*x^2)^(5/2))

sympy [B] time = 1.46, size = 413, normalized size = 7.12

$$\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}} + 45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}} + 45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}} + 15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}} + \frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}} + 45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**(7/2),x)

[Out] $15*a^{5/2}*x/(15*a^{17/2}*sqrt(1 + c*x^2/a) + 45*a^{15/2}*c*x^2*sqrt(1 + c*x^2/a) + 45*a^{13/2}*c^2*x^4*sqrt(1 + c*x^2/a) + 15*a^{11/2}*c^3*x^6*sqrt(1 + c*x^2/a)) + 35*a^4*c*x^3/(15*a^{17/2}*sqrt(1 + c*x^2/a) + 45*a^{15/2}*c*x^2*sqrt(1 + c*x^2/a) + 45*a^{13/2}*c^2*x^4*sqrt(1 + c*x^2/a) + 15*a^{11/2}*c^3*x^6*sqrt(1 + c*x^2/a)) + 28*a^3*c^2*x^5/(15*a^{17/2}*sqrt(1 + c*x^2/a) + 45*a^{15/2}*c*x^2*sqrt(1 + c*x^2/a) + 45*a^{13/2}*c^2*x^4*sqrt(1 + c*x^2/a) + 15*a^{11/2}*c^3*x^6*sqrt(1 + c*x^2/a)) + 8*a^2*c^3*x^7/(15*a^{17/2}*sqrt(1 + c*x^2/a) + 45*a^{15/2}*c*x^2*sqrt(1 + c*x^2/a) + 45*a^{13/2}*c^2*x^4*sqrt(1 + c*x^2/a) + 15*a^{11/2}*c^3*x^6*sqrt(1 + c*x^2/a))$

$$3.63 \quad \int \frac{1}{(a+cx^2)^{9/2}} dx$$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+cx^2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{x}{7a(a+cx^2)^{7/2}}$$

[Out] $1/7*x/a/(c*x^2+a)^{(7/2)}+6/35*x/a^2/(c*x^2+a)^{(5/2)}+8/35*x/a^3/(c*x^2+a)^{(3/2)}+16/35*x/a^4/(c*x^2+a)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {192, 191}

$$\frac{16x}{35a^4\sqrt{a+cx^2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{x}{7a(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)^(-9/2), x]

[Out] $x/(7*a*(a + c*x^2)^{(7/2)}) + (6*x)/(35*a^2*(a + c*x^2)^{(5/2)}) + (8*x)/(35*a^3*(a + c*x^2)^{(3/2)}) + (16*x)/(35*a^4*\text{Sqrt}[a + c*x^2])$

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^2)^{9/2}} dx &= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6 \int \frac{1}{(a+cx^2)^{7/2}} dx}{7a} \\
&= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{24 \int \frac{1}{(a+cx^2)^{5/2}} dx}{35a^2} \\
&= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{16 \int \frac{1}{(a+cx^2)^{3/2}} dx}{35a^3} \\
&= \frac{x}{7a(a+cx^2)^{7/2}} + \frac{6x}{35a^2(a+cx^2)^{5/2}} + \frac{8x}{35a^3(a+cx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.66

$$\frac{x(35a^3 + 70a^2cx^2 + 56ac^2x^4 + 16c^3x^6)}{35a^4(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)^(-9/2), x]

[Out] (x*(35*a^3 + 70*a^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6))/(35*a^4*(a + c*x^2)^(7/2))

fricas [A] time = 0.70, size = 91, normalized size = 1.18

$$\frac{(16c^3x^7 + 56ac^2x^5 + 70a^2cx^3 + 35a^3x)\sqrt{cx^2 + a}}{35(a^4c^4x^8 + 4a^5c^3x^6 + 6a^6c^2x^4 + 4a^7cx^2 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(9/2), x, algorithm="fricas")

[Out] 1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c*x^2 + a)/(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*x^2 + a^8)

giac [A] time = 0.41, size = 55, normalized size = 0.71

$$\frac{\left(2\left(4x^2\left(\frac{2c^3x^2}{a^4} + \frac{7c^2}{a^3}\right) + \frac{35c}{a^2}\right)x^2 + \frac{35}{a}\right)x}{35(cx^2 + a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="giac")

[Out] $\frac{1}{35} \cdot (2 \cdot (4 \cdot x^2 \cdot (2 \cdot c^3 \cdot x^2 / a^4 + 7 \cdot c^2 / a^3) + 35 \cdot c / a^2) \cdot x^2 + 35 / a) \cdot x / (c \cdot x^2 + a)^{7/2}$

maple [A] time = 0.05, size = 48, normalized size = 0.62

$$\frac{(16c^3x^6 + 56ac^2x^4 + 70a^2cx^2 + 35a^3)x}{35(c^2x^2 + a)^{7/2}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2+a)^(9/2),x)

[Out] $\frac{1}{35} \cdot x \cdot (16 \cdot c^3 \cdot x^6 + 56 \cdot a \cdot c^2 \cdot x^4 + 70 \cdot a^2 \cdot c \cdot x^2 + 35 \cdot a^3) / (c \cdot x^2 + a)^{7/2} / a^4$

maxima [A] time = 1.35, size = 61, normalized size = 0.79

$$\frac{16x}{35\sqrt{cx^2+a}a^4} + \frac{8x}{35(cx^2+a)^{3/2}a^3} + \frac{6x}{35(cx^2+a)^{5/2}a^2} + \frac{x}{7(cx^2+a)^{7/2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^2+a)^(9/2),x, algorithm="maxima")

[Out] $\frac{16}{35} \cdot x / (\sqrt{c \cdot x^2 + a} \cdot a^4) + \frac{8}{35} \cdot x / ((c \cdot x^2 + a)^{3/2} \cdot a^3) + \frac{6}{35} \cdot x / ((c \cdot x^2 + a)^{5/2} \cdot a^2) + \frac{1}{7} \cdot x / ((c \cdot x^2 + a)^{7/2} \cdot a)$

mupad [B] time = 0.20, size = 61, normalized size = 0.79

$$\frac{16x}{35a^4\sqrt{cx^2+a}} + \frac{8x}{35a^3(cx^2+a)^{3/2}} + \frac{6x}{35a^2(cx^2+a)^{5/2}} + \frac{x}{7a(cx^2+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^2)^(9/2),x)

[Out] $\frac{(16 \cdot x) / (35 \cdot a^4 \cdot (a + c \cdot x^2)^{1/2}) + (8 \cdot x) / (35 \cdot a^3 \cdot (a + c \cdot x^2)^{3/2}) + (6 \cdot x) / (35 \cdot a^2 \cdot (a + c \cdot x^2)^{5/2}) + x / (7 \cdot a \cdot (a + c \cdot x^2)^{7/2})$

sympy [B] time = 2.20, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**2+a)**(9/2),x)

[Out] $35*a^{14}*x/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a}) + 175*a^{13}*c*x^3/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a}) + 371*a^{12}*c^2*x^5/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a}) + 429*a^{11}*c^3*x^7/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a}) + 104*a^9*c^5*x^{11}/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a}) + 16*a^8*c^6*x^{13}/(35*a^{(37/2)}*\sqrt{1 + c*x^2/a} + 210*a^{(35/2)}*c*x^2*\sqrt{1 + c*x^2/a} + 525*a^{(33/2)}*c^2*x^4*\sqrt{1 + c*x^2/a} + 700*a^{(31/2)}*c^3*x^6*\sqrt{1 + c*x^2/a} + 525*a^{(29/2)}*c^4*x^8*\sqrt{1 + c*x^2/a} + 210*a^{(27/2)}*c^5*x^{10}*\sqrt{1 + c*x^2/a} + 35*a^{(25/2)}*c^6*x^{12}*\sqrt{1 + c*x^2/a})$

$$3.64 \quad \int (4 + 12x + 9x^2)^{3/2} dx$$

Optimal. Leaf size=23

$$\frac{1}{12}(3x + 2)(9x^2 + 12x + 4)^{3/2}$$

[Out] 1/12*(2+3*x)*(9*x^2+12*x+4)^(3/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {609}

$$\frac{1}{12}(3x + 2)(9x^2 + 12x + 4)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(4 + 12*x + 9*x^2)^(3/2), x]

[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rubi steps

$$\int (4 + 12x + 9x^2)^{3/2} dx = \frac{1}{12}(2 + 3x)(4 + 12x + 9x^2)^{3/2}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 0.87

$$\frac{1}{12}(3x + 2)((3x + 2)^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 12*x + 9*x^2)^(3/2), x]

[Out] ((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12

fricas [A] time = 1.04, size = 19, normalized size = 0.83

$$\frac{27}{4}x^4 + 18x^3 + 18x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="fricas")

[Out] 27/4*x^4 + 18*x^3 + 18*x^2 + 8*x

giac [B] time = 0.39, size = 45, normalized size = 1.96

$$\frac{3}{4} (3x^2 + 4x)^2 \operatorname{sgn}(3x + 2) + 2 (3x^2 + 4x) \operatorname{sgn}(3x + 2) + \frac{4}{3} \operatorname{sgn}(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="giac")

[Out] 3/4*(3*x^2 + 4*x)^2*sgn(3*x + 2) + 2*(3*x^2 + 4*x)*sgn(3*x + 2) + 4/3*sgn(3*x + 2)

maple [A] time = 0.05, size = 35, normalized size = 1.52

$$\frac{(27x^3 + 72x^2 + 72x + 32) \left((3x + 2)^2 \right)^{\frac{3}{2}} x}{4 (3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((9*x^2+12*x+4)^(3/2),x)

[Out] 1/4*x*(27*x^3+72*x^2+72*x+32)*((3*x+2)^2)^(3/2)/(3*x+2)^3

maxima [A] time = 2.96, size = 30, normalized size = 1.30

$$\frac{1}{4} (9x^2 + 12x + 4)^{\frac{3}{2}} x + \frac{1}{6} (9x^2 + 12x + 4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(3/2),x, algorithm="maxima")

[Out] 1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)

mupad [B] time = 0.05, size = 19, normalized size = 0.83

$$\frac{(9x + 6) (9x^2 + 12x + 4)^{3/2}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 9*x^2 + 4)^(3/2), x)
```

```
[Out] ((9*x + 6)*(12*x + 9*x^2 + 4)^(3/2))/36
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (9x^2 + 12x + 4)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2+12*x+4)**(3/2), x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(3/2), x)
```

3.65 $\int \sqrt{4 + 12x + 9x^2} dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x + 2)\sqrt{9x^2 + 12x + 4}$$

[Out] 1/6*(2+3*x)*((2+3*x)^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {609}

$$\frac{1}{6}(3x + 2)\sqrt{9x^2 + 12x + 4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4 + 12*x + 9*x^2], x]

[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rubi steps

$$\int \sqrt{4 + 12x + 9x^2} dx = \frac{1}{6}(2 + 3x)\sqrt{4 + 12x + 9x^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.09

$$\frac{x\sqrt{(3x + 2)^2(3x + 4)}}{6x + 4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 + 12*x + 9*x^2], x]

[Out] (x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)

fricas [A] time = 0.99, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2 + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="fricas")

[Out] 3/2*x^2 + 2*x

giac [A] time = 0.38, size = 26, normalized size = 1.13

$$\frac{1}{2}(3x^2 + 4x)\operatorname{sgn}(3x + 2) + \frac{2}{3}\operatorname{sgn}(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="giac")

[Out] 1/2*(3*x^2 + 4*x)*sgn(3*x + 2) + 2/3*sgn(3*x + 2)

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{(3x + 4)\sqrt{(3x + 2)^2 x}}{6x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((9*x^2+12*x+4)^(1/2),x)

[Out] 1/2*x*(3*x+4)*((3*x+2)^2)^(1/2)/(3*x+2)

maxima [A] time = 2.86, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{9x^2 + 12x + 4}x + \frac{1}{3}\sqrt{9x^2 + 12x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+12*x+4)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)

mupad [B] time = 0.05, size = 19, normalized size = 0.83

$$\frac{(3x + 2)\sqrt{9x^2 + 12x + 4}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12*x + 9*x^2 + 4)^(1/2),x)

[Out] $((3x + 2)(12x + 9x^2 + 4)^{1/2})/6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{9x^2 + 12x + 4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9*x**2+12*x+4)**(1/2),x)`

[Out] `Integral(sqrt(9*x**2 + 12*x + 4), x)`

$$3.66 \quad \int \frac{1}{\sqrt{4+12x+9x^2}} dx$$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

[Out] 1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 + 12*x + 9*x^2], x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{4+12x+9x^2}} dx &= \frac{(6+9x) \int \frac{1}{6+9x} dx}{\sqrt{4+12x+9x^2}} \\ &= \frac{(2+3x)\log(2+3x)}{3\sqrt{4+12x+9x^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4 + 12*x + 9*x^2], x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])

fricas [A] time = 0.79, size = 8, normalized size = 0.28

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(1/2), x, algorithm="fricas")

[Out] 1/3*log(3*x + 2)

giac [A] time = 0.29, size = 25, normalized size = 0.86

$$\frac{\log(|3x + 2| |\operatorname{sgn}(3x + 2)|)}{3 \operatorname{sgn}(3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(1/2), x, algorithm="giac")

[Out] 1/3*log(abs(3*x + 2)*abs(sgn(3*x + 2)))/sgn(3*x + 2)

maple [A] time = 0.05, size = 23, normalized size = 0.79

$$\frac{(3x + 2) \ln(3x + 2)}{3\sqrt{(3x + 2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2+12*x+4)^(1/2), x)

[Out] 1/3*(3*x+2)*ln(3*x+2)/((3*x+2)^2)^(1/2)

maxima [A] time = 2.83, size = 6, normalized size = 0.21

$$\frac{1}{3} \log\left(x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(1/2), x, algorithm="maxima")

[Out] $\frac{1}{3}\log(x + \frac{2}{3})$

mupad [B] time = 0.28, size = 14, normalized size = 0.48

$$\frac{\ln(9x + 6) \operatorname{sign}(18x + 12)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(12*x + 9*x^2 + 4)^(1/2),x)`

[Out] `(log(9*x + 6)*sign(18*x + 12))/3`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^2 + 12x + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(9*x**2+12*x+4)**(1/2),x)`

[Out] `Integral(1/sqrt(9*x**2 + 12*x + 4), x)`

$$3.67 \quad \int \frac{1}{(4+12x+9x^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

[Out] -1/6/(2+3*x)/((2+3*x)^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {607}

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

[In] Int[(4 + 12*x + 9*x^2)^(-3/2), x]

[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])

Rule 607

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(2*(a + b*x + c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\int \frac{1}{(4+12x+9x^2)^{3/2}} dx = -\frac{1}{6(2+3x)\sqrt{4+12x+9x^2}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{3x+2}{6((3x+2)^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 12*x + 9*x^2)^(-3/2), x]

[Out] -1/6*(2 + 3*x)/((2 + 3*x)^2)^(3/2)

fricas [A] time = 0.96, size = 14, normalized size = 0.56

$$-\frac{1}{6(9x^2 + 12x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="fricas")

[Out] -1/6/(9*x^2 + 12*x + 4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\text{sage0}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")

[Out] sage0*x

maple [A] time = 0.05, size = 17, normalized size = 0.68

$$-\frac{3x + 2}{6((3x + 2)^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2+12*x+4)^(3/2),x)

[Out] -1/6*(3*x+2)/((3*x+2)^2)^(3/2)

maxima [A] time = 2.89, size = 9, normalized size = 0.36

$$-\frac{1}{6(3x + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="maxima")

[Out] -1/6/(3*x + 2)^2

mupad [B] time = 0.17, size = 21, normalized size = 0.84

$$-\frac{\sqrt{9x^2 + 12x + 4}}{6(3x + 2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(12*x + 9*x^2 + 4)^(3/2), x)`

[Out] `-(12*x + 9*x^2 + 4)^(1/2)/(6*(3*x + 2)^3)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(9x^2 + 12x + 4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(9*x**2+12*x+4)**(3/2), x)`

[Out] `Integral((9*x**2 + 12*x + 4)**(-3/2), x)`

$$3.68 \quad \int \sqrt{4 - 12x + 9x^2} dx$$

Optimal. Leaf size=23

$$-\frac{1}{6}(2 - 3x)\sqrt{9x^2 - 12x + 4}$$

[Out] $-1/6*(2-3*x)*((-2+3*x)^2)^{(1/2)}$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {609}

$$-\frac{1}{6}(2 - 3x)\sqrt{9x^2 - 12x + 4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[4 - 12*x + 9*x^2], x]

[Out] $-((2 - 3*x)*\text{Sqrt}[4 - 12*x + 9*x^2])/6$

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rubi steps

$$\int \sqrt{4 - 12x + 9x^2} dx = -\frac{1}{6}(2 - 3x)\sqrt{4 - 12x + 9x^2}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.09

$$\frac{\sqrt{(2 - 3x)^2} x(3x - 4)}{6x - 4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[4 - 12*x + 9*x^2], x]

[Out] $(\text{Sqrt}[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)$

fricas [A] time = 0.96, size = 9, normalized size = 0.39

$$\frac{3}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−2+3*x)^2)^(1/2),x, algorithm="fricas")

[Out] 3/2*x^2 - 2*x

giac [A] time = 0.35, size = 26, normalized size = 1.13

$$\frac{1}{2} (3x^2 - 4x) \operatorname{sgn}(3x - 2) + \frac{2}{3} \operatorname{sgn}(3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−2+3*x)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*(3*x^2 - 4*x)*sgn(3*x - 2) + 2/3*sgn(3*x - 2)

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{(3x - 4) \sqrt{(3x - 2)^2} x}{6x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((−2+3*x)^2)^(1/2),x)

[Out] 1/2*x*(3*x-4)*((−2+3*x)^2)^(1/2)/(−2+3*x)

maxima [A] time = 3.01, size = 30, normalized size = 1.30

$$\frac{1}{2} \sqrt{9x^2 - 12x + 4} x - \frac{1}{3} \sqrt{9x^2 - 12x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((−2+3*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)

mupad [B] time = 0.11, size = 13, normalized size = 0.57

$$\frac{|3x - 2| (3x - 2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x - 2)^2)^(1/2),x)

[Out] (abs(3*x - 2)*(3*x - 2))/6

sympy [A] time = 0.08, size = 8, normalized size = 0.35

$$\frac{3x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((( -2+3*x)**2)**(1/2), x)
```

```
[Out] 3*x**2/2 - 2*x
```

$$3.69 \quad \int \frac{1}{\sqrt{4-12x+9x^2}} dx$$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

[Out] $-1/3*(2-3*x)*\ln(2-3*x)/((-2+3*x)^2)^{(1/2)}$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 - 12*x + 9*x^2], x]

[Out] $-((2 - 3*x)*\text{Log}[2 - 3*x])/(3*\text{Sqrt}[4 - 12*x + 9*x^2])$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{4-12x+9x^2}} dx &= \frac{(-6+9x) \int \frac{1}{-6+9x} dx}{\sqrt{4-12x+9x^2}} \\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{4-12x+9x^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]

[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[(2 - 3*x)^2]

fricas [A] time = 0.87, size = 8, normalized size = 0.28

$$\frac{1}{3} \log(3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/3*log(3*x - 2)

giac [A] time = 0.46, size = 15, normalized size = 0.52

$$\frac{1}{3} \log(|3x - 2|) \operatorname{sgn}(3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*log(abs(3*x - 2))*sgn(3*x - 2)

maple [A] time = 0.05, size = 23, normalized size = 0.79

$$\frac{(3x - 2) \ln(3x - 2)}{3\sqrt{(3x - 2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((3*x-2)^2)^(1/2),x)

[Out] 1/3/((3*x-2)^2)^(1/2)*(3*x-2)*ln(3*x-2)

maxima [A] time = 2.97, size = 6, normalized size = 0.21

$$\frac{1}{3} \log\left(x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2+3*x)^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{3}\log(x - 2/3)$

mupad [B] time = 0.35, size = 14, normalized size = 0.48

$$\frac{\ln(3x - 2) \operatorname{sign}(3x - 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((3*x - 2)^2)^(1/2), x)`

[Out] $(\log(3x - 2) * \operatorname{sign}(3x - 2)) / 3$

sympy [A] time = 0.08, size = 7, normalized size = 0.24

$$\frac{\log(3x - 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-2+3*x)**2)**(1/2), x)`

[Out] $\log(3x - 2) / 3$

3.70 $\int \sqrt{-4 + 12x - 9x^2} dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

[Out] $-1/6*(2-3*x)*(-(-2+3*x)^2)^{(1/2)}$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {609}

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[-4 + 12*x - 9*x^2], x]$

[Out] $-((2 - 3*x)*\text{Sqrt}[-4 + 12*x - 9*x^2])/6$

Rule 609

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] :> \text{Simp}[(b + 2*c*x) * (a + b*x + c*x^2)^p] / (2*c*(2*p + 1)), x] /;$ $\text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[p, -2^{(-1)}]$

Rubi steps

$$\int \sqrt{-4 + 12x - 9x^2} dx = -\frac{1}{6}(2-3x)\sqrt{-4 + 12x - 9x^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.17

$$\frac{\sqrt{-(2-3x)^2}x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sqrt}[-4 + 12*x - 9*x^2], x]$

[Out] $(\text{Sqrt}[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)$

fricas [C] time = 0.72, size = 9, normalized size = 0.39

$$\frac{3}{2}ix^2 - 2ix$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="fricas")

[Out] 3/2*I*x^2 - 2*I*x

giac [C] time = 0.38, size = 26, normalized size = 1.13

$$-\frac{1}{2}i(3x^2 - 4x)\operatorname{sgn}(-3x + 2) - \frac{2}{3}i\operatorname{sgn}(-3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*I*(3*x^2 - 4*x)*sgn(-3*x + 2) - 2/3*I*sgn(-3*x + 2)

maple [A] time = 0.05, size = 27, normalized size = 1.17

$$\frac{(3x - 4)\sqrt{-(3x - 2)^2}x}{6x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- (3*x-2)^2)^(1/2),x)

[Out] 1/2*x*(3*x-4)*(- (3*x-2)^2)^(1/2)/(3*x-2)

maxima [A] time = 3.02, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{-9x^2 + 12x - 4}x - \frac{1}{3}\sqrt{-9x^2 + 12x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-2+3*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)

mupad [B] time = 0.33, size = 18, normalized size = 0.78

$$\frac{(3x - 2)\sqrt{-(3x - 2)^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- (3*x - 2)^2)^(1/2),x)

```
[Out] ((3*x - 2)*(-(3*x - 2)^2)^(1/2))/6
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{-(3x - 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-2+3*x)**2)**(1/2), x)
```

```
[Out] Integral(sqrt(-(3*x - 2)**2), x)
```

$$3.71 \quad \int \frac{1}{\sqrt{-4+12x-9x^2}} dx$$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

[Out] $-1/3*(2-3*x)*\ln(2-3*x)/(-(-2+3*x)^2)^{(1/2)}$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {608, 31}

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-4 + 12*x - 9*x^2], x]

[Out] $-((2 - 3*x)*\text{Log}[2 - 3*x])/(3*\text{Sqrt}[-4 + 12*x - 9*x^2])$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-4+12x-9x^2}} dx &= \frac{(6-9x) \int \frac{1}{6-9x} dx}{\sqrt{-4+12x-9x^2}} \\ &= -\frac{(2-3x)\log(2-3x)}{3\sqrt{-4+12x-9x^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.97

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-4 + 12*x - 9*x^2], x]

[Out] -1/3*((2 - 3*x)*Log[2 - 3*x])/Sqrt[-(2 - 3*x)^2]

fricas [C] time = 1.11, size = 6, normalized size = 0.21

$$-\frac{1}{3}i \log\left(x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(-2+3*x)^2)^(1/2), x, algorithm="fricas")

[Out] -1/3*I*log(x - 2/3)

giac [C] time = 0.45, size = 23, normalized size = 0.79

$$\frac{i \log\left((-3ix + 2i)\operatorname{sgn}(-3x + 2)\right)}{3 \operatorname{sgn}(-3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(-2+3*x)^2)^(1/2), x, algorithm="giac")

[Out] 1/3*I*log((-3*I*x + 2*I)*sgn(-3*x + 2))/sgn(-3*x + 2)

maple [A] time = 0.05, size = 25, normalized size = 0.86

$$\frac{(3x - 2) \ln(3x - 2)}{3\sqrt{-(3x - 2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-(3*x-2)^2)^(1/2), x)

[Out] 1/3/(-(3*x-2)^2)^(1/2)*(3*x-2)*ln(3*x-2)

maxima [C] time = 2.90, size = 6, normalized size = 0.21

$$\frac{1}{3}i \log\left(x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(-2+3*x)^2)^(1/2), x, algorithm="maxima")

[Out] $1/3*I*\log(x - 2/3)$

mupad [B] time = 0.25, size = 15, normalized size = 0.52

$$\frac{\ln(2 - 3x) \operatorname{sign}(3x - 2) i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-(3*x - 2)^2)^(1/2), x)`

[Out] $-(\log(2 - 3*x)*\operatorname{sign}(3*x - 2)*i)/3$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(3x - 2)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-(-2+3*x)**2)**(1/2), x)`

[Out] `Integral(1/sqrt(-(3*x - 2)**2), x)`

$$3.72 \quad \int \sqrt{-4 - 12x - 9x^2} dx$$

Optimal. Leaf size=23

$$\frac{1}{6}(3x + 2)\sqrt{-9x^2 - 12x - 4}$$

[Out] 1/6*(2+3*x)*(-(2+3*x)^2)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {609}

$$\frac{1}{6}(3x + 2)\sqrt{-9x^2 - 12x - 4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-4 - 12*x - 9*x^2], x]

[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x) * (a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rubi steps

$$\int \sqrt{-4 - 12x - 9x^2} dx = \frac{1}{6}(2 + 3x)\sqrt{-4 - 12x - 9x^2}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.17

$$\frac{x\sqrt{-(3x+2)^2}(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-4 - 12*x - 9*x^2], x]

[Out] (x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)

fricas [C] time = 0.85, size = 9, normalized size = 0.39

$$\frac{3}{2}ix^2 + 2ix$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)^2)^(1/2),x, algorithm="fricas")

[Out] 3/2*I*x^2 + 2*I*x

giac [C] time = 0.41, size = 26, normalized size = 1.13

$$-\frac{1}{2}i(3x^2 + 4x)\operatorname{sgn}(-3x - 2) - \frac{2}{3}i\operatorname{sgn}(-3x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*I*(3*x^2 + 4*x)*sgn(-3*x - 2) - 2/3*I*sgn(-3*x - 2)

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$\frac{(3x + 4)\sqrt{-(3x + 2)^2} x}{6x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x+2)^2)^(1/2),x)

[Out] 1/2*x*(3*x+4)*(-3*x+2)^2)^(1/2)/(3*x+2)

maxima [A] time = 2.96, size = 30, normalized size = 1.30

$$\frac{1}{2}\sqrt{-9x^2 - 12x - 4}x + \frac{1}{3}\sqrt{-9x^2 - 12x - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+3*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)

mupad [B] time = 0.07, size = 18, normalized size = 0.78

$$\frac{(3x + 2)\sqrt{-(3x + 2)^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x + 2)^2)^(1/2),x)

```
[Out] ((3*x + 2)*(-(3*x + 2)^2)^(1/2))/6
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{-(3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)**2)**(1/2),x
```

```
[Out] Integral(sqrt(-(3*x + 2)**2), x)
```

$$3.73 \quad \int \frac{1}{\sqrt{-4-12x-9x^2}} dx$$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {608, 31}

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-4 - 12*x - 9*x^2], x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 608

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(b/2 + c*x)/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-4-12x-9x^2}} dx = - \left(- \frac{(-6-9x) \int \frac{1}{-6-9x} dx}{\sqrt{-4-12x-9x^2}} \right) = \frac{(2+3x)\log(2+3x)}{3\sqrt{-4-12x-9x^2}}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 0.97

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])

fricas [C] time = 0.86, size = 6, normalized size = 0.21

$$-\frac{1}{3}i \log\left(x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/3*I*log(x + 2/3)

giac [C] time = 0.34, size = 23, normalized size = 0.79

$$\frac{i \log\left((-3ix - 2i)\operatorname{sgn}(-3x - 2)\right)}{3 \operatorname{sgn}(-3x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)

maple [A] time = 0.05, size = 25, normalized size = 0.86

$$\frac{(3x + 2) \ln(3x + 2)}{3\sqrt{-(3x + 2)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-(3*x+2)^2)^(1/2),x)

[Out] 1/3*(3*x+2)/(-(3*x+2)^2)^(1/2)*ln(3*x+2)

maxima [C] time = 2.78, size = 6, normalized size = 0.21

$$\frac{1}{3}i \log\left(x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")

[Out] $1/3*I*\log(x + 2/3)$

mupad [B] time = 0.29, size = 15, normalized size = 0.52

$$\frac{\ln(-3x - 2) \operatorname{sign}(3x + 2) i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-(3*x + 2)^2)^(1/2), x)`

[Out] $-(\log(-3x - 2) * \operatorname{sign}(3x + 2) * i) / 3$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(3x + 2)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-(2+3*x)**2)**(1/2), x)`

[Out] `Integral(1/sqrt(-(3*x + 2)**2), x)`

$$3.74 \quad \int \left(\frac{-1+b^2}{4c} + bx + cx^2 \right)^5 dx$$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+1)^{11}}{22528c^6} + \frac{(-b-2cx+1)^{10}}{2048c^6} - \frac{5(-b-2cx+1)^9}{2304c^6} + \frac{5(-b-2cx+1)^8}{1024c^6} - \frac{5(-b-2cx+1)^7}{896c^6} + \frac{(-b-2cx+1)^6}{384c^6}$$

[Out] 1/384*(-2*c*x-b+1)^6/c^6-5/896*(-2*c*x-b+1)^7/c^6+5/1024*(-2*c*x-b+1)^8/c^6-5/2304*(-2*c*x-b+1)^9/c^6+1/2048*(-2*c*x-b+1)^10/c^6-1/22528*(-2*c*x-b+1)^11/c^6

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {610, 43}

$$-\frac{(-b-2cx+1)^{11}}{22528c^6} + \frac{(-b-2cx+1)^{10}}{2048c^6} - \frac{5(-b-2cx+1)^9}{2304c^6} + \frac{5(-b-2cx+1)^8}{1024c^6} - \frac{5(-b-2cx+1)^7}{896c^6} + \frac{(-b-2cx+1)^6}{384c^6}$$

Antiderivative was successfully verified.

[In] Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (5*(1 - b - 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6) + (1 - b - 2*c*x)^10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 610

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c*x, x]^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\int \left(\frac{-1+b^2}{4c} + bx + cx^2 \right)^5 dx = \frac{\int \left(\frac{1}{2}(-1+b) + cx \right)^5 \left(\frac{1+b}{2} + cx \right)^5 dx}{c^5}$$

$$= \frac{\int \left(\left(\frac{1}{2}(-1+b) + cx \right)^5 + 5 \left(\frac{1}{2}(-1+b) + cx \right)^6 + 10 \left(\frac{1}{2}(-1+b) + cx \right)^7 + 10 \left(\frac{1}{2}(-1+b) + cx \right)^8 + 5 \left(\frac{1}{2}(-1+b) + cx \right)^9 + \left(\frac{1}{2}(-1+b) + cx \right)^{10} \right) dx}{c^5}$$

$$= \frac{(1-b-2cx)^6}{384c^6} - \frac{5(1-b-2cx)^7}{896c^6} + \frac{5(1-b-2cx)^8}{1024c^6} - \frac{5(1-b-2cx)^9}{2304c^6} + \frac{(1-b-2cx)^{10}}{2048c^6}$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.90

$$\frac{5}{8} (3b^3 - b) c^2 x^8 + \frac{(b^2 - 1)^5 x}{1024 c^5} + \frac{5b(b^2 - 1)^4 x^2}{512 c^4} + \frac{5}{36} (9b^2 - 1) c^3 x^9 + \frac{5(b^2 - 1)^3 (9b^2 - 1) x^3}{768 c^3} + \frac{5b(b^2 - 1)^2 (3b^2 - 1)}{64 c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] ((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) + (5*(-1 + b^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(-1 + 3*b^2)*x^4)/(64*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5)/(32*c) + (b*(15 - 70*b^2 + 63*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 21*b^4)*c*x^7)/56 + (5*(-b + 3*b^3)*c^2*x^8)/8 + (5*(-1 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11

fricas [B] time = 0.79, size = 233, normalized size = 2.14

$$64512 c^{10} x^{11} + 354816 b c^9 x^{10} + 98560 (9 b^2 - 1) c^8 x^9 + 443520 (3 b^3 - b) c^7 x^8 + 63360 (21 b^4 - 14 b^2 + 1) c^6 x^7 + 14784 (63 b^5 - 70 b^3 + 15 b) c^5 x^6 + 22176 (21 b^6 - 35 b^4 + 15 b^2 - 1) c^4 x^5 + 55440 (3 b^7 - 7 b^5 + 5 b^3 - b) c^3 x^4 + 4620 (9 b^8 - 28 b^6 + 30 b^4 - 12 b^2 + 1) c^2 x^3 + 6930 (b^9 - 4 b^7 + 6 b^5 - 4 b^3 + b) c x^2 + 693 (b^{10} - 5 b^8 + 10 b^6 - 10 b^4 + 5 b^2 - 1) x) / c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="fricas")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)*c^8*x^9 + 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 + 1)*c^6*x^7 + 14784*(63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b^6 - 35*b^4 + 15*b^2 - 1)*c^4*x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3 - b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6 + 30*b^4 - 12*b^2 + 1)*c^2*x^3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*x^2 + 693*(b^10 - 5*b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5

giac [B] time = 0.43, size = 334, normalized size = 3.06

$$64512 c^{10} x^{11} + 354816 b c^9 x^{10} + 887040 b^2 c^8 x^9 + 1330560 b^3 c^7 x^8 + 1330560 b^4 c^6 x^7 - 98560 c^8 x^9 + 931392 b^5 c^5 x^6 + 22176 (21 b^6 - 35 b^4 + 15 b^2 - 1) c^4 x^5 + 55440 (3 b^7 - 7 b^5 + 5 b^3 - b) c^3 x^4 + 4620 (9 b^8 - 28 b^6 + 30 b^4 - 12 b^2 + 1) c^2 x^3 + 6930 (b^9 - 4 b^7 + 6 b^5 - 4 b^3 + b) c x^2 + 693 (b^{10} - 5 b^8 + 10 b^6 - 10 b^4 + 5 b^2 - 1) x) / c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="giac")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 1330560*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 98560*c^8*x^9 + 931392*b^5*c^5*x^6 - 443520*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 887040*b^2*c^6*x^7 + 166320*b^7*c^3*x^4 - 1034880*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 776160*b^4*c^4*x^5 + 63360*c^6*x^7 + 6930*b^9*c*x^2 - 388080*b^5*c^3*x^4 + 221760*b*c^5*x^6 + 693*b^10*x - 129360*b^6*c^2*x^3 + 332640*b^2*c^4*x^5 - 27720*b^7*c*x^2 + 277200*b^3*c^3*x^4 - 3465*b^8*x + 138600*b^4*c^2*x^3 - 22176*c^4*x^5 + 41580*b^5*c*x^2 - 55440*b*c^3*x^4 + 6930*b^6*x - 55440*b^2*c^2*x^3 - 27720*b^3*c*x^2 - 6930*b^4*x + 4620*c^2*x^3 + 6930*b*c*x^2 + 3465*b^2*x - 693*x)/c^5

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^5 x^{11}}{11} + \frac{b c^4 x^{10}}{2} + \frac{\left(4b^2 c^3 + \frac{(b^2-1)c^3}{4} + \left(4b^2 c^2 + 2\left(\frac{3b^2}{2} - \frac{1}{2}\right)c^2\right)c\right) x^9}{9} + \frac{\left((b^2-1) b c^2 + \left(4b^2 c^2 + 2\left(\frac{3b^2}{2} - \frac{1}{2}\right)c^2\right) b\right) b + \dots}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x)

[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-1)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-1)*c^2*b+b*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2)+c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c))*x^8+1/7*(1/4*(b^2-1)/c*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2)+b*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+c*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2))*x^7+1/6*(1/4*(b^2-1)/c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+b*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2)))*x^6+1/5*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+b*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-1)/c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3/2*b^2-1/2))+b*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16/c^2*(b^2-1)^3*b)*x^4+1/3*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16*b^2*(b^2-1)^3/c^3+1/256/c^3*(b^2-1)^4)*x^3+5/512*(b^2-1)^4/c^4*b*x^2+1/1024*(b^2-1)^5/c^5*x

maxima [B] time = 1.41, size = 234, normalized size = 2.15

$$\frac{1}{11} c^5 x^{11} + \frac{1}{2} b c^4 x^{10} + \frac{10}{9} b^2 c^3 x^9 + \frac{5}{4} b^3 c^2 x^8 + \frac{5}{7} b^4 c x^7 + \frac{1}{6} b^5 x^6 + \frac{5(2cx^3 + 3bx^2)(b^2-1)^4}{1536c^4} + \frac{(6c^2x^5 + 15bcx^4 + 10b^2x^3)}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-1)/c+b*x+c*x^2)^5,x, algorithm="maxima")

[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 1)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 1)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 1)/c + 1/1024*(b^2 - 1)^5*x/c^5

mupad [B] time = 0.31, size = 184, normalized size = 1.69

$$\frac{c^5 x^{11}}{11} + \frac{x(b^2 - 1)^5}{1024 c^5} + \frac{b x^6 (63 b^4 - 70 b^2 + 15)}{48} + \frac{5 c x^7 (21 b^4 - 14 b^2 + 1)}{56} + \frac{b c^4 x^{10}}{2} + \frac{5 c^3 x^9 (9 b^2 - 1)}{36} + \frac{x^5 (21 b^6 - 21 b^4 + 15 b^2 - 1)}{1024 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2 + (b^2/4 - 1/4)/c)^5,x)

[Out] (c^5*x^11)/11 + (x*(b^2 - 1)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 70*b^2 + 15))/48 + (5*c*x^7*(21*b^4 - 14*b^2 + 1))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9*b^2 - 1))/36 + (x^5*(15*b^2 - 35*b^4 + 21*b^6 - 1))/(32*c) + (5*b*c^2*x^8*(3*b^2 - 1))/8 + (5*b*x^2*(b^2 - 1)^4)/(512*c^4) + (5*x^3*(b^2 - 1)^3*(9*b^2 - 1))/(768*c^3) + (5*b*x^4*(b^2 - 1)^2*(3*b^2 - 1))/(64*c^2)

sympy [B] time = 0.19, size = 253, normalized size = 2.32

$$\frac{b c^4 x^{10}}{2} + \frac{c^5 x^{11}}{11} + x^9 \left(\frac{5 b^2 c^3}{4} - \frac{5 c^3}{36} \right) + x^8 \left(\frac{15 b^3 c^2}{8} - \frac{5 b c^2}{8} \right) + x^7 \left(\frac{15 b^4 c}{8} - \frac{5 b^2 c}{4} + \frac{5 c}{56} \right) + x^6 \left(\frac{21 b^5}{16} - \frac{35 b^3}{24} + \frac{5 b}{16} \right) + \frac{x^5 (21 b^6 - 21 b^4 + 15 b^2 - 1)}{1024 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)

[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36) + x**8*(15*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**2*c/4 + 5*c/56) + x**6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*(21*b**6 - 35*b**4 + 15*b**2 - 1)/(32*c) + x**4*(15*b**7 - 35*b**5 + 25*b**3 - 5*b)/(64*c**2) + x**3*(45*b**8 - 140*b**6 + 150*b**4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 - 20*b**7 + 30*b**5 - 20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**6 - 10*b**4 + 5*b**2 - 1)/(1024*c**5)

$$3.75 \quad \int \left(\frac{-4+b^2}{4c} + bx + cx^2 \right)^5 dx$$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+2)^{11}}{22528c^6} + \frac{(-b-2cx+2)^{10}}{1024c^6} - \frac{5(-b-2cx+2)^9}{576c^6} + \frac{5(-b-2cx+2)^8}{128c^6} - \frac{5(-b-2cx+2)^7}{56c^6} + \frac{(-b-2cx+2)^6}{12c^6}$$

[Out] 1/12*(-2*c*x-b+2)^6/c^6-5/56*(-2*c*x-b+2)^7/c^6+5/128*(-2*c*x-b+2)^8/c^6-5/576*(-2*c*x-b+2)^9/c^6+1/1024*(-2*c*x-b+2)^10/c^6-1/22528*(-2*c*x-b+2)^11/c^6

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {610, 43}

$$-\frac{(-b-2cx+2)^{11}}{22528c^6} + \frac{(-b-2cx+2)^{10}}{1024c^6} - \frac{5(-b-2cx+2)^9}{576c^6} + \frac{5(-b-2cx+2)^8}{128c^6} - \frac{5(-b-2cx+2)^7}{56c^6} + \frac{(-b-2cx+2)^6}{12c^6}$$

Antiderivative was successfully verified.

[In] Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]

[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*(2 - b - 2*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) + (2 - b - 2*c*x)^10/(1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 610

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c*x, x]^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\int \left(\frac{-4 + b^2}{4c} + bx + cx^2 \right)^5 dx = \frac{\int \left(\frac{1}{2}(-2 + b) + cx \right)^5 \left(\frac{2+b}{2} + cx \right)^5 dx}{c^5}$$

$$= \frac{\int \left(32 \left(\frac{1}{2}(-2 + b) + cx \right)^5 + 80 \left(\frac{1}{2}(-2 + b) + cx \right)^6 + 80 \left(\frac{1}{2}(-2 + b) + cx \right)^7 + 40 \left(\frac{1}{2}(-2 + b) + cx \right)^8 + 10 \left(\frac{1}{2}(-2 + b) + cx \right)^9 \right) dx}{c^5}$$

$$= \frac{(2 - b - 2cx)^6}{12c^6} - \frac{5(2 - b - 2cx)^7}{56c^6} + \frac{5(2 - b - 2cx)^8}{128c^6} - \frac{5(2 - b - 2cx)^9}{576c^6} + \frac{(2 - b - 2cx)^{10}}{1024c^6}$$

Mathematica [A] time = 0.04, size = 207, normalized size = 1.90

$$\frac{5}{8} (3b^3 - 4b) c^2 x^8 + \frac{(b^2 - 4)^5 x}{1024c^5} + \frac{5b(b^2 - 4)^4 x^2}{512c^4} + \frac{5}{36} (9b^2 - 4) c^3 x^9 + \frac{5(b^2 - 4)^3 (9b^2 - 4) x^3}{768c^3} + \frac{5b(b^2 - 4)^2 (3b^2 - 4) x^4}{64c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] ((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) + (5*(-4 + b^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(-4 + 3*b^2)*x^4)/(64*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^5)/(32*c) + (b*(240 - 280*b^2 + 63*b^4)*x^6)/48 + (5*(16 - 56*b^2 + 21*b^4)*c*x^7)/56 + (5*(-4*b + 3*b^3)*c^2*x^8)/8 + (5*(-4 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11

fricas [B] time = 0.92, size = 235, normalized size = 2.16

$$\frac{64512 c^{10} x^{11} + 354816 b c^9 x^{10} + 98560 (9 b^2 - 4) c^8 x^9 + 443520 (3 b^3 - 4 b) c^7 x^8 + 63360 (21 b^4 - 56 b^2 + 16) c^6 x^7 + 14784 (63 b^5 - 280 b^3 + 240 b) c^5 x^6 + 22176 (21 b^6 - 140 b^4 + 240 b^2 - 64) c^4 x^5 + 55440 (3 b^7 - 28 b^5 + 80 b^3 - 64 b) c^3 x^4 + 4620 (9 b^8 - 112 b^6 + 480 b^4 - 768 b^2 + 256) c^2 x^3 + 6930 (b^9 - 16 b^7 + 96 b^5 - 256 b^3 + 256 b) c x^2 + 693 (b^{10} - 20 b^8 + 160 b^6 - 640 b^4 + 1280 b^2 - 1024) x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="fricas")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)*c^8*x^9 + 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2 + 16)*c^6*x^7 + 14784*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*(21*b^6 - 140*b^4 + 240*b^2 - 64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*b^8 - 112*b^6 + 480*b^4 - 768*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96*b^5 - 256*b^3 + 256*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1280*b^2 - 1024)*x)/c^5

giac [B] time = 0.41, size = 334, normalized size = 3.06

$$\frac{64512c^{10}x^{11} + 354816bc^9x^{10} + 887040b^2c^8x^9 + 1330560b^3c^7x^8 + 1330560b^4c^6x^7 - 394240c^8x^9 + 931392b^5c^5x^6 - 1774080bc^7x^8 + 465696b^6c^4x^5 - 3548160b^2c^6x^7 + 166320b^7c^3x^4 - 4139520b^3c^5x^6 + 41580b^8c^2x^3 - 3104640b^4c^4x^5 + 1013760c^6x^7 + 6930b^9cx^2 - 1552320b^5c^3x^4 + 3548160b^6c^5x^6 + 693b^{10}x - 517440b^6c^2x^3 + 5322240b^2c^4x^5 - 110880b^7cx^2 + 4435200b^3c^3x^4 - 13860b^8x + 2217600b^4c^2x^3 - 1419264c^4x^5 + 665280b^5cx^2 - 3548160b^6c^3x^4 + 110880b^6x - 3548160b^2c^2x^3 - 1774080b^3cx^2 - 443520b^4x + 1182720c^2x^3 + 1774080b^2cx^2 + 887040b^2x - 709632x)/c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="giac")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 1330560*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 394240*c^8*x^9 + 931392*b^5*c^5*x^6 - 1774080*b*c^7*x^8 + 465696*b^6*c^4*x^5 - 3548160*b^2*c^6*x^7 + 166320*b^7*c^3*x^4 - 4139520*b^3*c^5*x^6 + 41580*b^8*c^2*x^3 - 3104640*b^4*c^4*x^5 + 1013760*c^6*x^7 + 6930*b^9*c*x^2 - 1552320*b^5*c^3*x^4 + 3548160*b^6*c^5*x^6 + 693*b^{10}*x - 517440*b^6*c^2*x^3 + 5322240*b^2*c^4*x^5 - 110880*b^7*c*x^2 + 4435200*b^3*c^3*x^4 - 13860*b^8*x + 2217600*b^4*c^2*x^3 - 1419264*c^4*x^5 + 665280*b^5*c*x^2 - 3548160*b^6*c^3*x^4 + 110880*b^6*x - 3548160*b^2*c^2*x^3 - 1774080*b^3*c*x^2 - 443520*b^4*x + 1182720*c^2*x^3 + 1774080*b^2*c*x^2 + 887040*b^2*x - 709632*x)/c^5

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^5x^{11}}{11} + \frac{bc^4x^{10}}{2} + \frac{\left(4b^2c^3 + \frac{(b^2-4)c^3}{4} + \left(4b^2c^2 + 2\left(\frac{3b^2}{2} - 2\right)c^2\right)c\right)x^9}{9} + \frac{\left((b^2-4)bc^2 + \left(4b^2c^2 + 2\left(\frac{3b^2}{2} - 2\right)c^2\right)b\right)b + \dots}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x)

[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-4)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-4)*c^2*b+b*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c))*x^8+1/7*(1/4*(b^2-4)/c*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+b*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2))*x^7+1/6*(1/4*(b^2-4)/c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+b*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2)+c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2)))*x^6+1/5*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2)+b*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+c*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-4)/c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+b*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16/c^2*(b^2-4)^3*b))*x^4+1/3*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16*b^2*(b^2-4)^3/c^3+1/256/c^3*(b^2-4)^4)*x^3+5/512*(b^2-4)^4/c^4*b*x^2+1/1024*(b^2-4)^5/c^5*x

maxima [B] time = 1.52, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11} + \frac{1}{2}bc^4x^{10} + \frac{10}{9}b^2c^3x^9 + \frac{5}{4}b^3c^2x^8 + \frac{5}{7}b^4cx^7 + \frac{1}{6}b^5x^6 + \frac{5(2cx^3 + 3bx^2)(b^2 - 4)^4}{1536c^4} + \frac{(6c^2x^5 + 15bcx^4 + 10b^2x^3)(b^2 - 4)^3}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-4)/c+b*x+c*x^2)^5,x, algorithm="maxima")

[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 4)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 4)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 4)/c + 1/1024*(b^2 - 4)^5*x/c^5

mupad [B] time = 0.33, size = 184, normalized size = 1.69

$$\frac{c^5x^{11}}{11} + \frac{x(b^2 - 4)^5}{1024c^5} + \frac{bx^6(63b^4 - 280b^2 + 240)}{48} + \frac{5cx^7(21b^4 - 56b^2 + 16)}{56} + \frac{bc^4x^{10}}{2} + \frac{5c^3x^9(9b^2 - 4)}{36} + \frac{x^5(21b^5 - 35b^3 + 5b)}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2 + (b^2/4 - 1)/c)^5,x)

[Out] (c^5*x^11)/11 + (x*(b^2 - 4)^5)/(1024*c^5) + (b*x^6*(63*b^4 - 280*b^2 + 240))/48 + (5*c*x^7*(21*b^4 - 56*b^2 + 16))/56 + (b*c^4*x^10)/2 + (5*c^3*x^9*(9*b^2 - 4))/36 + (x^5*(240*b^2 - 140*b^4 + 21*b^6 - 64))/(32*c) + (5*b*c^2*x^8*(3*b^2 - 4))/8 + (5*b*x^2*(b^2 - 4)^4)/(512*c^4) + (5*x^3*(b^2 - 4)^3*(9*b^2 - 4))/(768*c^3) + (5*b*x^4*(b^2 - 4)^2*(3*b^2 - 4))/(64*c^2)

sympy [B] time = 0.20, size = 250, normalized size = 2.29

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{9}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{5bc^2}{2}\right) + x^7\left(\frac{15b^4c}{8} - 5b^2c + \frac{10c}{7}\right) + x^6\left(\frac{21b^5}{16} - \frac{35b^3}{6} + 5b\right) + \frac{x^5(21b^5 - 35b^3 + 5b)}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)

[Out] b**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9) + x**8*(15*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2*c + 10*c/7) + x**6*(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**6 - 140*b**4 + 240*b**2 - 64)/(32*c) + x**4*(15*b**7 - 140*b**5 + 400*b**3 - 320*b)/(64*c**2) + x**3*(45*b**8 - 560*b**6 + 2400*b**4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b**9 - 80*b**7 + 480*b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*b**8 + 160*b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)

$$3.76 \quad \int \left(\frac{-9+b^2}{4c} + bx + cx^2 \right)^5 dx$$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+3)^{11}}{22528c^6} + \frac{3(-b-2cx+3)^{10}}{2048c^6} - \frac{5(-b-2cx+3)^9}{256c^6} + \frac{135(-b-2cx+3)^8}{1024c^6} - \frac{405(-b-2cx+3)^7}{896c^6} + \frac{81(-b-2cx+3)^6}{128c^6}$$

[Out] 81/128*(-2*c*x-b+3)^6/c^6-405/896*(-2*c*x-b+3)^7/c^6+135/1024*(-2*c*x-b+3)^8/c^6-5/256*(-2*c*x-b+3)^9/c^6+3/2048*(-2*c*x-b+3)^10/c^6-1/22528*(-2*c*x-b+3)^11/c^6

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {610, 43}

$$-\frac{(-b-2cx+3)^{11}}{22528c^6} + \frac{3(-b-2cx+3)^{10}}{2048c^6} - \frac{5(-b-2cx+3)^9}{256c^6} + \frac{135(-b-2cx+3)^8}{1024c^6} - \frac{405(-b-2cx+3)^7}{896c^6} + \frac{81(-b-2cx+3)^6}{128c^6}$$

Antiderivative was successfully verified.

[In] Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c^6) + (135*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/(256*c^6) + (3*(3 - b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^11/(22528*c^6)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 610

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c*x, x]^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\int \left(\frac{-9 + b^2}{4c} + bx + cx^2 \right)^5 dx = \frac{\int \left(\frac{1}{2}(-3 + b) + cx \right)^5 \left(\frac{3+b}{2} + cx \right)^5 dx}{c^5}$$

$$= \frac{\int \left(243 \left(\frac{1}{2}(-3 + b) + cx \right)^5 + 405 \left(\frac{1}{2}(-3 + b) + cx \right)^6 + 270 \left(\frac{1}{2}(-3 + b) + cx \right)^7 + 90 \left(\frac{1}{2}(-3 + b) + cx \right)^8 + 27 \left(\frac{1}{2}(-3 + b) + cx \right)^9 \right) dx}{c^5}$$

$$= \frac{81(3 - b - 2cx)^6}{128c^6} - \frac{405(3 - b - 2cx)^7}{896c^6} + \frac{135(3 - b - 2cx)^8}{1024c^6} - \frac{5(3 - b - 2cx)^9}{256c^6} + \frac{3(3 - b - 2cx)^{10}}{128c^6}$$

Mathematica [A] time = 0.03, size = 199, normalized size = 1.83

$$\frac{15}{8} (b^3 - 3b) c^2 x^8 + \frac{(b^2 - 9)^5 x}{1024 c^5} + \frac{5b (b^2 - 9)^4 x^2}{512 c^4} + \frac{5}{4} (b^2 - 1) c^3 x^9 + \frac{15 (b^2 - 9)^3 (b^2 - 1) x^3}{256 c^3} + \frac{15b (b^2 - 9)^2 (b^2 - 3) x}{64 c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] ((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) + (15*(-9 + b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(-3 + b^2)*x^4)/(64*c^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5)/(32*c) + (3*b*(135 - 70*b^2 + 7*b^4)*x^6)/16 + (15*(27 - 42*b^2 + 7*b^4)*c*x^7)/56 + (15*(-3*b + b^3)*c^2*x^8)/8 + (5*(-1 + b^2)*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11

fricas [B] time = 0.96, size = 227, normalized size = 2.08

$$\frac{7168 c^{10} x^{11} + 39424 b c^9 x^{10} + 98560 (b^2 - 1) c^8 x^9 + 147840 (b^3 - 3b) c^7 x^8 + 21120 (7b^4 - 42b^2 + 27) c^6 x^7 + 14784 (7b^5 - 70b^3 + 135b) c^5 x^6 + 7392 (7b^6 - 105b^4 + 405b^2 - 243) c^4 x^5 + 18480 (b^7 - 21b^5 + 135b^3 - 243b) c^3 x^4 + 4620 (b^8 - 28b^6 + 270b^4 - 972b^2 + 729) c^2 x^3 + 770 (b^9 - 36b^7 + 486b^5 - 2916b^3 + 6561b) c x^2 + 77 (b^{10} - 45b^8 + 810b^6 - 7290b^4 + 32805b^2 - 59049) x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="fricas")

[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*x^9 + 147840*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^6*x^7 + 14784*(7*b^5 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 105*b^4 + 405*b^2 - 243)*c^4*x^5 + 18480*(b^7 - 21*b^5 + 135*b^3 - 243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6 + 270*b^4 - 972*b^2 + 729)*c^2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^3 + 6561*b)*c*x^2 + 77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 59049)*x)/c^5

giac [B] time = 0.45, size = 334, normalized size = 3.06

$$\frac{7168 c^{10} x^{11} + 39424 b c^9 x^{10} + 98560 b^2 c^8 x^9 + 147840 b^3 c^7 x^8 + 147840 b^4 c^6 x^7 - 98560 c^8 x^9 + 103488 b^5 c^5 x^6 - 443520 b^6 c^4 x^5 + 51744 b^7 c^3 x^4 - 1034880 b^8 c^2 x^3 + 4620 b^9 c x^2 - 388080 b^{10} x + 129360 b^{11}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="giac")

[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*b^2*c^8*x^9 + 147840*b^3*c^7*x^8 + 147840*b^4*c^6*x^7 - 98560*c^8*x^9 + 103488*b^5*c^5*x^6 - 443520*b^6*c^4*x^5 + 51744*b^7*c^3*x^4 - 1034880*b^8*c^2*x^3 + 4620*b^9*c*x^2 - 388080*b^10*x + 129360*b^11)/c^5

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^5 x^{11}}{11} + \frac{b c^4 x^{10}}{2} + \frac{\left(4b^2 c^3 + \frac{(b^2-9)c^3}{4} + \left(4b^2 c^2 + 2\left(\frac{3b^2}{2} - \frac{9}{2}\right)c^2\right)c\right) x^9}{9} + \frac{\left((b^2-9) b c^2 + \left(4b^2 c^2 + 2\left(\frac{3b^2}{2} - \frac{9}{2}\right)c^2\right) b + \left(\frac{3b^2}{2} - \frac{9}{2}\right) c\right) x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x)

[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-9)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-9)*c^2*b+b*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2)+c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c))*x^8+1/7*(1/4*(b^2-9)/c*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2)+b*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+c*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2))*x^7+1/6*(1/4*(b^2-9)/c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+b*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2)))*x^6+1/5*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+b*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-9)/c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3/2*b^2-9/2))+b*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16/c^2*(b^2-9)^3*b)*x^4+1/3*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16*b^2*(b^2-9)^3)

$$-9)^3/c^3+1/256/c^3*(b^2-9)^4)*x^3+5/512*(b^2-9)^4/c^4*b*x^2+1/1024*(b^2-9)^5/c^5*x$$

maxima [B] time = 1.41, size = 234, normalized size = 2.15

$$\frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6+\frac{5(2cx^3+3bx^2)(b^2-9)^4}{1536c^4}+\frac{(6c^2x^5+15bcx^4+10b^2x^3)(b^2-9)^3}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-9)/c+b*x+c*x^2)^5,x, algorithm="maxima")

[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^2 - 9)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 - 9)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 9)/c + 1/1024*(b^2 - 9)^5*x/c^5

mupad [B] time = 0.34, size = 176, normalized size = 1.61

$$\frac{c^5 x^{11}}{11} + \frac{5 c^3 x^9 (b^2 - 1)}{4} + \frac{x (b^2 - 9)^5}{1024 c^5} + \frac{3 b x^6 (7 b^4 - 70 b^2 + 135)}{16} + \frac{15 c x^7 (7 b^4 - 42 b^2 + 27)}{56} + \frac{b c^4 x^{10}}{2} + \frac{3 x^5 (7 b^4 - 70 b^2 + 135)}{1024 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2 + (b^2/4 - 9/4)/c)^5,x)

[Out] (c^5*x^11)/11 + (5*c^3*x^9*(b^2 - 1))/4 + (x*(b^2 - 9)^5)/(1024*c^5) + (3*b*x^6*(7*b^4 - 70*b^2 + 135))/16 + (15*c*x^7*(7*b^4 - 42*b^2 + 27))/56 + (b*c^4*x^10)/2 + (3*x^5*(405*b^2 - 105*b^4 + 7*b^6 - 243))/(32*c) + (15*b*c^2*x^8*(b^2 - 3))/8 + (15*x^3*(b^2 - 1)*(b^2 - 9)^3)/(256*c^3) + (5*b*x^2*(b^2 - 9)^4)/(512*c^4) + (15*b*x^4*(b^2 - 3)*(b^2 - 9)^2)/(64*c^2)

sympy [B] time = 0.20, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9 \left(\frac{5b^2c^3}{4} - \frac{5c^3}{4} \right) + x^8 \left(\frac{15b^3c^2}{8} - \frac{45bc^2}{8} \right) + x^7 \left(\frac{15b^4c}{8} - \frac{45b^2c}{4} + \frac{405c}{56} \right) + x^6 \left(\frac{21b^5}{16} - \frac{105b^3}{8} + \frac{405b}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)

[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4) + x**8*(15*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b**2*c/4 + 405*c/56) + x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) + x**5*(21*b**6 - 315*b**4 + 121

$$\begin{aligned} & 5b^{**2} - 729)/(32*c) + x^{**4}*(15b^{**7} - 315b^{**5} + 2025b^{**3} - 3645*b)/(64*c \\ & **2) + x^{**3}*(15b^{**8} - 420b^{**6} + 4050b^{**4} - 14580b^{**2} + 10935)/(256*c^{**3} \\ &) + x^{**2}*(5b^{**9} - 180b^{**7} + 2430b^{**5} - 14580b^{**3} + 32805*b)/(512*c^{**4}) \\ & + x*(b^{**10} - 45b^{**8} + 810b^{**6} - 7290b^{**4} + 32805b^{**2} - 59049)/(1024*c^{**} \\ & 5) \end{aligned}$$

$$3.77 \quad \int \left(\frac{-16+b^2}{4c} + bx + cx^2 \right)^5 dx$$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+4)^{11}}{22528c^6} + \frac{(-b-2cx+4)^{10}}{512c^6} - \frac{5(-b-2cx+4)^9}{144c^6} + \frac{5(-b-2cx+4)^8}{16c^6} - \frac{10(-b-2cx+4)^7}{7c^6} + \frac{8(-b-2cx+4)^6}{3c^6}$$

[Out] $8/3*(-2*c*x-b+4)^6/c^6-10/7*(-2*c*x-b+4)^7/c^6+5/16*(-2*c*x-b+4)^8/c^6-5/14$
 $4*(-2*c*x-b+4)^9/c^6+1/512*(-2*c*x-b+4)^{10}/c^6-1/22528*(-2*c*x-b+4)^{11}/c^6$

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {610, 43}

$$-\frac{(-b-2cx+4)^{11}}{22528c^6} + \frac{(-b-2cx+4)^{10}}{512c^6} - \frac{5(-b-2cx+4)^9}{144c^6} + \frac{5(-b-2cx+4)^8}{16c^6} - \frac{10(-b-2cx+4)^7}{7c^6} + \frac{8(-b-2cx+4)^6}{3c^6}$$

Antiderivative was successfully verified.

[In] Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] $(8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) + (5*(4 - b - 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6) + (4 - b - 2*c*x)^{10}/(512*c^6) - (4 - b - 2*c*x)^{11}/(22528*c^6)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 610

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[1/c^p, Int[Simp[b/2 - q/2 + c*x, x]^p*Simp[b/2 + q/2 + c*x, x]^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\int \left(\frac{-16 + b^2}{4c} + bx + cx^2 \right)^5 dx = \frac{\int \left(\frac{1}{2}(-4 + b) + cx \right)^5 \left(\frac{4+b}{2} + cx \right)^5 dx}{c^5}$$

$$= \frac{\int \left(1024 \left(\frac{1}{2}(-4 + b) + cx \right)^5 + 1280 \left(\frac{1}{2}(-4 + b) + cx \right)^6 + 640 \left(\frac{1}{2}(-4 + b) + cx \right)^7 \right)}{c^5}$$

$$= \frac{8(4 - b - 2cx)^6}{3c^6} - \frac{10(4 - b - 2cx)^7}{7c^6} + \frac{5(4 - b - 2cx)^8}{16c^6} - \frac{5(4 - b - 2cx)^9}{144c^6} + \frac{(4 - b - 2cx)^{10}}{144c^6}$$

Mathematica [A] time = 0.04, size = 207, normalized size = 1.90

$$\frac{5}{8} (3b^3 - 16b) c^2 x^8 + \frac{(b^2 - 16)^5 x}{1024 c^5} + \frac{5b(b^2 - 16)^4 x^2}{512 c^4} + \frac{5}{36} (9b^2 - 16) c^3 x^9 + \frac{5(b^2 - 16)^3 (9b^2 - 16) x^3}{768 c^3} + \frac{5b(b^2 - 16)}{6}$$

Antiderivative was successfully verified.

[In] Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5, x]

[Out] ((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4) + (5*(-16 + b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2)^2*(-16 + 3*b^2)*x^4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 21*b^4)*x^5)/(32*c) + (b*(3840 - 1120*b^2 + 63*b^4)*x^6)/48 + (5*(256 - 224*b^2 + 21*b^4)*c*x^7)/56 + (5*(-16*b + 3*b^3)*c^2*x^8)/8 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11

fricas [B] time = 0.95, size = 235, normalized size = 2.16

$$\frac{64512 c^{10} x^{11} + 354816 b c^9 x^{10} + 98560 (9 b^2 - 16) c^8 x^9 + 443520 (3 b^3 - 16 b) c^7 x^8 + 63360 (21 b^4 - 224 b^2 + 256) c^6 x^7 + 14784 (63 b^5 - 1120 b^3 + 3840 b) c^5 x^6 + 22176 (21 b^6 - 560 b^4 + 3840 b^2 - 4096) c^4 x^5 + 55440 (3 b^7 - 112 b^5 + 1280 b^3 - 4096 b) c^3 x^4 + 4620 (9 b^8 - 448 b^6 + 7680 b^4 - 49152 b^2 + 65536) c^2 x^3 + 6930 (b^9 - 64 b^7 + 1536 b^5 - 16384 b^3 + 65536 b) c x^2 + 693 (b^{10} - 80 b^8 + 2560 b^6 - 40960 b^4 + 327680 b^2 - 1048576) x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="fricas")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16)*c^8*x^9 + 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b^2 + 256)*c^6*x^7 + 14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 + 22176*(21*b^6 - 560*b^4 + 3840*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*x^4 + 4620*(9*b^8 - 448*b^6 + 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930*(b^9 - 64*b^7 + 1536*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^8 + 2560*b^6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5

giac [B] time = 0.53, size = 334, normalized size = 3.06

$$\frac{64512c^{10}x^{11} + 354816bc^9x^{10} + 887040b^2c^8x^9 + 1330560b^3c^7x^8 + 1330560b^4c^6x^7 - 1576960c^8x^9 + 931392b^5c^5x^6 - 7096320b^6c^4x^5 - 14192640b^7c^3x^4 - 166320b^8c^2x^3 - 12418560b^9cx^2 + 16220160c^5x^6 + 6930b^{10}x - 2069760b^{11} - 6209280b^{12}c^3x^4 + 56770560b^{13}c^4x^5 - 443520b^{14}c^5x^6 + 693b^{15}x - 2069760b^{16}c^2x^3 + 85155840b^{17}c^3x^4 - 443520b^{18}c^4x^5 - 443520b^{19}c^5x^6 + 70963200b^{20}c^6x^7 - 55440b^{21}c^7x^8 + 35481600b^{22}c^8x^9 - 90832896b^{23}c^9x^{10} + 10644480b^{24}c^{10}x^{11} - 227082240b^{25}c^{11}x^{12} + 1774080b^{26}c^{12}x^{13} - 227082240b^{27}c^{13}x^{14} - 113541120b^{28}c^{14}x^{15} - 28385280b^{29}c^{15}x^{16} + 302776320b^{30}c^{16}x^{17} + 454164480b^{31}c^{17}x^{18} + 227082240b^{32}c^{18}x^{19} - 726663168b^{33}c^{19}x^{20})/c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="giac")

[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 887040*b^2*c^8*x^9 + 1330560*b^3*c^7*x^8 + 1330560*b^4*c^6*x^7 - 1576960*c^8*x^9 + 931392*b^5*c^5*x^6 - 7096320*b^6*c^4*x^5 - 14192640*b^7*c^3*x^4 + 166320*b^8*c^2*x^3 - 12418560*b^9*c*x^2 + 16220160*c^5*x^6 + 6930*b^10*x - 2069760*b^11 - 6209280*b^12*c^3*x^4 + 56770560*b^13*c^4*x^5 - 443520*b^14*c^5*x^6 + 693*b^15*x - 2069760*b^16*c^2*x^3 + 85155840*b^17*c^3*x^4 - 443520*b^18*c^4*x^5 - 443520*b^19*c^5*x^6 + 70963200*b^20*c^6*x^7 - 55440*b^21*c^7*x^8 + 35481600*b^22*c^8*x^9 - 90832896*b^23*c^9*x^10 + 10644480*b^24*c^10*x^11 - 227082240*b^25*c^11*x^12 + 1774080*b^26*c^12*x^13 - 227082240*b^27*c^13*x^14 - 113541120*b^28*c^14*x^15 - 28385280*b^29*c^15*x^16 + 302776320*b^30*c^16*x^17 + 454164480*b^31*c^17*x^18 + 227082240*b^32*c^18*x^19 - 726663168*b^33*c^19*x^20)/c^5

maple [B] time = 0.04, size = 636, normalized size = 5.83

$$\frac{c^5x^{11}}{11} + \frac{bc^4x^{10}}{2} + \frac{\left(4b^2c^3 + \frac{(b^2-16)c^3}{4} + \left(4b^2c^2 + 2\left(\frac{3b^2}{2} - 8\right)c^2\right)c\right)x^9}{9} + \frac{\left((b^2-16)bc^2 + \left(4b^2c^2 + 2\left(\frac{3b^2}{2} - 8\right)c^2\right)b\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x)

[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-16)*c^3+4*b^2*c^3+c*(2*(3/2*b^2-8)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-16)*c^2*b+b*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c))*x^8+1/7*(1/4*(b^2-16)/c*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+b*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+c*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2))*x^7+1/6*(1/4*(b^2-16)/c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+b*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8)))*x^6+1/5*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+b*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2))*x^5+1/4*(1/4*(b^2-16)/c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/2*b^2-8))+b*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16/c^2*(b^2-16)^3*b)*x^4+1/3*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16*b^2*(b^2-

$(b^2 - 16)^3 / c^3 + 1/256 c^3 (b^2 - 16)^4 * x^3 + 5/512 (b^2 - 16)^4 / c^4 * b * x^2 + 1/1024 (b^2 - 16)^5 / c^5 * x$

maxima [B] time = 1.40, size = 234, normalized size = 2.15

$$\frac{1}{11} c^5 x^{11} + \frac{1}{2} b c^4 x^{10} + \frac{10}{9} b^2 c^3 x^9 + \frac{5}{4} b^3 c^2 x^8 + \frac{5}{7} b^4 c x^7 + \frac{1}{6} b^5 x^6 + \frac{5(2cx^3 + 3bx^2)(b^2 - 16)^4}{1536c^4} + \frac{(6c^2x^5 + 15bcx^4 + 10b^2x^3)(b^2 - 16)^3}{192c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b^2-16)/c+b*x+c*x^2)^5,x, algorithm="maxima")

[Out] $\frac{1}{11} c^5 x^{11} + \frac{1}{2} b c^4 x^{10} + \frac{10}{9} b^2 c^3 x^9 + \frac{5}{4} b^3 c^2 x^8 + \frac{5}{7} b^4 c x^7 + \frac{1}{6} b^5 x^6 + \frac{5(2cx^3 + 3bx^2)(b^2 - 16)^4}{1536c^4} + \frac{1}{19} \frac{2(6c^2x^5 + 15bcx^4 + 10b^2x^3)(b^2 - 16)^3}{c^3} + \frac{1}{224} \frac{20c^3x^7 + 70b^2c^2x^6 + 84b^2c^2x^5 + 35b^3x^4}{c^2} + \frac{1}{504} \frac{70c^4x^9 + 315b^2c^3x^8 + 540b^2c^2x^7 + 420b^3c^2x^6 + 126b^4x^5}{c} + \frac{1}{1024} \frac{(b^2 - 16)^5 x}{c^5}$

mupad [B] time = 0.32, size = 184, normalized size = 1.69

$$\frac{c^5 x^{11}}{11} + \frac{x(b^2 - 16)^5}{1024 c^5} + \frac{b x^6 (63 b^4 - 1120 b^2 + 3840)}{48} + \frac{5 c x^7 (21 b^4 - 224 b^2 + 256)}{56} + \frac{b c^4 x^{10}}{2} + \frac{5 c^3 x^9 (9 b^2 - 16)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2 + (b^2/4 - 4)/c)^5,x)

[Out] $\frac{c^5 x^{11}}{11} + \frac{x(b^2 - 16)^5}{1024 c^5} + \frac{b x^6 (63 b^4 - 1120 b^2 + 3840)}{48} + \frac{5 c x^7 (21 b^4 - 224 b^2 + 256)}{56} + \frac{b c^4 x^{10}}{2} + \frac{5 c^3 x^9 (9 b^2 - 16)}{36} + \frac{x^5 (3840 b^2 - 560 b^4 + 21 b^6 - 4096)}{32 c} + \frac{5 b c^2 x^8 (3 b^2 - 16)}{8} + \frac{5 b x^2 (b^2 - 16)^4}{512 c^4} + \frac{5 x^3 (b^2 - 16)^3 (9 b^2 - 16)}{768 c^3} + \frac{5 b x^4 (b^2 - 16)^2 (3 b^2 - 16)}{64 c^2}$

sympy [B] time = 0.20, size = 248, normalized size = 2.28

$$\frac{b c^4 x^{10}}{2} + \frac{c^5 x^{11}}{11} + x^9 \left(\frac{5 b^2 c^3}{4} - \frac{20 c^3}{9} \right) + x^8 \left(\frac{15 b^3 c^2}{8} - 10 b c^2 \right) + x^7 \left(\frac{15 b^4 c}{8} - 20 b^2 c + \frac{160 c}{7} \right) + x^6 \left(\frac{21 b^5}{16} - \frac{70 b^3}{3} + 80 b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)

[Out] $b c^4 x^{10} / 2 + c^5 x^{11} / 11 + x^9 (5 b^2 c^3 / 4 - 20 c^3 / 9) + x^8 (15 b^3 c^2 / 8 - 10 b c^2) + x^7 (15 b^4 c / 8 - 20 b^2 c + 160 c / 7) + x^6 (21 b^5 / 16 - 70 b^3 / 3 + 80 b)$

$$\begin{aligned} &6*(21*b^{5/16} - 70*b^{3/3} + 80*b) + x^{5}*(21*b^{6} - 560*b^{4} + 3840*b^{2} - \\ &4096)/(32*c) + x^{4}*(15*b^{7} - 560*b^{5} + 6400*b^{3} - 20480*b)/(64*c^{2}) + \\ &x^{3}*(45*b^{8} - 2240*b^{6} + 38400*b^{4} - 245760*b^{2} + 327680)/(768*c^{3}) + \\ &x^{2}*(5*b^{9} - 320*b^{7} + 7680*b^{5} - 81920*b^{3} + 327680*b)/(512*c^{4}) + \\ &x*(b^{10} - 80*b^{8} + 2560*b^{6} - 40960*b^{4} + 327680*b^{2} - 1048576)/(1024* \\ &c^{5}) \end{aligned}$$

$$3.78 \quad \int \frac{1}{2+4x+3x^2} dx$$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 204}

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x + 3*x^2)^(-1), x]

[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2+4x+3x^2} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{-8-x^2} dx, x, 4+6x\right)\right) \\ &= \frac{\tan^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x + 3*x^2)^(-1), x]

[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]

fricas [A] time = 1.05, size = 16, normalized size = 0.89

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3x + 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))

giac [A] time = 0.37, size = 16, normalized size = 0.89

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3x + 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2), x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))

maple [A] time = 0.04, size = 17, normalized size = 0.94

$$\frac{\sqrt{2} \arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+4*x+2), x)

[Out] 1/2*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))

maxima [A] time = 2.95, size = 16, normalized size = 0.89

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (3x + 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))

mupad [B] time = 0.14, size = 16, normalized size = 0.89

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}(3x+2)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 3*x^2 + 2),x)

[Out] (2^(1/2)*atan((2^(1/2)*(3*x + 2))/2))/2

sympy [A] time = 0.11, size = 22, normalized size = 1.22

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{3\sqrt{2}x}{2} + \sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+4*x+2),x)

[Out] sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2

$$3.79 \quad \int \frac{1}{4-2\sqrt{3}x+x^2} dx$$

Optimal. Leaf size=12

$$-\tan^{-1}(\sqrt{3}-x)$$

[Out] arctan(x-3^(1/2))

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {618, 204}

$$-\tan^{-1}(\sqrt{3}-x)$$

Antiderivative was successfully verified.

[In] Int[(4 - 2*Sqrt[3]*x + x^2)^(-1), x]

[Out] -ArcTan[Sqrt[3] - x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{4-2\sqrt{3}x+x^2} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{-4-x^2} dx, x, -2\sqrt{3}+2x\right)\right) \\ &= -\tan^{-1}(\sqrt{3}-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\tan^{-1}(\sqrt{3}-x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]

[Out] -ArcTan[Sqrt[3] - x]

fricas [A] time = 0.90, size = 8, normalized size = 0.67

$$\arctan(x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="fricas")

[Out] arctan(x - sqrt(3))

giac [A] time = 0.41, size = 8, normalized size = 0.67

$$\arctan(x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="giac")

[Out] arctan(x - sqrt(3))

maple [A] time = 0.10, size = 9, normalized size = 0.75

$$\arctan(x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4+x^2-2*3^(1/2)*x),x)

[Out] arctan(x-3^(1/2))

maxima [A] time = 2.97, size = 8, normalized size = 0.67

$$\arctan(x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x^2-2*x*3^(1/2)),x, algorithm="maxima")

[Out] arctan(x - sqrt(3))

mupad [B] time = 0.27, size = 8, normalized size = 0.67

$$\operatorname{atan}(x - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*3^(1/2)*x + 4),x)
```

```
[Out] atan(x - 3^(1/2))
```

sympy [A] time = 0.16, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x - \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

```
[Out] atan(x - sqrt(3))
```

$$3.80 \quad \int \frac{1}{2+4x-3x^2} dx$$

Optimal. Leaf size=19

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

[Out] $-1/10*\operatorname{arctanh}(1/10*(2-3*x)*10^{(1/2)})*10^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(2 + 4*x - 3*x^2)^{-1}, x]$

[Out] $-(\operatorname{ArcTanh}[(2 - 3*x)/\operatorname{Sqrt}[10]]/\operatorname{Sqrt}[10])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{2+4x-3x^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{40-x^2} dx, x, 4-6x\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.79

$$\frac{\log(3x + \sqrt{10} - 2) - \log(-3x + \sqrt{10} + 2)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x - 3*x^2)^(-1), x]

[Out] (-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10])

fricas [B] time = 0.88, size = 39, normalized size = 2.05

$$\frac{1}{20} \sqrt{10} \log\left(\frac{9x^2 + 2\sqrt{10}(3x - 2) - 12x + 14}{3x^2 - 4x - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2), x, algorithm="fricas")

[Out] 1/20*sqrt(10)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x - 2))

giac [A] time = 0.46, size = 31, normalized size = 1.63

$$-\frac{1}{20} \sqrt{10} \log\left(\frac{|6x - 2\sqrt{10} - 4|}{|6x + 2\sqrt{10} - 4|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2), x, algorithm="giac")

[Out] -1/20*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4))

maple [A] time = 0.04, size = 17, normalized size = 0.89

$$\frac{\sqrt{10} \operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+4*x+2), x)

[Out] 1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))

maxima [A] time = 2.95, size = 27, normalized size = 1.42

$$-\frac{1}{20} \sqrt{10} \log\left(\frac{3x - \sqrt{10} - 2}{3x + \sqrt{10} - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2),x, algorithm="maxima")

[Out] -1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))

mupad [B] time = 0.21, size = 15, normalized size = 0.79

$$\frac{\sqrt{10} \operatorname{atanh}\left(\sqrt{10} \left(\frac{3x}{10} - \frac{1}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 3*x^2 + 2),x)

[Out] (10^(1/2)*atanh(10^(1/2)*((3*x)/10 - 1/5)))/10

sympy [A] time = 0.12, size = 39, normalized size = 2.05

$$\frac{\sqrt{10} \log\left(x - \frac{2}{3} + \frac{\sqrt{10}}{3}\right)}{20} - \frac{\sqrt{10} \log\left(x - \frac{\sqrt{10}}{3} - \frac{2}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+4*x+2),x)

[Out] sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)/3 - 2/3)/20

$$3.81 \quad \int \frac{1}{2+5x+3x^2} dx$$

Optimal. Leaf size=13

$$\log(3x + 2) - \log(x + 1)$$

[Out] -ln(1+x)+ln(2+3*x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {616, 31}

$$\log(3x + 2) - \log(x + 1)$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x + 3*x^2)^(-1), x]

[Out] -Log[1 + x] + Log[2 + 3*x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{2+5x+3x^2} dx &= 3 \int \frac{1}{2+3x} dx - 3 \int \frac{1}{3+3x} dx \\ &= -\log(1+x) + \log(2+3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\log(3x + 2) - \log(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x + 3*x^2)^(-1),x]

[Out] -Log[1 + x] + Log[2 + 3*x]

fricas [A] time = 1.04, size = 13, normalized size = 1.00

$$\log(3x + 2) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2),x, algorithm="fricas")

[Out] log(3*x + 2) - log(x + 1)

giac [A] time = 0.51, size = 15, normalized size = 1.15

$$\log(|3x + 2|) - \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2),x, algorithm="giac")

[Out] log(abs(3*x + 2)) - log(abs(x + 1))

maple [A] time = 0.05, size = 14, normalized size = 1.08

$$\ln(3x + 2) - \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+5*x+2),x)

[Out] -ln(x+1)+ln(3*x+2)

maxima [A] time = 1.33, size = 13, normalized size = 1.00

$$\log(3x + 2) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2),x, algorithm="maxima")

[Out] log(3*x + 2) - log(x + 1)

mupad [B] time = 0.08, size = 8, normalized size = 0.62

$$-2 \operatorname{atanh}(6x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x + 3*x^2 + 2),x)
```

```
[Out] -2*atanh(6*x + 5)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log\left(x + \frac{2}{3}\right) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+5*x+2),x)
```

```
[Out] log(x + 2/3) - log(x + 1)
```

$$3.82 \quad \int \frac{1}{2+5x-3x^2} dx$$

Optimal. Leaf size=21

$$\frac{1}{7} \log(3x+1) - \frac{1}{7} \log(2-x)$$

[Out] -1/7*ln(2-x)+1/7*ln(1+3*x)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {616, 31}

$$\frac{1}{7} \log(3x+1) - \frac{1}{7} \log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x - 3*x^2)^(-1), x]

[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{2+5x-3x^2} dx &= -\left(\frac{3}{7} \int \frac{1}{-1-3x} dx\right) + \frac{3}{7} \int \frac{1}{6-3x} dx \\ &= -\frac{1}{7} \log(2-x) + \frac{1}{7} \log(1+3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7} \log(3x+1) - \frac{1}{7} \log(2-x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x - 3*x^2)^(-1), x]

[Out] -1/7*Log[2 - x] + Log[1 + 3*x]/7

fricas [A] time = 1.05, size = 15, normalized size = 0.71

$$\frac{1}{7} \log(3x + 1) - \frac{1}{7} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2), x, algorithm="fricas")

[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)

giac [A] time = 0.46, size = 17, normalized size = 0.81

$$\frac{1}{7} \log(|3x + 1|) - \frac{1}{7} \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2), x, algorithm="giac")

[Out] 1/7*log(abs(3*x + 1)) - 1/7*log(abs(x - 2))

maple [A] time = 0.05, size = 16, normalized size = 0.76

$$\frac{\ln(3x + 1)}{7} - \frac{\ln(x - 2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+5*x+2), x)

[Out] -1/7*ln(x-2)+1/7*ln(1+3*x)

maxima [A] time = 1.31, size = 15, normalized size = 0.71

$$\frac{1}{7} \log(3x + 1) - \frac{1}{7} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2), x, algorithm="maxima")

[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)

mupad [B] time = 0.09, size = 8, normalized size = 0.38

$$\frac{2 \operatorname{atanh}\left(\frac{6x}{7} - \frac{5}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x - 3*x^2 + 2),x)`

[Out] `(2*atanh((6*x)/7 - 5/7))/7`

sympy [A] time = 0.11, size = 14, normalized size = 0.67

$$-\frac{\log(x-2)}{7} + \frac{\log\left(x + \frac{1}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+5*x+2),x)`

[Out] `-log(x - 2)/7 + log(x + 1/3)/7`

$$3.83 \quad \int \frac{1}{3+4x+x^2} dx$$

Optimal. Leaf size=6

$$-\tanh^{-1}(x+2)$$

[Out] -arctanh(2+x)

Rubi [B] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {616, 31}

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + x^2)^(-1), x]

[Out] Log[1 + x]/2 - Log[3 + x]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{3+4x+x^2} dx &= \frac{1}{2} \int \frac{1}{1+x} dx - \frac{1}{2} \int \frac{1}{3+x} dx \\ &= \frac{1}{2} \log(1+x) - \frac{1}{2} \log(3+x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 17, normalized size = 2.83

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + x^2)^(-1),x]

[Out] Log[1 + x]/2 - Log[3 + x]/2

fricas [B] time = 1.06, size = 13, normalized size = 2.17

$$-\frac{1}{2} \log(x + 3) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x+3),x, algorithm="fricas")

[Out] -1/2*log(x + 3) + 1/2*log(x + 1)

giac [B] time = 0.33, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log(|x + 3|) + \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x+3),x, algorithm="giac")

[Out] -1/2*log(abs(x + 3)) + 1/2*log(abs(x + 1))

maple [B] time = 0.05, size = 14, normalized size = 2.33

$$-\frac{\ln(x + 3)}{2} + \frac{\ln(x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+4*x+3),x)

[Out] 1/2*ln(x+1)-1/2*ln(3+x)

maxima [B] time = 1.29, size = 13, normalized size = 2.17

$$-\frac{1}{2} \log(x + 3) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+4*x+3),x, algorithm="maxima")

[Out] -1/2*log(x + 3) + 1/2*log(x + 1)

mupad [B] time = 0.18, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x + x^2 + 3),x)`

[Out] `-atanh(x + 2)`

sympy [B] time = 0.10, size = 12, normalized size = 2.00

$$\frac{\log(x + 1)}{2} - \frac{\log(x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+4*x+3),x)`

[Out] `log(x + 1)/2 - log(x + 3)/2`

$$3.84 \quad \int \frac{1}{1+\pi x+2x^2} dx$$

Optimal. Leaf size=27

$$-\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

[Out] $-2*\operatorname{arctanh}((\pi+4*x)/(\pi^2-8)^{(1/2))}/(\pi^2-8)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 + \pi*x + 2*x^2)^{-1}, x]$

[Out] $(-2*\operatorname{ArcTanh}[(\pi + 4*x)/\operatorname{Sqrt}[-8 + \pi^2]])/\operatorname{Sqrt}[-8 + \pi^2]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x\} \ \&\& \ \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\pi x+2x^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{-8+\pi^2-x^2} dx, x, \pi+4x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{\pi+4x}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Pi*x + 2*x^2)^(-1),x]

[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]

fricas [B] time = 1.03, size = 50, normalized size = 1.85

$$\frac{\log\left(\frac{\pi^2+4\pi x+8x^2-(\pi+4x)\sqrt{\pi^2-8}-4}{\pi x+2x^2+1}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="fricas")

[Out] log((pi^2 + 4*pi*x + 8*x^2 - (pi + 4*x)*sqrt(pi^2 - 8) - 4)/(pi*x + 2*x^2 + 1))/sqrt(pi^2 - 8)

giac [A] time = 0.35, size = 40, normalized size = 1.48

$$\frac{\log\left(\frac{|\pi+4x-\sqrt{\pi^2-8}|}{|\pi+4x+\sqrt{\pi^2-8}|}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+2*x^2+1),x, algorithm="giac")

[Out] log(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))/sqrt(pi^2 - 8)

maple [A] time = 0.06, size = 24, normalized size = 0.89

$$\frac{2 \operatorname{arctanh}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(Pi*x+2*x^2+1),x)

[Out] $-2*\operatorname{arctanh}((\pi+4*x)/(\pi^2-8)^{(1/2)})/(\pi^2-8)^{(1/2)}$

maxima [A] time = 1.31, size = 38, normalized size = 1.41

$$\frac{\log\left(\frac{\pi+4x-\sqrt{\pi^2-8}}{\pi+4x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x+2*x^2+1),x, algorithm="maxima")`

[Out] $\log((\pi + 4*x - \sqrt{\pi^2 - 8})/(\pi + 4*x + \sqrt{\pi^2 - 8}))/\sqrt{\pi^2 - 8}$

mupad [B] time = 0.36, size = 23, normalized size = 0.85

$$-\frac{2 \operatorname{atanh}\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(Pi*x + 2*x^2 + 1),x)`

[Out] $-(2*\operatorname{atanh}((\pi + 4*x)/(\pi^2 - 8)^{(1/2)}))/(\pi^2 - 8)^{(1/2)}$

sympy [B] time = 0.22, size = 76, normalized size = 2.81

$$\frac{\log\left(x - \frac{\pi^2}{4\sqrt{-8+\pi^2}} + \frac{\pi}{4} + \frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}} - \frac{\log\left(x - \frac{2}{\sqrt{-8+\pi^2}} + \frac{\pi}{4} + \frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x+2*x**2+1),x)`

[Out] $\log(x - \pi^2/(4*\sqrt{-8 + \pi^2}) + \pi/4 + 2/\sqrt{-8 + \pi^2})/\sqrt{-8 + \pi^2} - \log(x - 2/\sqrt{-8 + \pi^2} + \pi/4 + \pi^2/(4*\sqrt{-8 + \pi^2}))/\sqrt{-8 + \pi^2}$

$$3.85 \quad \int \frac{1}{1+\pi x-2x^2} dx$$

Optimal. Leaf size=27

$$-\frac{2 \tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

[Out] $-2*\operatorname{arctanh}((\pi-4*x)/(\pi^2+8)^{(1/2)})/(\pi^2+8)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 + \pi*x - 2*x^2)^{-1}, x]$

[Out] $(-2*\operatorname{ArcTanh}[(\pi - 4*x)/\operatorname{Sqrt}[8 + \pi^2]])/\operatorname{Sqrt}[8 + \pi^2]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\pi x-2x^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{8+\pi^2-x^2} dx, x, \pi-4x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{2 \tanh^{-1}\left(\frac{4x-\pi}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Pi*x - 2*x^2)^(-1), x]

[Out] (2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]

fricas [B] time = 0.99, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-4\pi x+8x^2-(\pi-4x)\sqrt{\pi^2+8}+4}{\pi x-2x^2+1}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x-2*x^2+1), x, algorithm="fricas")

[Out] log(-(pi^2 - 4*pi*x + 8*x^2 - (pi - 4*x)*sqrt(pi^2 + 8) + 4)/(pi*x - 2*x^2 + 1))/sqrt(pi^2 + 8)

giac [A] time = 0.38, size = 45, normalized size = 1.67

$$-\frac{\log\left(\frac{|\pi-4x-\sqrt{\pi^2+8}|}{|\pi-4x+\sqrt{\pi^2+8}|}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x-2*x^2+1), x, algorithm="giac")

[Out] -log(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8)))/sqrt(pi^2 + 8)

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$\frac{2 \operatorname{arctanh}\left(\frac{4x-\pi}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(Pi*x-2*x^2+1), x)

[Out] $2/(\pi^2+8)^{(1/2)}*\operatorname{arctanh}((4*x-\pi)/(\pi^2+8)^{(1/2)})$

maxima [A] time = 1.33, size = 39, normalized size = 1.44

$$-\frac{\log\left(\frac{\pi-4x+\sqrt{\pi^2+8}}{\pi-4x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x-2*x^2+1),x, algorithm="maxima")`

[Out] $-\log((\pi - 4*x + \sqrt{\pi^2 + 8})/(\pi - 4*x - \sqrt{\pi^2 + 8}))/\sqrt{\pi^2 + 8}$

mupad [B] time = 0.39, size = 23, normalized size = 0.85

$$-\frac{2 \operatorname{atanh}\left(\frac{\pi-4x}{\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(pi*x - 2*x^2 + 1),x)`

[Out] $-(2*\operatorname{atanh}((\pi - 4*x)/(\pi^2 + 8)^{(1/2)}))/(\pi^2 + 8)^{(1/2)}$

sympy [B] time = 0.23, size = 76, normalized size = 2.81

$$-\frac{\log\left(x - \frac{\pi}{4} - \frac{\pi^2}{4\sqrt{8+\pi^2}} - \frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}} + \frac{\log\left(x - \frac{\pi}{4} + \frac{2}{\sqrt{8+\pi^2}} + \frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x-2*x**2+1),x)`

[Out] $-\log(x - \pi/4 - \pi^2/(4*\sqrt{8 + \pi^2}) - 2/\sqrt{8 + \pi^2})/\sqrt{8 + \pi^2} + \log(x - \pi/4 + 2/\sqrt{8 + \pi^2} + \pi^2/(4*\sqrt{8 + \pi^2}))/\sqrt{8 + \pi^2}$

$$3.86 \quad \int \frac{1}{1+\pi x+3x^2} dx$$

Optimal. Leaf size=31

$$\frac{2 \tan^{-1} \left(\frac{6x+\pi}{\sqrt{12-\pi^2}} \right)}{\sqrt{12-\pi^2}}$$

[Out] $2*\arctan((\text{Pi}+6*x)/(-\text{Pi}^2+12)^{(1/2)))/(-\text{Pi}^2+12)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 204}

$$\frac{2 \tan^{-1} \left(\frac{6x+\pi}{\sqrt{12-\pi^2}} \right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + \text{Pi}*x + 3*x^2)^{-1}, x]$

[Out] $(2*\text{ArcTan}[(\text{Pi} + 6*x)/\text{Sqrt}[12 - \text{Pi}^2]])/\text{Sqrt}[12 - \text{Pi}^2]$

Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\pi x+3x^2} dx &= -\left(2 \text{Subst} \left(\int \frac{1}{-12+\pi^2-x^2} dx, x, \pi+6x \right)\right) \\ &= \frac{2 \tan^{-1} \left(\frac{\pi+6x}{\sqrt{12-\pi^2}} \right)}{\sqrt{12-\pi^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Pi*x + 3*x^2)^(-1),x]

[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]

fricas [A] time = 0.80, size = 41, normalized size = 1.32

$$\frac{2 \sqrt{-\pi^2 + 12} \arctan\left(\frac{(\pi+6x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\pi^2 - 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="fricas")

[Out] 2*sqrt(-pi^2 + 12)*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/(pi^2 - 12)

giac [A] time = 0.48, size = 27, normalized size = 0.87

$$\frac{2 \arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="giac")

[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)

maple [A] time = 0.05, size = 28, normalized size = 0.90

$$\frac{2 \arctan\left(\frac{6x+\pi}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(Pi*x+3*x^2+1),x)

[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)

maxima [A] time = 1.32, size = 27, normalized size = 0.87

$$\frac{2 \arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+3*x^2+1),x, algorithm="maxima")

[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)

mupad [B] time = 0.38, size = 23, normalized size = 0.74

$$-\frac{2 \operatorname{atanh}\left(\frac{\Pi+6x}{\sqrt{\Pi^2-12}}\right)}{\sqrt{\Pi^2-12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(Pi*x + 3*x^2 + 1),x)

[Out] -(2*atanh((Pi + 6*x)/(Pi^2 - 12)^(1/2)))/(Pi^2 - 12)^(1/2)

sympy [C] time = 0.20, size = 87, normalized size = 2.81

$$-\frac{i \log\left(x + \frac{\pi}{6} - \frac{2i}{\sqrt{12-\pi^2}} + \frac{i\pi^2}{6\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}} + \frac{i \log\left(x + \frac{\pi}{6} - \frac{i\pi^2}{6\sqrt{12-\pi^2}} + \frac{2i}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x+3*x**2+1),x)

[Out] -I*log(x + pi/6 - 2*I/sqrt(12 - pi**2) + I*pi**2/(6*sqrt(12 - pi**2)))/sqrt(12 - pi**2) + I*log(x + pi/6 - I*pi**2/(6*sqrt(12 - pi**2)) + 2*I/sqrt(12 - pi**2))/sqrt(12 - pi**2)

$$3.87 \quad \int \frac{1}{1+\pi x-3x^2} dx$$

Optimal. Leaf size=27

$$-\frac{2 \tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

[Out] $-2*\operatorname{arctanh}((\pi-6*x)/(\pi^2+12)^{(1/2)})/(\pi^2+12)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(1 + \pi*x - 3*x^2)^{-1}, x]$

[Out] $(-2*\operatorname{ArcTanh}[(\pi - 6*x)/\operatorname{Sqrt}[12 + \pi^2]])/\operatorname{Sqrt}[12 + \pi^2]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\pi x-3x^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{12+\pi^2-x^2} dx, x, \pi-6x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{2 \tanh^{-1}\left(\frac{6x-\pi}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Pi*x - 3*x^2)^(-1), x]

[Out] (2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]

fricas [B] time = 0.93, size = 51, normalized size = 1.89

$$\frac{\log\left(-\frac{\pi^2-6\pi x+18x^2-(\pi-6x)\sqrt{\pi^2+12}+6}{\pi x-3x^2+1}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x-3*x^2+1), x, algorithm="fricas")

[Out] log(-(pi^2 - 6*pi*x + 18*x^2 - (pi - 6*x)*sqrt(pi^2 + 12) + 6)/(pi*x - 3*x^2 + 1))/sqrt(pi^2 + 12)

giac [A] time = 0.53, size = 45, normalized size = 1.67

$$\frac{\log\left(\frac{|\pi-6x-\sqrt{\pi^2+12}|}{|\pi-6x+\sqrt{\pi^2+12}|}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(pi*x-3*x^2+1), x, algorithm="giac")

[Out] -log(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 + 12)))/sqrt(pi^2 + 12)

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$\frac{2 \operatorname{arctanh}\left(\frac{6x-\pi}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(Pi*x-3*x^2+1), x)

[Out] $2/(\pi^2+12)^{(1/2)}*\operatorname{arctanh}((6*x-\pi)/(\pi^2+12)^{(1/2)})$

maxima [A] time = 1.44, size = 39, normalized size = 1.44

$$\frac{\log\left(\frac{\pi-6x+\sqrt{\pi^2+12}}{\pi-6x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x-3*x^2+1),x, algorithm="maxima")`

[Out] $-\log((\pi - 6*x + \sqrt{\pi^2 + 12})/(\pi - 6*x - \sqrt{\pi^2 + 12}))/\sqrt{\pi^2 + 12}$

mupad [B] time = 0.42, size = 23, normalized size = 0.85

$$\frac{2 \operatorname{atanh}\left(\frac{\pi-6x}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(Pi*x - 3*x^2 + 1),x)`

[Out] $-(2*\operatorname{atanh}((\pi - 6*x)/(\pi^2 + 12)^{(1/2)}))/(\pi^2 + 12)^{(1/2)}$

sympy [B] time = 0.23, size = 76, normalized size = 2.81

$$\frac{\log\left(x - \frac{\pi}{6} + \frac{\pi^2}{6\sqrt{\pi^2+12}} + \frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}} - \frac{\log\left(x - \frac{\pi}{6} - \frac{2}{\sqrt{\pi^2+12}} - \frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(pi*x-3*x**2+1),x)`

[Out] $\log(x - \pi/6 + \pi^2/(6*\sqrt{\pi^2 + 12}) + 2/\sqrt{\pi^2 + 12})/\sqrt{\pi^2 + 12} - \log(x - \pi/6 - 2/\sqrt{\pi^2 + 12} - \pi^2/(6*\sqrt{\pi^2 + 12}))/\sqrt{\pi^2 + 12}$

$$3.88 \quad \int \frac{1}{a+cx+bx^2} dx$$

Optimal. Leaf size=38

$$\frac{2 \tan^{-1} \left(\frac{2bx+c}{\sqrt{4ab-c^2}} \right)}{\sqrt{4ab-c^2}}$$

[Out] $2 \cdot \arctan((2 \cdot b \cdot x + c) / (4 \cdot a \cdot b - c^2)^{(1/2)}) / (4 \cdot a \cdot b - c^2)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {618, 204}

$$\frac{2 \tan^{-1} \left(\frac{2bx+c}{\sqrt{4ab-c^2}} \right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x + b*x^2)^(-1), x]

[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+cx+bx^2} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{-4ab+c^2-x^2} dx, x, c+2bx \right) \right) \\ &= \frac{2 \tan^{-1} \left(\frac{c+2bx}{\sqrt{4ab-c^2}} \right)}{\sqrt{4ab-c^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x + b*x^2)^(-1), x]

[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]

fricas [A] time = 0.92, size = 113, normalized size = 2.97

$$\left[-\frac{\sqrt{-4ab+c^2} \log\left(\frac{2b^2x^2+2bcx-2ab+c^2-\sqrt{-4ab+c^2}(2bx+c)}{bx^2+cx+a}\right)}{4ab-c^2}, -\frac{2 \arctan\left(-\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+c*x+a), x, algorithm="fricas")

[Out] [-sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 - sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x + a))/(4*a*b - c^2), -2*arctan(-(2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)]

giac [A] time = 0.43, size = 34, normalized size = 0.89

$$\frac{2 \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+c*x+a), x, algorithm="giac")

[Out] 2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)

maple [A] time = 0.06, size = 35, normalized size = 0.92

$$\frac{2 \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+c*x+a),x)`

[Out] `2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^2+c*x+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for more details)Is c^2-4*a*b positive or negative?

mupad [B] time = 0.23, size = 46, normalized size = 1.21

$$\frac{2 \operatorname{atan}\left(\frac{c}{\sqrt{4ab-c^2}} + \frac{2bx}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x + b*x^2),x)`

[Out] `(2*atan(c/(4*a*b - c^2)^(1/2) + (2*b*x)/(4*a*b - c^2)^(1/2)))/(4*a*b - c^2)^(1/2)`

sympy [B] time = 0.22, size = 124, normalized size = 3.26

$$-\sqrt{-\frac{1}{4ab-c^2}} \log\left(x + \frac{-4ab\sqrt{-\frac{1}{4ab-c^2}} + c^2\sqrt{-\frac{1}{4ab-c^2}} + c}{2b}\right) + \sqrt{-\frac{1}{4ab-c^2}} \log\left(x + \frac{4ab\sqrt{-\frac{1}{4ab-c^2}} - c^2\sqrt{-\frac{1}{4ab-c^2}}}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+c*x+a),x)`

[Out] `-sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2)) + c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**2))*log(x + (4*a*b*sqrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b))`

$$3.89 \quad \int \frac{1}{b+2ax+bx^2} dx$$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

[Out] $-\arctanh((b*x+a)/(a^2-b^2)^{(1/2)})/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b + 2*a*x + b*x^2)^{-1}, x]$

[Out] $-(\text{ArcTanh}[(a + b*x)/\text{Sqrt}[a^2 - b^2]]/\text{Sqrt}[a^2 - b^2])$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{b+2ax+bx^2} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{4(a^2-b^2)-x^2} dx, x, 2a+2bx\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*a*x + b*x^2)^(-1),x]

[Out] ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]

fricas [A] time = 1.11, size = 124, normalized size = 3.54

$$\left[\frac{\log\left(\frac{b^2x^2+2abx+2a^2-b^2-2\sqrt{a^2-b^2}(bx+a)}{bx^2+2ax+b}\right)}{2\sqrt{a^2-b^2}}, -\frac{\sqrt{-a^2+b^2} \arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)}{a^2-b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="fricas")

[Out] [1/2*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*x + b))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2 - b^2))/(a^2 - b^2)]

giac [A] time = 0.37, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+2*a*x+b),x, algorithm="giac")

[Out] arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)

maple [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+2*a*x+b),x)`

[Out] `1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^2+2*a*x+b),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.27, size = 33, normalized size = 0.94

$$\frac{\operatorname{atanh}\left(\frac{a+bx}{\sqrt{a+b}\sqrt{a-b}}\right)}{\sqrt{a+b}\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b + 2*a*x + b*x^2),x)`

[Out] `-atanh((a + b*x)/((a + b)^(1/2)*(a - b)^(1/2)))/((a + b)^(1/2)*(a - b)^(1/2))`

sympy [B] time = 0.23, size = 100, normalized size = 2.86

$$\frac{\sqrt{\frac{1}{(a-b)(a+b)}} \log\left(x + \frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}} + a + b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2} - \frac{\sqrt{\frac{1}{(a-b)(a+b)}} \log\left(x + \frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}} + a - b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+2*a*x+b),x)`

[Out] `sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))) + a + b**2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a + b)))*log(x + (a**2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1/((a - b)*(a + b))))/b)/2`

$$3.90 \quad \int \frac{1}{b+2ax-bx^2} dx$$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

[Out] $-\operatorname{arctanh}((-b*x+a)/(a^2+b^2)^{(1/2)})/(a^2+b^2)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {618, 206}

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(b + 2*a*x - b*x^2)^{-1}, x]$

[Out] $-(\operatorname{ArcTanh}[(a - b*x)/\operatorname{Sqrt}[a^2 + b^2]])/\operatorname{Sqrt}[a^2 + b^2]$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{b+2ax-bx^2} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2} dx, x, 2a-2bx\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.28

$$\frac{\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*a*x - b*x^2)^(-1), x]

[Out] -(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])

fricas [B] time = 0.90, size = 67, normalized size = 2.09

$$\frac{\log\left(\frac{b^2x^2-2abx+2a^2+b^2+2\sqrt{a^2+b^2}(bx-a)}{bx^2-2ax-b}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+2*a*x+b), x, algorithm="fricas")

[Out] 1/2*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x - b))/sqrt(a^2 + b^2)

giac [A] time = 0.45, size = 55, normalized size = 1.72

$$-\frac{\log\left(\frac{|2bx-2a-2\sqrt{a^2+b^2}|}{|2bx-2a+2\sqrt{a^2+b^2}|}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+2*a*x+b), x, algorithm="giac")

[Out] -1/2*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)

maple [A] time = 0.07, size = 31, normalized size = 0.97

$$\frac{\operatorname{arctanh}\left(\frac{2bx-2a}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+2*a*x+b), x)

[Out] $1/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*x-2*a)/(a^2+b^2)^{(1/2)})$

maxima [A] time = 2.91, size = 49, normalized size = 1.53

$$-\frac{\log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x^2+2*a*x+b),x, algorithm="maxima")`

[Out] $-1/2*\log((b*x - a - \sqrt{a^2 + b^2})/(b*x - a + \sqrt{a^2 + b^2}))/\sqrt{a^2 + b^2}$

mupad [B] time = 0.23, size = 28, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b + 2*a*x - b*x^2),x)`

[Out] $-\operatorname{atanh}((a - b*x)/(a^2 + b^2)^{(1/2)})/(a^2 + b^2)^{(1/2)}$

sympy [B] time = 0.25, size = 102, normalized size = 3.19

$$-\frac{\sqrt{\frac{1}{a^2+b^2}} \log\left(x + \frac{-a^2\sqrt{\frac{1}{a^2+b^2}} - a - b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2} + \frac{\sqrt{\frac{1}{a^2+b^2}} \log\left(x + \frac{a^2\sqrt{\frac{1}{a^2+b^2}} - a + b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+2*a*x+b),x)`

[Out] $-\sqrt{1/(a**2 + b**2)}*\log(x + (-a**2*\sqrt{1/(a**2 + b**2)} - a - b**2*\sqrt{1/(a**2 + b**2)})/b)/2 + \sqrt{1/(a**2 + b**2)}*\log(x + (a**2*\sqrt{1/(a**2 + b**2)} - a + b**2*\sqrt{1/(a**2 + b**2)})/b)/2$

$$3.91 \quad \int \frac{1}{(2+4x+3x^2)^2} dx$$

Optimal. Leaf size=43

$$\frac{3x+2}{4(3x^2+4x+2)} + \frac{3 \tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] 1/4*(2+3*x)/(3*x^2+4*x+2)+3/8*arctan(1/2*(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {614, 618, 204}

$$\frac{3x+2}{4(3x^2+4x+2)} + \frac{3 \tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x + 3*x^2)^(-2), x]

[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+4x+3x^2)^2} dx &= \frac{2+3x}{4(2+4x+3x^2)} + \frac{3}{4} \int \frac{1}{2+4x+3x^2} dx \\
&= \frac{2+3x}{4(2+4x+3x^2)} - \frac{3}{2} \text{Subst} \left(\int \frac{1}{-8-x^2} dx, x, 4+6x \right) \\
&= \frac{2+3x}{4(2+4x+3x^2)} + \frac{3 \tan^{-1} \left(\frac{2+3x}{\sqrt{2}} \right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{3x+2}{4(3x^2+4x+2)} + \frac{3 \tan^{-1} \left(\frac{3x+2}{\sqrt{2}} \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x + 3*x^2)^(-2), x]

[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(4*Sqrt[2])

fricas [A] time = 0.91, size = 45, normalized size = 1.05

$$\frac{3\sqrt{2}(3x^2+4x+2) \arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + 6x+4}{8(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="fricas")

[Out] 1/8*(3*sqrt(2)*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 6*x + 4)/(3*x^2 + 4*x + 2)

giac [A] time = 0.58, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2)^2,x, algorithm="giac")

[Out] $3/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)$

maple [A] time = 0.04, size = 37, normalized size = 0.86

$$\frac{3\sqrt{2} \arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)}{8} + \frac{6x + 4}{24x^2 + 32x + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x^2+4*x+2)^2,x)`

[Out] $1/8*(6*x+4)/(3*x^2+4*x+2)+3/8*2^{(1/2)}*\arctan(1/4*(6*x+4)*2^{(1/2)})$

maxima [A] time = 2.27, size = 36, normalized size = 0.84

$$\frac{3}{8}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{3x+2}{4(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x^2+4*x+2)^2,x, algorithm="maxima")`

[Out] $3/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2 + 4*x + 2)$

mupad [B] time = 0.04, size = 33, normalized size = 0.77

$$\frac{\frac{x}{4} + \frac{1}{6}}{x^2 + \frac{4x}{3} + \frac{2}{3}} + \frac{3\sqrt{2} \operatorname{atan}\left(\frac{3\sqrt{2}x}{2} + \sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x + 3*x^2 + 2)^2,x)`

[Out] $(x/4 + 1/6)/((4*x)/3 + x^2 + 2/3) + (3*2^{(1/2)}*\operatorname{atan}((3*2^{(1/2)}*x)/2 + 2^{(1/2)}))/8$

sympy [A] time = 0.15, size = 39, normalized size = 0.91

$$\frac{3x + 2}{12x^2 + 16x + 8} + \frac{3\sqrt{2} \operatorname{atan}\left(\frac{3\sqrt{2}x}{2} + \sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x**2+4*x+2)**2,x)`

[Out] $(3*x + 2)/(12*x**2 + 16*x + 8) + 3*\sqrt{2}*\operatorname{atan}(3*\sqrt{2}*x/2 + \sqrt{2})/8$

$$3.92 \quad \int \frac{1}{(2+4x-3x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{2-3x}{20(-3x^2+4x+2)} - \frac{3 \tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

[Out] 1/20*(-2+3*x)/(-3*x^2+4*x+2)-3/200*arctanh(1/10*(2-3*x)*10^(1/2))*10^(1/2)

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {614, 618, 206}

$$-\frac{2-3x}{20(-3x^2+4x+2)} - \frac{3 \tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x - 3*x^2)^(-2), x]

[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sqrt[10])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+4x-3x^2)^2} dx &= -\frac{2-3x}{20(2+4x-3x^2)} + \frac{3}{20} \int \frac{1}{2+4x-3x^2} dx \\
&= -\frac{2-3x}{20(2+4x-3x^2)} - \frac{3}{10} \operatorname{Subst} \left(\int \frac{1}{40-x^2} dx, x, 4-6x \right) \\
&= -\frac{2-3x}{20(2+4x-3x^2)} - \frac{3 \tanh^{-1} \left(\frac{2-3x}{\sqrt{10}} \right)}{20\sqrt{10}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.44

$$\frac{2-3x}{20(3x^2-4x-2)} - \frac{3 \log(-3x + \sqrt{10} + 2)}{40\sqrt{10}} + \frac{3 \log(3x + \sqrt{10} - 2)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x - 3*x^2)^(-2), x]

[Out] (2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[10]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])

fricas [A] time = 0.85, size = 68, normalized size = 1.58

$$\frac{3\sqrt{10}(3x^2-4x-2) \log\left(\frac{9x^2+2\sqrt{10}(3x-2)-12x+14}{3x^2-4x-2}\right) - 60x + 40}{400(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="fricas")

[Out] 1/400*(3*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*x + 14)/(3*x^2 - 4*x - 2)) - 60*x + 40)/(3*x^2 - 4*x - 2)

giac [A] time = 0.52, size = 51, normalized size = 1.19

$$-\frac{3}{400} \sqrt{10} \log \left(\frac{|6x - 2\sqrt{10} - 4|}{|6x + 2\sqrt{10} - 4|} \right) - \frac{3x - 2}{20(3x^2 - 4x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="giac")

[Out] $-\frac{3}{400}\sqrt{10}\log\left(\frac{\text{abs}(6x - 2\sqrt{10} - 4)}{\text{abs}(6x + 2\sqrt{10} - 4)}\right) - \frac{1}{20}\frac{(3x - 2)}{(3x^2 - 4x - 2)}$

maple [A] time = 0.06, size = 37, normalized size = 0.86

$$\frac{3\sqrt{10} \operatorname{arctanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)}{200} - \frac{6x-4}{40(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+4*x+2)^2,x)

[Out] $-\frac{1}{40}\frac{(6x-4)}{(3x^2-4x-2)} + \frac{3}{200}10^{(1/2)}\operatorname{arctanh}\left(\frac{1}{20}(6x-4)10^{(1/2)}\right)$

maxima [A] time = 2.81, size = 47, normalized size = 1.09

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right) - \frac{3x-2}{20(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^2,x, algorithm="maxima")

[Out] $-\frac{3}{400}\sqrt{10}\log\left(\frac{(3x - \sqrt{10} - 2)}{(3x + \sqrt{10} - 2)}\right) - \frac{1}{20}\frac{(3x - 2)}{(3x^2 - 4x - 2)}$

mupad [B] time = 0.16, size = 34, normalized size = 0.79

$$\frac{3\sqrt{10} \operatorname{atanh}\left(\sqrt{10}\left(\frac{3x}{10} - \frac{1}{5}\right)\right)}{200} + \frac{\frac{x}{20} - \frac{1}{30}}{-x^2 + \frac{4x}{3} + \frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 3*x^2 + 2)^2,x)

[Out] $\frac{(3\sqrt{10}\operatorname{atanh}(10^{(1/2)}((3x)/10 - 1/5)))}{200} + \frac{(x/20 - 1/30)}{(4x)/3 - x^2 + 2/3}$

sympy [A] time = 0.15, size = 58, normalized size = 1.35

$$\frac{2-3x}{60x^2-80x-40} + \frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400} - \frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] (2 - 3*x)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/400  
- 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

$$3.93 \quad \int \frac{1}{(2+5x+3x^2)^2} dx$$

Optimal. Leaf size=34

$$-\frac{6x+5}{3x^2+5x+2} + 6\log(x+1) - 6\log(3x+2)$$

[Out] $(-5-6*x)/(3*x^2+5*x+2)+6*\ln(1+x)-6*\ln(2+3*x)$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {614, 616, 31}

$$-\frac{6x+5}{3x^2+5x+2} + 6\log(x+1) - 6\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x + 3*x^2)^(-2), x]

[Out] $-((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*\text{Log}[1 + x] - 6*\text{Log}[2 + 3*x]$

Rule 31

Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(2+5x+3x^2)^2} dx &= -\frac{5+6x}{2+5x+3x^2} - 6 \int \frac{1}{2+5x+3x^2} dx \\
 &= -\frac{5+6x}{2+5x+3x^2} - 18 \int \frac{1}{2+3x} dx + 18 \int \frac{1}{3+3x} dx \\
 &= -\frac{5+6x}{2+5x+3x^2} + 6 \log(1+x) - 6 \log(2+3x)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.97

$$\frac{-6x-5}{3x^2+5x+2} + 6 \log(x+1) - 6 \log(3x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x + 3*x^2)^(-2), x]

[Out] (-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]

fricas [A] time = 0.93, size = 53, normalized size = 1.56

$$\frac{6(3x^2+5x+2)\log(3x+2) - 6(3x^2+5x+2)\log(x+1) + 6x+5}{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="fricas")

[Out] -(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x + 1) + 6*x + 5)/(3*x^2 + 5*x + 2)

giac [A] time = 0.45, size = 36, normalized size = 1.06

$$-\frac{6x+5}{3x^2+5x+2} - 6 \log(|3x+2|) + 6 \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2)^2,x, algorithm="giac")

[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(abs(3*x + 2)) + 6*log(abs(x + 1))

maple [A] time = 0.05, size = 32, normalized size = 0.94

$$-6 \ln(3x+2) + 6 \ln(x+1) - \frac{3}{3x+2} - \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x^2+5*x+2)^2,x)`

[Out] $-3/(3*x+2)-6*\ln(3*x+2)-1/(x+1)+6*\ln(x+1)$

maxima [A] time = 1.32, size = 34, normalized size = 1.00

$$-\frac{6x+5}{3x^2+5x+2} - 6 \log(3x+2) + 6 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x^2+5*x+2)^2,x, algorithm="maxima")`

[Out] $-(6*x+5)/(3*x^2+5*x+2) - 6*\log(3*x+2) + 6*\log(x+1)$

mupad [B] time = 0.20, size = 34, normalized size = 1.00

$$-6 \ln\left(\frac{3x+2}{x+1}\right) - \frac{2\left(3x+\frac{5}{2}\right)}{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x+3*x^2+2)^2,x)`

[Out] $-6*\log((3*x+2)/(x+1)) - (2*(3*x+5/2))/(5*x+3*x^2+2)$

sympy [A] time = 0.14, size = 31, normalized size = 0.91

$$\frac{-6x-5}{3x^2+5x+2} - 6 \log\left(x+\frac{2}{3}\right) + 6 \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x**2+5*x+2)**2,x)`

[Out] $(-6*x-5)/(3*x**2+5*x+2) - 6*\log(x+2/3) + 6*\log(x+1)$

$$3.94 \quad \int \frac{1}{(2+5x-3x^2)^2} dx$$

Optimal. Leaf size=42

$$-\frac{5-6x}{49(-3x^2+5x+2)} - \frac{6}{343} \log(2-x) + \frac{6}{343} \log(3x+1)$$

[Out] 1/49*(-5+6*x)/(-3*x^2+5*x+2)-6/343*ln(2-x)+6/343*ln(1+3*x)

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {614, 616, 31}

$$-\frac{5-6x}{49(-3x^2+5x+2)} - \frac{6}{343} \log(2-x) + \frac{6}{343} \log(3x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x - 3*x^2)^(-2), x]

[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/343

Rule 31

Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+5x-3x^2)^2} dx &= -\frac{5-6x}{49(2+5x-3x^2)} + \frac{6}{49} \int \frac{1}{2+5x-3x^2} dx \\
&= -\frac{5-6x}{49(2+5x-3x^2)} - \frac{18}{343} \int \frac{1}{-1-3x} dx + \frac{18}{343} \int \frac{1}{6-3x} dx \\
&= -\frac{5-6x}{49(2+5x-3x^2)} - \frac{6}{343} \log(2-x) + \frac{6}{343} \log(1+3x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{5-6x}{49(3x^2-5x-2)} - \frac{6}{343} \log(2-x) + \frac{6}{343} \log(3x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x - 3*x^2)^(-2), x]

[Out] (5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1 + 3*x])/343

fricas [A] time = 1.07, size = 53, normalized size = 1.26

$$\frac{6(3x^2 - 5x - 2) \log(3x + 1) - 6(3x^2 - 5x - 2) \log(x - 2) - 42x + 35}{343(3x^2 - 5x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="fricas")

[Out] 1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log(x - 2) - 42*x + 35)/(3*x^2 - 5*x - 2)

giac [A] time = 0.32, size = 36, normalized size = 0.86

$$-\frac{6x-5}{49(3x^2-5x-2)} + \frac{6}{343} \log(|3x+1|) - \frac{6}{343} \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="giac")

[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(abs(3*x + 1)) - 6/343*log(abs(x - 2))

maple [A] time = 0.05, size = 32, normalized size = 0.76

$$\frac{6 \ln(3x+1)}{343} - \frac{6 \ln(x-2)}{343} - \frac{1}{49(x-2)} - \frac{3}{49(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3*x^2+5*x+2)^2,x)`

[Out] `-1/49/(x-2)-6/343*ln(x-2)-3/49/(3*x+1)+6/343*ln(3*x+1)`

maxima [A] time = 1.35, size = 34, normalized size = 0.81

$$-\frac{6x-5}{49(3x^2-5x-2)} + \frac{6}{343} \log(3x+1) - \frac{6}{343} \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x^2+5*x+2)^2,x, algorithm="maxima")`

[Out] `-1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*log(x - 2)`

mupad [B] time = 0.08, size = 34, normalized size = 0.81

$$\frac{6 \ln\left(\frac{3x+1}{x-2}\right)}{343} + \frac{2\left(3x - \frac{5}{2}\right)}{49(-3x^2 + 5x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x - 3*x^2 + 2)^2,x)`

[Out] `(6*log((3*x + 1)/(x - 2)))/343 + (2*(3*x - 5/2))/(49*(5*x - 3*x^2 + 2))`

sympy [A] time = 0.15, size = 32, normalized size = 0.76

$$\frac{5-6x}{147x^2-245x-98} - \frac{6 \log(x-2)}{343} + \frac{6 \log\left(x + \frac{1}{3}\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+5*x+2)**2,x)`

[Out] `(5 - 6*x)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x + 1/3)/343`

$$3.95 \quad \int \frac{1}{(a+cx+bx^2)^2} dx$$

Optimal. Leaf size=71

$$\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)} + \frac{4b \tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

[Out] $(2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*\arctan((2*b*x+c)/(4*a*b-c^2)^{(1/2)})/(4*a*b-c^2)^{(3/2)}$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {614, 618, 204}

$$\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)} + \frac{4b \tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x + b*x^2)^(-2), x]

[Out] $(c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*\text{ArcTan}[(c + 2*b*x)/\text{Sqrt}[4*a*b - c^2]])/(4*a*b - c^2)^{(3/2)}$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+cx+bx^2)^2} dx &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)} + \frac{(2b) \int \frac{1}{a+cx+bx^2} dx}{4ab-c^2} \\ &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)} - \frac{(4b) \text{Subst}\left(\int \frac{1}{-4ab+c^2-x^2} dx, x, c+2bx\right)}{4ab-c^2} \\ &= \frac{c+2bx}{(4ab-c^2)(a+cx+bx^2)} + \frac{4b \tan^{-1}\left(\frac{c+2bx}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.99

$$\frac{2bx+c}{(4ab-c^2)(a+x(bx+c))} + \frac{4b \tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x + b*x^2)^(-2), x]

[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)

fricas [B] time = 1.08, size = 334, normalized size = 4.70

$$\left[\frac{4abc - c^3 + 2(b^2x^2 + bcx + ab)\sqrt{-4ab + c^2} \log\left(\frac{2b^2x^2 + 2bcx - 2ab + c^2 + \sqrt{-4ab + c^2}(2bx + c)}{bx^2 + cx + a}\right) + 2(4ab^2 - bc^2)x}{16a^3b^2 - 8a^2bc^2 + ac^4 + (16a^2b^3 - 8ab^2c^2 + bc^4)x^2 + (16a^2b^2c - 8abc^3 + c^5)x} \right], \frac{4abc - c^3}{16a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="fricas")

[Out] [(4*a*b*c - c^3 + 2*(b^2*x^2 + b*c*x + a*b)*sqrt(-4*a*b + c^2)*log((2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2 + sqrt(-4*a*b + c^2)*(2*b*x + c))/(b*x^2 + c*x + a)) + 2*(4*a*b^2 - b*c^2)*x]/(16*a^3*b^2 - 8*a^2*b*c^2 + a*c^4 + (16*a^2*b^2*c - 8*a*b*c^3 + c^5)*x)

$\sqrt{3 - 8ab^2c^2 + bc^4}x^2 + (16a^2b^2c - 8abc^3 + c^5)x$, $(4ab^2c - c^3 - 4(b^2x^2 + bcx + ab)\sqrt{4ab - c^2})\arctan\left(\frac{2bx + c}{\sqrt{4ab - c^2}}\right) + 2(4ab^2 - bc^2)x / (16a^3b^2 - 8a^2b^2c^2 + abc^4 + (16a^2b^3 - 8ab^2c^2 + bc^4)x^2 + (16a^2b^2c - 8abc^3 + c^5)x]$

giac [A] time = 0.40, size = 67, normalized size = 0.94

$$\frac{4b \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{\frac{3}{2}}} + \frac{2bx+c}{(bx^2+cx+a)(4ab-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="giac")

[Out] $4b \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right) / (4ab-c^2)^{3/2} + (2bx+c) / ((b^2x^2+cx+a)(4ab-c^2))$

maple [A] time = 0.06, size = 68, normalized size = 0.96

$$\frac{4b \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{\frac{3}{2}}} + \frac{2bx+c}{(4ab-c^2)(bx^2+cx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+c*x+a)^2,x)

[Out] $(2bx+c)/(4ab-c^2)/(b^2x^2+cx+a) + 4b \arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right) / (4ab-c^2)^{3/2}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+c*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-4*a*b>0)', see `assume?` for more details)Is c^2-4*a*b positive or negative?

mupad [B] time = 0.17, size = 119, normalized size = 1.68

$$\frac{\frac{c}{4ab-c^2} + \frac{2bx}{4ab-c^2}}{bx^2 + cx + a} - \frac{4b \operatorname{atan}\left(\frac{\left(\frac{2b(c^3-4abc)}{(4ab-c^2)^{5/2}} - \frac{4b^2x}{(4ab-c^2)^{3/2}}\right)(4ab-c^2)}{2b}\right)}{(4ab-c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x + b*x^2)^2,x)`

[Out] $(c/(4*a*b - c^2) + (2*b*x)/(4*a*b - c^2))/(a + c*x + b*x^2) - (4*b*\operatorname{atan}\left(\left(\frac{2*b*(c^3 - 4*a*b*c)}{(4*a*b - c^2)^{5/2}} - \frac{4*b^2*x}{(4*a*b - c^2)^{3/2}}\right)*(4*a*b - c^2)\right)/(2*b))/(4*a*b - c^2)^{3/2}$

sympy [B] time = 0.59, size = 265, normalized size = 3.73

$$-2b \sqrt{-\frac{1}{(4ab-c^2)^3}} \log\left(x + \frac{-32a^2b^3 \sqrt{-\frac{1}{(4ab-c^2)^3}} + 16ab^2c^2 \sqrt{-\frac{1}{(4ab-c^2)^3}} - 2bc^4 \sqrt{-\frac{1}{(4ab-c^2)^3}} + 2bc}{4b^2}\right) + 2b \sqrt{-\frac{1}{(4ab-c^2)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+c*x+a)**2,x)`

[Out] $-2*b*\sqrt{-1/(4*a*b - c**2)**3}*\log(x + (-32*a**2*b**3*\sqrt{-1/(4*a*b - c**2)**3} + 16*a*b**2*c**2*\sqrt{-1/(4*a*b - c**2)**3} - 2*b*c**4*\sqrt{-1/(4*a*b - c**2)**3} + 2*b*c)/(4*b**2)) + 2*b*\sqrt{-1/(4*a*b - c**2)**3}*\log(x + (32*a**2*b**3*\sqrt{-1/(4*a*b - c**2)**3} - 16*a*b**2*c**2*\sqrt{-1/(4*a*b - c**2)**3} + 2*b*c**4*\sqrt{-1/(4*a*b - c**2)**3} + 2*b*c)/(4*b**2)) + (2*b*x + c)/(4*a**2*b - a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**3))$

$$3.96 \quad \int \frac{1}{(b+2ax+bx^2)^2} dx$$

Optimal. Leaf size=72

$$\frac{b \tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}} - \frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

[Out] 1/2*(-b*x-a)/(a^2-b^2)/(b*x^2+2*a*x+b)+1/2*b*arctanh((b*x+a)/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {614, 618, 206}

$$\frac{b \tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}} - \frac{a+bx}{2(a^2-b^2)(2ax+bx^2+b)}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*a*x + b*x^2)^(-2), x]

[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(b+2ax+bx^2)^2} dx &= -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} - \frac{b \int \frac{1}{b+2ax+bx^2} dx}{2(a^2-b^2)} \\ &= -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4(a^2-b^2)-x^2} dx, x, 2a+2bx\right)}{a^2-b^2} \\ &= -\frac{a+bx}{2(a^2-b^2)(b+2ax+bx^2)} + \frac{b \tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2(a^2-b^2)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 1.00

$$\frac{a+bx}{2(b^2-a^2)(2ax+bx^2+b)} + \frac{b \tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{2(b^2-a^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*a*x + b*x^2)^(-2), x]

[Out] (a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))

fricas [B] time = 0.95, size = 317, normalized size = 4.40

$$\left[\frac{2a^3 - 2ab^2 + (b^2x^2 + 2abx + b^2)\sqrt{a^2 - b^2} \log\left(\frac{b^2x^2 + 2abx + 2a^2 - b^2 - 2\sqrt{a^2 - b^2}(bx+a)}{bx^2 + 2ax + b}\right) + 2(a^2b - b^3)x}{4(a^4b - 2a^2b^3 + b^5 + (a^4b - 2a^2b^3 + b^5)x^2 + 2(a^5 - 2a^3b^2 + ab^4)x)}, -\frac{a^3 - ab^2 - (b^2x^2 + 2abx + b^2)\sqrt{a^2 - b^2}}{2(a^4b - 2a^2b^3 + b^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="fricas")

[Out] [-1/4*(2*a^3 - 2*a*b^2 + (b^2*x^2 + 2*a*b*x + b^2)*sqrt(a^2 - b^2)*log((b^2*x^2 + 2*a*b*x + 2*a^2 - b^2 - 2*sqrt(a^2 - b^2)*(b*x + a))/(b*x^2 + 2*a*x + b)) + 2*(a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 +

$b^5*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x$, $-1/2*(a^3 - a*b^2 - (b^2*x^2 + 2*a*b*x + b^2)*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*x + a)/(a^2 - b^2)) + (a^2*b - b^3)*x)/(a^4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*x^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*x]$

giac [A] time = 0.51, size = 75, normalized size = 1.04

$$-\frac{b \arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{bx+a}{2(bx^2+2ax+b)(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="giac")

[Out] $-1/2*b*\arctan((b*x + a)/\sqrt{-a^2 + b^2})/((a^2 - b^2)*\sqrt{-a^2 + b^2}) - 1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))$

maple [A] time = 0.05, size = 86, normalized size = 1.19

$$\frac{2b \arctan\left(\frac{2bx+2a}{2\sqrt{-a^2+b^2}}\right)}{(-4a^2+4b^2)\sqrt{-a^2+b^2}} + \frac{2bx+2a}{(-4a^2+4b^2)(bx^2+2ax+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+2*a*x+b)^2,x)

[Out] $(2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a^2+b^2)^(1/2)*\arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+2*a*x+b)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.32, size = 107, normalized size = 1.49

$$-\frac{\frac{a}{2(a^2-b^2)} + \frac{bx}{2(a^2-b^2)}}{bx^2+2ax+b} + \frac{b \operatorname{atan}\left(\frac{-a^3 \operatorname{li}(-\operatorname{li} x a^2 b + a b^2 \operatorname{li} + \operatorname{li} x b^3)}{(a+b)^{3/2} (a-b)^{3/2}}\right) \operatorname{li}}{2(a+b)^{3/2} (a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b + 2*a*x + b*x^2)^2,x)`

[Out] $(b \operatorname{atan}\left(\frac{a^2 b^2 + b^3 x - a^3 - a^2 b x}{(a+b)^{3/2}(a-b)^{3/2}}\right) + \frac{a}{2(a^2 - b^2)} + \frac{b x}{2(a^2 - b^2)}) / (b + 2 a x + b x^2)$

sympy [B] time = 0.55, size = 230, normalized size = 3.19

$$\frac{b \sqrt{\frac{1}{(a-b)^3(a+b)^3}} \log\left(x + \frac{-a^4 b \sqrt{\frac{1}{(a-b)^3(a+b)^3}} + 2a^2 b^3 \sqrt{\frac{1}{(a-b)^3(a+b)^3}} + ab - b^5 \sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right) + b \sqrt{\frac{1}{(a-b)^3(a+b)^3}} \log\left(x + \frac{a^4 b \sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+2*a*x+b)**2,x)`

[Out] $-b \sqrt{1/((a-b)^3(a+b)^3)} \log(x + (-a^4 b \sqrt{1/((a-b)^3(a+b)^3)} + 2a^2 b^3 \sqrt{1/((a-b)^3(a+b)^3)} + ab - b^5 \sqrt{1/((a-b)^3(a+b)^3)})/b^2)/4 + b \sqrt{1/((a-b)^3(a+b)^3)} \log(x + (a^4 b \sqrt{1/((a-b)^3(a+b)^3)} - 2a^2 b^3 \sqrt{1/((a-b)^3(a+b)^3)} + ab + b^5 \sqrt{1/((a-b)^3(a+b)^3)})/b^2)/4 + (-a - b x)/(2a^2 b - 2b^3 + x^2(2a^2 b - 2b^3) + x(4a^3 - 4ab^2))$

$$3.97 \quad \int \frac{1}{(b+2ax-bx^2)^2} dx$$

Optimal. Leaf size=69

$$-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)} - \frac{b \tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

[Out] 1/2*(b*x-a)/(a^2+b^2)/(-b*x^2+2*a*x+b)-1/2*b*arctanh((-b*x+a)/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {614, 618, 206}

$$-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)} - \frac{b \tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*a*x - b*x^2)^(-2), x]

[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(b+2ax-bx^2)^2} dx &= -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} + \frac{b \int \frac{1}{b+2ax-bx^2} dx}{2(a^2+b^2)} \\ &= -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b \operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2} dx, x, 2a-2bx\right)}{a^2+b^2} \\ &= -\frac{a-bx}{2(a^2+b^2)(b+2ax-bx^2)} - \frac{b \tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.13

$$\frac{\frac{bx-a}{2ax-bx^2+b} - \frac{b \tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*a*x - b*x^2)^(-2), x]

[Out] ((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/(2*(a^2 + b^2))

fricas [B] time = 0.96, size = 171, normalized size = 2.48

$$\frac{2a^3 + 2ab^2 + (b^2x^2 - 2abx - b^2)\sqrt{a^2 + b^2} \log\left(\frac{b^2x^2 - 2abx + 2a^2 + b^2 + 2\sqrt{a^2 + b^2}(bx - a)}{bx^2 - 2ax - b}\right) - 2(a^2b + b^3)x}{4(a^4b + 2a^2b^3 + b^5 - (a^4b + 2a^2b^3 + b^5)x^2 + 2(a^5 + 2a^3b^2 + ab^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="fricas")

[Out] -1/4*(2*a^3 + 2*a*b^2 + (b^2*x^2 - 2*a*b*x - b^2)*sqrt(a^2 + b^2)*log((b^2*x^2 - 2*a*b*x + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*(b*x - a))/(b*x^2 - 2*a*x - b)) - 2*(a^2*b + b^3)*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*x^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*x)

giac [A] time = 0.53, size = 90, normalized size = 1.30

$$-\frac{b \log\left(\frac{|2bx-2a-2\sqrt{a^2+b^2}|}{|2bx-2a+2\sqrt{a^2+b^2}|}\right)}{4(a^2+b^2)^{\frac{3}{2}}} - \frac{bx-a}{2(bx^2-2ax-b)(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="giac")

[Out] -1/4*b*log(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2*a*x - b)*(a^2 + b^2))

maple [A] time = 0.05, size = 84, normalized size = 1.22

$$-\frac{2b \operatorname{arctanh}\left(\frac{2bx-2a}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}} + \frac{2bx-2a}{(-4a^2-4b^2)(bx^2-2ax-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+2*a*x+b)^2,x)

[Out] (2*b*x-2*a)/(-4*a^2-4*b^2)/(b*x^2-2*a*x-b)-2*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))

maxima [A] time = 2.84, size = 97, normalized size = 1.41

$$-\frac{b \log\left(\frac{bx-a-\sqrt{a^2+b^2}}{bx-a+\sqrt{a^2+b^2}}\right)}{4(a^2+b^2)^{\frac{3}{2}}} + \frac{bx-a}{2(a^2b+b^3-(a^2b+b^3)x^2+2(a^3+ab^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+2*a*x+b)^2,x, algorithm="maxima")

[Out] -1/4*b*log((b*x - a - sqrt(a^2 + b^2))/(b*x - a + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 1/2*(b*x - a)/(a^2*b + b^3 - (a^2*b + b^3)*x^2 + 2*(a^3 + a*b^2)*x)

mupad [B] time = 0.32, size = 100, normalized size = 1.45

$$-\frac{\frac{a}{2(a^2+b^2)} - \frac{bx}{2(a^2+b^2)}}{-bx^2 + 2ax + b} + \frac{b \operatorname{atan}\left(\frac{ab^2 + a^3 - bx(a^2+b^2)}{(a^2+b^2)^{3/2}}\right)}{2(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b + 2*a*x - b*x^2)^2,x)`

[Out] $(b \operatorname{atan}\left(\frac{a^2 b^2 + a^3 + b^2 x - b^2 x^2}{(a^2 + b^2)^{3/2}}\right) - (a^2 + b^2)^{3/2} - (a/(2(a^2 + b^2)) - (b^2 x)/(2(a^2 + b^2))) / (b + 2ax - b^2 x^2))$

sympy [B] time = 0.60, size = 218, normalized size = 3.16

$$\frac{b \sqrt{\frac{1}{(a^2+b^2)^3}} \log\left(x + \frac{-a^4 b \sqrt{\frac{1}{(a^2+b^2)^3}} - 2a^2 b^3 \sqrt{\frac{1}{(a^2+b^2)^3}} - ab - b^5 \sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4} + \frac{b \sqrt{\frac{1}{(a^2+b^2)^3}} \log\left(x + \frac{a^4 b \sqrt{\frac{1}{(a^2+b^2)^3}} + 2a^2 b^3 \sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+2*a*x+b)**2,x)`

[Out] $-b \sqrt{(a^2 + b^2)^{-3}} \log(x + (-a^4 b \sqrt{(a^2 + b^2)^{-3}} - 2a^2 b^3 \sqrt{(a^2 + b^2)^{-3}} - ab - b^5 \sqrt{(a^2 + b^2)^{-3}}) / b^2) / 4 + b \sqrt{(a^2 + b^2)^{-3}} \log(x + (a^4 b \sqrt{(a^2 + b^2)^{-3}} + 2a^2 b^3 \sqrt{(a^2 + b^2)^{-3}} + ab + b^5 \sqrt{(a^2 + b^2)^{-3}}) / b^2) / 4 + (a - bx) / (-2a^2 b - 2b^3 + x^2(2a^2 b + 2b^3) + x(-4a^3 - 4ab^2))$

$$3.98 \quad \int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{1/n} x \cos\left(\frac{\pi - 2k\pi}{n}\right)} dx$$

Optimal. Leaf size=62

$$-\left(\frac{a}{b}\right)^{-1/n} \csc\left(\frac{\pi - 2\pi k}{n}\right) \tan^{-1}\left(\cot\left(\frac{\pi - 2\pi k}{n}\right) - x\left(\frac{a}{b}\right)^{-1/n} \csc\left(\frac{\pi - 2\pi k}{n}\right)\right)$$

[Out] arctan(-cot((-2*Pi*k+Pi)/n)+x*csc((-2*Pi*k+Pi)/n)/((a/b)^(1/n)))*csc((-2*Pi*k+Pi)/n)/((a/b)^(1/n))

Rubi [A] time = 0.16, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {618, 204}

$$-\left(\frac{a}{b}\right)^{-1/n} \csc\left(\frac{\pi - 2\pi k}{n}\right) \tan^{-1}\left(\cot\left(\frac{\pi - 2\pi k}{n}\right) - x\left(\frac{a}{b}\right)^{-1/n} \csc\left(\frac{\pi - 2\pi k}{n}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1), x]

[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{\left(\frac{a}{b}\right)^{2/n} + x^2 - 2\left(\frac{a}{b}\right)^{1/n} x \cos\left(\frac{\pi-2k\pi}{n}\right)} dx = - \left(2 \text{Subst} \left(\int \frac{1}{-x^2 - 4\left(\frac{a}{b}\right)^{2/n} \left(1 - \cos^2\left(\frac{\pi-2k\pi}{n}\right)\right)} dx, x, 2x - 2\left(\frac{a}{b}\right)^{1/n} \cos\left(\frac{\pi-2k\pi}{n}\right) \right) \right. \\ \left. = -\left(\frac{a}{b}\right)^{-1/n} \tan^{-1}\left(\cot\left(\frac{\pi-2k\pi}{n}\right) - \left(\frac{a}{b}\right)^{-1/n} x \csc\left(\frac{\pi-2k\pi}{n}\right)\right) \csc\left(\frac{\pi-2k\pi}{n}\right) \right)$$

Mathematica [A] time = 0.10, size = 65, normalized size = 1.05

$$\left(\frac{a}{b}\right)^{-1/n} \csc\left(\frac{\pi-2\pi k}{n}\right) \tan^{-1}\left(\frac{\tan\left(\frac{\pi-2\pi k}{2n}\right)\left(\left(\frac{a}{b}\right)^{1/n} + x\right)}{\left(\frac{a}{b}\right)^{1/n} - x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-1), x]

[Out] (ArcTan[(((a/b)^n^(-1) + x)*Tan[(Pi - 2*k*Pi)/(2*n)])/((a/b)^n^(-1) - x)]*Csc[(Pi - 2*k*Pi)/n])/((a/b)^n^(-1))

fricas [A] time = 1.26, size = 89, normalized size = 1.44

$$\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)} \cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right) - x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)} \sin\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)} \sin\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algorithm="fricas")

[Out] -arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2*pi*k/n - pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))

giac [A] time = 0.57, size = 100, normalized size = 1.61

$$\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{n}} \cos\left(-\frac{2\pi k}{n} + \frac{\pi}{n}\right) - x}{\sqrt{-\cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)^2 + 1} \left(\frac{a}{b}\right)^{\frac{1}{n}}}\right)}{\sqrt{-\cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right)^2 + 1} \left(\frac{a}{b}\right)^{\frac{1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algorithm="giac")

[Out] arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n)))/sqrt(-cos(2*pi*k/n - pi/n)^2 + 1)*(a/b)^(1/n))

maple [A] time = 0.57, size = 111, normalized size = 1.79

$$\frac{\arctan\left(\frac{-2\left(\frac{a}{b}\right)^{\frac{1}{n}} \cos\left(\frac{\pi(2k-1)}{n}\right) + 2x}{2\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}} \left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right) + \left(\frac{a}{b}\right)^{\frac{2}{n}}}\right)}{\sqrt{-\left(\frac{a}{b}\right)^{\frac{2}{n}} \left(\cos^2\left(\frac{\pi(2k-1)}{n}\right)\right) + \left(\frac{a}{b}\right)^{\frac{2}{n}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x)

[Out] 1/(-cos(Pi*(2*k-1)/n)^2*((a/b)^(1/n))^2+(a/b)^(2/n))^(1/2)*arctan(1/2*(2*x-2*cos(Pi*(2*k-1)/n)*(a/b)^(1/n))/(-cos(Pi*(2*k-1)/n)^2*((a/b)^(1/n))^2+(a/b)^(2/n))^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*pi*k+pi)/n)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(1>0)', see `assume?` for more details)Is 1 zero or nonzero?

mupad [B] time = 0.25, size = 110, normalized size = 1.77

$$\frac{\operatorname{atanh}\left(\frac{x - \cos\left(\frac{\pi(2k-1)}{n}\right)\left(\frac{a}{b}\right)^{1/n}}{\sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)-1} \sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)+1} \left(\frac{a}{b}\right)^{1/n}}\right)}{\sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)-1} \sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)+1} \left(\frac{a}{b}\right)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a/b)^(2/n) + x^2 - 2*x*cos((Pi - 2*Pi*k)/n)*(a/b)^(1/n)),x)`

[Out] `-atanh((x - cos((Pi*(2*k - 1))/n)*(a/b)^(1/n))/((cos((Pi*(2*k - 1))/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n)))/((cos((Pi*(2*k - 1))/n) - 1)^(1/2)*(cos((Pi*(2*k - 1))/n) + 1)^(1/2)*(a/b)^(1/n))`

sympy [B] time = 0.98, size = 212, normalized size = 3.42

$$\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}} \log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{n}} \cos\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right) - \frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}} \left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}} \cos^2\left(\frac{2\pi k}{n} - \frac{\pi}{n}\right) + 2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}} \log\left(\dots\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)`

[Out] `-sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(2*pi*k/n - pi/n) - sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2 + sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**(1/n)*cos(2*pi*k/n - pi/n) + sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**(2/n))/2)/2`

$$3.99 \quad \int \frac{1}{ab + \sqrt{b^2 - 4ab^3}x - b^2x^2} dx$$

Optimal. Leaf size=33

$$\frac{2 \tanh^{-1}\left(\frac{2b^2x - \sqrt{b^2 - 4ab^3}}{b}\right)}{b}$$

[Out] $2*\operatorname{arctanh}((2*b^2*x - (-4*a*b^3 + b^2)^{(1/2)})/b)/b$

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {616, 31}

$$\frac{\log\left(-\sqrt{b^2 - 4ab^3} + 2b^2x + b\right)}{b} - \frac{\log\left(\sqrt{b^2 - 4ab^3} - 2b^2x + b\right)}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a*b + \operatorname{Sqrt}[b^2 - 4*a*b^3]*x - b^2*x^2)^{-1}, x]$

[Out] $-(\operatorname{Log}[b + \operatorname{Sqrt}[b^2 - 4*a*b^3] - 2*b^2*x]/b) + \operatorname{Log}[b - \operatorname{Sqrt}[b^2 - 4*a*b^3] + 2*b^2*x]/b$

Rule 31

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 616

$\operatorname{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/\operatorname{Simp}[b/2 - q/2 + c*x, x], x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/\operatorname{Simp}[b/2 + q/2 + c*x, x], x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{PosQ}[b^2 - 4*a*c] \&\& \operatorname{PerfectSquareQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{1}{ab + \sqrt{b^2 - 4ab^3}x - b^2x^2} dx &= - \left(b \int \frac{1}{\frac{1}{2}(-b + \sqrt{b^2 - 4ab^3}) - b^2x} dx \right) + b \int \frac{1}{\frac{1}{2}(b + \sqrt{b^2 - 4ab^3}) - b^2x} dx \\ &= - \frac{\log\left(b + \sqrt{b^2 - 4ab^3} - 2b^2x\right)}{b} + \frac{\log\left(b - \sqrt{b^2 - 4ab^3} + 2b^2x\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 1.03

$$\frac{2 \tanh^{-1} \left(\frac{2b^2x - \sqrt{-b^2(4ab-1)}}{b} \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1), x]

[Out] (2*ArcTanh[(-Sqrt[-(b^2*(-1 + 4*a*b))]) + 2*b^2*x)/b])/b

fricas [B] time = 1.29, size = 63, normalized size = 1.91

$$\frac{\log \left(\frac{2b^2x + b - \sqrt{-4ab^3 + b^2}}{b} \right) - \log \left(\frac{2b^2x - b - \sqrt{-4ab^3 + b^2}}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)), x, algorithm="fricas")

[Out] (log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b - sqrt(-4*a*b^3 + b^2))/b))/b

giac [A] time = 0.53, size = 56, normalized size = 1.70

$$\frac{\log \left(\frac{|2b^2x - \sqrt{-4ab+1}|b| - |b|}{|2b^2x - \sqrt{-4ab+1}|b| + |b|} \right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)), x, algorithm="giac")

[Out] -log(abs(2*b^2*x - sqrt(-4*a*b + 1))*abs(b) - abs(b))/abs(2*b^2*x - sqrt(-4*a*b + 1))*abs(b) + abs(b))/abs(b)

maple [A] time = 0.08, size = 31, normalized size = 0.94

$$\frac{2 \operatorname{arctanh} \left(\frac{-2b^2x + \sqrt{-(4ab-1)b^2}}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)), x)

[Out] $-2/b \cdot \operatorname{arctanh}((-2*b^2*x + (-b^2*(4*a*b-1))^{1/2})/b)$

maxima [A] time = 1.39, size = 55, normalized size = 1.67

$$-\frac{\log\left(\frac{2b^2x-b-\sqrt{-4ab^3+b^2}}{2b^2x+b-\sqrt{-4ab^3+b^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")`

[Out] $-\log((2*b^2*x - b - \sqrt{-4*a*b^3 + b^2})/(2*b^2*x + b - \sqrt{-4*a*b^3 + b^2}))/b$

mupad [B] time = 0.26, size = 38, normalized size = 1.15

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{b^2-4ab^3}}{\sqrt{b^2}} - \frac{2b^2x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*b + x*(b^2 - 4*a*b^3)^(1/2) - b^2*x^2),x)`

[Out] $-(2*\operatorname{atanh}((b^2 - 4*a*b^3)^{1/2}/(b^2)^{1/2} - (2*b^2*x)/(b^2)^{1/2}))/b^{1/2}$

sympy [B] time = 0.29, size = 56, normalized size = 1.70

$$\frac{\log\left(x - \frac{1}{2b} - \frac{\sqrt{-4ab^3+b^2}}{2b^2}\right) - \log\left(x + \frac{1}{2b} - \frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)`

[Out] $-(\log(x - 1/(2*b) - \sqrt{-4*a*b**3 + b**2}/(2*b**2)) - \log(x + 1/(2*b) - \sqrt{-4*a*b**3 + b**2}/(2*b**2)))/b$

$$3.100 \quad \int \frac{1}{ab - \sqrt{b^2 - 4ab^3}x - b^2x^2} dx$$

Optimal. Leaf size=31

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b^2 - 4ab^3} + 2b^2x}{b} \right)}{b}$$

[Out] 2*arctanh((2*b^2*x+(-4*a*b^3+b^2)^(1/2))/b)/b

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {616, 31}

$$\frac{\log \left(\sqrt{b^2 - 4ab^3} + 2b^2x + b \right)}{b} - \frac{\log \left(-\sqrt{b^2 - 4ab^3} - 2b^2x + b \right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1), x]

[Out] -(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b + Sqrt[b^2 - 4*a*b^3] + 2*b^2*x]/b

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{ab - \sqrt{b^2 - 4ab^3}x - b^2x^2} dx &= - \left(b \int \frac{1}{\frac{1}{2}(-b - \sqrt{b^2 - 4ab^3}) - b^2x} dx \right) + b \int \frac{1}{\frac{1}{2}(b - \sqrt{b^2 - 4ab^3}) - b^2x} dx \\ &= - \frac{\log(b - \sqrt{b^2 - 4ab^3} - 2b^2x)}{b} + \frac{\log(b + \sqrt{b^2 - 4ab^3} + 2b^2x)}{b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.03

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{-b^2(4ab-1)} + 2b^2x}{b} \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1), x]

[Out] (2*ArcTanh[(Sqrt[-(b^2*(-1 + 4*a*b))]] + 2*b^2*x)/b])/b

fricas [B] time = 0.86, size = 59, normalized size = 1.90

$$\frac{\log \left(\frac{2b^2x + b + \sqrt{-4ab^3 + b^2}}{b} \right) - \log \left(\frac{2b^2x - b + \sqrt{-4ab^3 + b^2}}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)), x, algorithm="fricas")

[Out] (log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b + sqrt(-4*a*b^3 + b^2))/b))/b

giac [A] time = 0.55, size = 54, normalized size = 1.74

$$\frac{\log \left(\frac{|2b^2x + \sqrt{-4ab+1}|b - |b|}{|2b^2x + \sqrt{-4ab+1}|b + |b|} \right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)), x, algorithm="giac")

[Out] -log(abs(2*b^2*x + sqrt(-4*a*b + 1))*abs(b) - abs(b))/abs(2*b^2*x + sqrt(-4*a*b + 1))*abs(b) + abs(b))/abs(b)

maple [A] time = 0.05, size = 31, normalized size = 1.00

$$\frac{2 \operatorname{arctanh} \left(\frac{2b^2x + \sqrt{-(4ab-1)b^2}}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)), x)

[Out] $2/b \cdot \operatorname{arctanh}\left(\frac{2b^2x + (-4ab - 1)b^2}{b}\right)$

maxima [A] time = 1.32, size = 51, normalized size = 1.65

$$\frac{\log\left(\frac{2b^2x - b + \sqrt{-4ab^3 + b^2}}{2b^2x + b + \sqrt{-4ab^3 + b^2}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x, algorithm="maxima")`

[Out] $-\log\left(\frac{2b^2x - b + \sqrt{-4ab^3 + b^2}}{2b^2x + b + \sqrt{-4ab^3 + b^2}}\right)/b$

mupad [B] time = 0.13, size = 38, normalized size = 1.23

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{b^2 - 4ab^3}}{\sqrt{b^2}} + \frac{2b^2x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(x*(b^2 - 4*a*b^3)^(1/2) - a*b + b^2*x^2),x)`

[Out] $\frac{2 \operatorname{atanh}\left(\frac{(b^2 - 4ab^3)^{1/2}}{(b^2)^{1/2}} + \frac{2b^2x}{(b^2)^{1/2}}\right)}{(b^2)^{1/2}}$

sympy [B] time = 0.28, size = 56, normalized size = 1.81

$$\frac{\log\left(x - \frac{1}{2b} + \frac{\sqrt{-4ab^3 + b^2}}{2b^2}\right) - \log\left(x + \frac{1}{2b} + \frac{\sqrt{-4ab^3 + b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)`

[Out] $-(\log(x - 1/(2b) + \sqrt{-4ab^3 + b^2}/(2b^2)) - \log(x + 1/(2b) + \sqrt{-4ab^3 + b^2}/(2b^2)))/b$

$$3.101 \quad \int \frac{1}{1+x^2+2x \cos\left(\frac{1}{7}\right)} dx$$

Optimal. Leaf size=17

$$\csc\left(\frac{1}{7}\right) \tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x + \cos\left(\frac{1}{7}\right)\right)\right)$$

[Out] arctan((x+cos(1/7))*csc(1/7))*csc(1/7)

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {618, 204}

$$\csc\left(\frac{1}{7}\right) \tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x + \cos\left(\frac{1}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + 2*x*Cos[1/7])^(-1), x]

[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^2+2x \cos\left(\frac{1}{7}\right)} dx &= -\left(2 \text{Subst}\left[\int \frac{1}{-x^2-4 \sin^2\left(\frac{1}{7}\right)} dx, x, 2x+2 \cos\left(\frac{1}{7}\right)\right]\right) \\ &= \tan^{-1}\left(\left(x + \cos\left(\frac{1}{7}\right)\right) \csc\left(\frac{1}{7}\right)\right) \csc\left(\frac{1}{7}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.12

$$\csc\left(\frac{1}{7}\right) \tan^{-1}\left(\frac{(x-1) \tan\left(\frac{1}{14}\right)}{x+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1), x]

[Out] ArcTan[((-1 + x)*Tan[1/14])/(1 + x)]*Csc[1/7]

fricas [A] time = 1.11, size = 15, normalized size = 0.88

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="fricas")

[Out] arctan((x + cos(1/7))/sin(1/7))/sin(1/7)

giac [B] time = 0.47, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="giac")

[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1)

maple [B] time = 0.24, size = 33, normalized size = 1.94

$$\frac{\arctan\left(\frac{2x+2\cos\left(\frac{1}{7}\right)}{2\sqrt{1-\left(\cos^2\left(\frac{1}{7}\right)\right)}}\right)}{\sqrt{1-\left(\cos^2\left(\frac{1}{7}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+x^2+2*x*cos(1/7)),x)`

[Out] $1/(1-\cos(1/7)^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2x+2\cos(1/7))/(1-\cos(1/7)^2)^{(1/2)})$

maxima [B] time = 2.95, size = 27, normalized size = 1.59

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x^2+2*x*cos(1/7)),x, algorithm="maxima")`

[Out] $\arctan((x + \cos(1/7))/\sqrt{-\cos(1/7)^2 + 1})/\sqrt{-\cos(1/7)^2 + 1}$

mupad [B] time = 0.12, size = 27, normalized size = 1.59

$$\frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*x*cos(1/7) + x^2 + 1),x)`

[Out] $\operatorname{atan}((x + \cos(1/7))/(1 - \cos(1/7)^2)^{(1/2)})/(1 - \cos(1/7)^2)^{(1/2)}$

sympy [C] time = 0.19, size = 165, normalized size = 9.71

$$\frac{i \log\left(x + \cos\left(\frac{1}{7}\right) - \frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}} + \frac{i \cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}} + \frac{i \log\left(x + \cos\left(\frac{1}{7}\right) - \frac{i \cos^2\left(\frac{1}{7}\right)}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}} + \frac{i}{\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{1-\cos\left(\frac{1}{7}\right)}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x**2+2*x*cos(1/7)),x)`

```
[Out] -I*log(x + cos(1/7) - I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(1/7)**2/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)) + I/(sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1)))/(2*sqrt(1 - cos(1/7))*sqrt(cos(1/7) + 1))
```

$$3.102 \quad \int \frac{1}{1+x^2+2x \cos\left(\frac{\pi}{7}\right)} dx$$

Optimal. Leaf size=23

$$\csc\left(\frac{\pi}{7}\right) \tan^{-1}\left(x \csc\left(\frac{\pi}{7}\right) + \cot\left(\frac{\pi}{7}\right)\right)$$

[Out] arctan(cot(1/7*Pi)+x*csc(1/7*Pi))*csc(1/7*Pi)

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {618, 204}

$$\csc\left(\frac{\pi}{7}\right) \tan^{-1}\left(\csc\left(\frac{\pi}{7}\right)\left(x + \cos\left(\frac{\pi}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1), x]

[Out] ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Csc[Pi/7]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^2+2x \cos\left(\frac{\pi}{7}\right)} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{-x^2-4 \sin^2\left(\frac{\pi}{7}\right)} dx, x, 2x+2 \cos\left(\frac{\pi}{7}\right)\right)\right) \\ &= \tan^{-1}\left(\left(x + \cos\left(\frac{\pi}{7}\right)\right) \csc\left(\frac{\pi}{7}\right)\right) \csc\left(\frac{\pi}{7}\right) \end{aligned}$$

Mathematica [B] time = 0.04, size = 56, normalized size = 2.43

$$\frac{2 \tan^{-1} \left(\frac{2x - (-1)^{6/7} + \sqrt[7]{-1}}{\sqrt{2 - (-1)^{2/7} + (-1)^{5/7}}} \right)}{\sqrt{2 - (-1)^{2/7} + (-1)^{5/7}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1), x]

[Out] (2*ArcTan[((-1)^(1/7) - (-1)^(6/7) + 2*x)/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)])/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]

fricas [A] time = 0.87, size = 21, normalized size = 0.91

$$\frac{\arctan \left(\frac{x + \cos\left(\frac{1}{7}\pi\right)}{\sin\left(\frac{1}{7}\pi\right)} \right)}{\sin\left(\frac{1}{7}\pi\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)), x, algorithm="fricas")

[Out] arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)

giac [A] time = 0.47, size = 33, normalized size = 1.43

$$\frac{\arctan \left(\frac{x + \cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2 + 1}} \right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x^2+2*x*cos(1/7*pi)), x, algorithm="giac")

[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*pi)^2 + 1)

maple [B] time = 0.20, size = 39, normalized size = 1.70

$$\frac{\arctan \left(\frac{2x + 2 \cos\left(\frac{\pi}{7}\right)}{2 \sqrt{1 - \left(\cos\left(\frac{\pi}{7}\right)\right)^2}} \right)}{\sqrt{1 - \left(\cos\left(\frac{\pi}{7}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+x^2+2*x*cos(1/7*Pi)),x)`

[Out] $1/((1-\cos(1/7\pi))^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2x+2\cos(1/7\pi))/(1-\cos(1/7\pi))^2)^{(1/2)}$

maxima [A] time = 3.02, size = 33, normalized size = 1.43

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x^2+2*x*cos(1/7*pi)),x, algorithm="maxima")`

[Out] $\arctan((x + \cos(1/7\pi))/\sqrt{-\cos(1/7\pi)^2 + 1})/\sqrt{-\cos(1/7\pi)^2 + 1}$

mupad [B] time = 0.30, size = 42, normalized size = 1.83

$$-\frac{\operatorname{atanh}\left(\frac{x+\cos\left(\frac{\pi}{7}\right)}{\sqrt{\cos\left(\frac{\pi}{7}\right)-1}\sqrt{\cos\left(\frac{\pi}{7}\right)+1}}\right)}{\sqrt{\cos\left(\frac{\pi}{7}\right)-1}\sqrt{\cos\left(\frac{\pi}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 + 2*x*cos(Pi/7) + 1),x)`

[Out] $-\operatorname{atanh}\left(\frac{x + \cos(\pi/7)}{((\cos(\pi/7) - 1)^{(1/2)} \cdot (\cos(\pi/7) + 1)^{(1/2)})} / ((\cos(\pi/7) - 1)^{(1/2)} \cdot (\cos(\pi/7) + 1)^{(1/2)})\right)$

sympy [C] time = 0.59, size = 70, normalized size = 3.04

$$-\frac{i \log\left(x + \cos\left(\frac{\pi}{7}\right) - \frac{i(2-2\cos^2\left(\frac{\pi}{7}\right))}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)} + \frac{i \log\left(x + \cos\left(\frac{\pi}{7}\right) + \frac{i(2-2\cos^2\left(\frac{\pi}{7}\right))}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)`

[Out] $-I \cdot \log(x + \cos(\pi/7) - I \cdot (2 - 2 \cdot \cos(\pi/7)^2) / (2 \cdot \sin(\pi/7))) / (2 \cdot \sin(\pi/7)) + I \cdot \log(x + \cos(\pi/7) + I \cdot (2 - 2 \cdot \cos(\pi/7)^2) / (2 \cdot \sin(\pi/7))) / (2 \cdot \sin(\pi/7))$

3.103 $\int \sqrt{5 - 6x + 9x^2} dx$

Optimal. Leaf size=38

$$\frac{2}{3} \sinh^{-1} \left(\frac{1}{2}(3x - 1) \right) - \frac{1}{6}(1 - 3x)\sqrt{9x^2 - 6x + 5}$$

[Out] 2/3*arcsinh(-1/2+3/2*x)-1/6*(1-3*x)*(9*x^2-6*x+5)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 215}

$$\frac{2}{3} \sinh^{-1} \left(\frac{1}{2}(3x - 1) \right) - \frac{1}{6}(1 - 3x)\sqrt{9x^2 - 6x + 5}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[5 - 6*x + 9*x^2], x]

[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/3

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{5-6x+9x^2} \, dx &= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + 2 \int \frac{1}{\sqrt{5-6x+9x^2}} \, dx \\
&= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + \frac{1}{18} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{144}}} \, dx, x, -6+18x \right) \\
&= -\frac{1}{6}(1-3x)\sqrt{5-6x+9x^2} + \frac{2}{3} \sinh^{-1} \left(\frac{1}{2}(-1+3x) \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.03

$$\sqrt{9x^2 - 6x + 5} \left(\frac{x}{2} - \frac{1}{6} \right) + \frac{2}{3} \sinh^{-1} \left(\frac{1}{2}(3x - 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[5 - 6*x + 9*x^2], x]

[Out] (-1/6 + x/2)*Sqrt[5 - 6*x + 9*x^2] + (2*ArcSinh[(-1 + 3*x)/2])/3

fricas [A] time = 0.91, size = 40, normalized size = 1.05

$$\frac{1}{6} \sqrt{9x^2 - 6x + 5} (3x - 1) - \frac{2}{3} \log(-3x + \sqrt{9x^2 - 6x + 5} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2-6*x+5)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)

giac [A] time = 0.47, size = 40, normalized size = 1.05

$$\frac{1}{6} \sqrt{9x^2 - 6x + 5} (3x - 1) - \frac{2}{3} \log(-3x + \sqrt{9x^2 - 6x + 5} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2-6*x+5)^(1/2), x, algorithm="giac")

[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)

maple [A] time = 0.06, size = 29, normalized size = 0.76

$$\frac{2 \operatorname{arcsinh}\left(\frac{3x}{2} - \frac{1}{2}\right)}{3} + \frac{(18x - 6) \sqrt{9x^2 - 6x + 5}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((9*x^2-6*x+5)^(1/2),x)`

[Out] `1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)`

maxima [A] time = 3.04, size = 38, normalized size = 1.00

$$\frac{1}{2} \sqrt{9x^2 - 6x + 5}x - \frac{1}{6} \sqrt{9x^2 - 6x + 5} + \frac{2}{3} \operatorname{arsinh}\left(\frac{3}{2}x - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9*x^2-6*x+5)^(1/2),x, algorithm="maxima")`

[Out] `1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arcsinh(3/2*x - 1/2)`

mupad [B] time = 0.08, size = 39, normalized size = 1.03

$$\frac{2 \ln\left(3x + \sqrt{9x^2 - 6x + 5} - 1\right)}{3} + \left(\frac{x}{2} - \frac{1}{6}\right) \sqrt{9x^2 - 6x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((9*x^2 - 6*x + 5)^(1/2),x)`

[Out] `(2*log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1))/3 + (x/2 - 1/6)*(9*x^2 - 6*x + 5)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{9x^2 - 6x + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9*x**2-6*x+5)**(1/2),x)`

[Out] `Integral(sqrt(9*x**2 - 6*x + 5), x)`

3.104 $\int \sqrt{3 - 4x - 4x^2} dx$

Optimal. Leaf size=30

$$\frac{1}{4}\sqrt{-4x^2 - 4x + 3}(2x + 1) + \sin^{-1}\left(x + \frac{1}{2}\right)$$

[Out] arcsin(1/2+x)+1/4*(1+2*x)*(-4*x^2-4*x+3)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 216}

$$\frac{1}{4}\sqrt{-4x^2 - 4x + 3}(2x + 1) + \sin^{-1}\left(x + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3 - 4*x - 4*x^2], x]

[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{3-4x-4x^2} \, dx &= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + 2 \int \frac{1}{\sqrt{3-4x-4x^2}} \, dx \\
&= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} - \frac{1}{8} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} \, dx, x, -4-8x \right) \\
&= \frac{1}{4}(1+2x)\sqrt{3-4x-4x^2} + \sin^{-1} \left(\frac{1}{2} + x \right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1) + \sin^{-1}\left(x + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3 - 4*x - 4*x^2], x]

[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]

fricas [B] time = 0.78, size = 53, normalized size = 1.77

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1) - \arctan\left(\frac{\sqrt{-4x^2-4x+3}(2x+1)}{4x^2+4x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-4*x+3)^(1/2), x, algorithm="fricas")

[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) - arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))

giac [A] time = 1.60, size = 24, normalized size = 0.80

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1) + \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-4*x+3)^(1/2), x, algorithm="giac")

[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\arcsin\left(x + \frac{1}{2}\right) - \frac{(-8x - 4)\sqrt{-4x^2 - 4x + 3}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2-4*x+3)^(1/2),x)

[Out] -1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(x+1/2)

maxima [A] time = 3.00, size = 38, normalized size = 1.27

$$\frac{1}{2}\sqrt{-4x^2 - 4x + 3}x + \frac{1}{4}\sqrt{-4x^2 - 4x + 3} - \arcsin\left(-x - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsin(-x - 1/2)

mupad [B] time = 0.05, size = 23, normalized size = 0.77

$$\operatorname{asin}\left(x + \frac{1}{2}\right) + \left(\frac{x}{2} + \frac{1}{4}\right)\sqrt{-4x^2 - 4x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3 - 4*x^2 - 4*x)^(1/2),x)

[Out] asin(x + 1/2) + (x/2 + 1/4)*(3 - 4*x^2 - 4*x)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-4x^2 - 4x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x**2-4*x+3)**(1/2),x)

[Out] Integral(sqrt(-4*x**2 - 4*x + 3), x)

3.105 $\int \sqrt{-8 + 6x + 9x^2} dx$

Optimal. Leaf size=49

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8} - \frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

[Out] $-3/2*\operatorname{arctanh}((1+3*x)/(9*x^2+6*x-8)^{(1/2)})+1/6*(1+3*x)*(9*x^2+6*x-8)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8} - \frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-8 + 6*x + 9*x^2], x]

[Out] $((1 + 3*x)*\operatorname{Sqrt}[-8 + 6*x + 9*x^2])/6 - (3*\operatorname{ArcTanh}[(1 + 3*x)/\operatorname{Sqrt}[-8 + 6*x + 9*x^2]])/2$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{-8 + 6x + 9x^2} \, dx &= \frac{1}{6}(1 + 3x)\sqrt{-8 + 6x + 9x^2} - \frac{9}{2} \int \frac{1}{\sqrt{-8 + 6x + 9x^2}} \, dx \\
&= \frac{1}{6}(1 + 3x)\sqrt{-8 + 6x + 9x^2} - 9 \operatorname{Subst} \left(\int \frac{1}{36 - x^2} \, dx, x, \frac{6 + 18x}{\sqrt{-8 + 6x + 9x^2}} \right) \\
&= \frac{1}{6}(1 + 3x)\sqrt{-8 + 6x + 9x^2} - \frac{3}{2} \tanh^{-1} \left(\frac{1 + 3x}{\sqrt{-8 + 6x + 9x^2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\left(\frac{x}{2} + \frac{1}{6}\right)\sqrt{9x^2 + 6x - 8} - \frac{3}{2} \log\left(\sqrt{9x^2 + 6x - 8} + 3x + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-8 + 6*x + 9*x^2], x]

[Out] (1/6 + x/2)*Sqrt[-8 + 6*x + 9*x^2] - (3*Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]])/2

fricas [A] time = 0.90, size = 40, normalized size = 0.82

$$\frac{1}{6} \sqrt{9x^2 + 6x - 8} (3x + 1) + \frac{3}{2} \log\left(-3x + \sqrt{9x^2 + 6x - 8} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+6*x-8)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)

giac [A] time = 0.43, size = 41, normalized size = 0.84

$$\frac{1}{6} \sqrt{9x^2 + 6x - 8} (3x + 1) + \frac{3}{2} \log\left(\left|-3x + \sqrt{9x^2 + 6x - 8} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+6*x-8)^(1/2), x, algorithm="giac")

[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))

maple [A] time = 0.05, size = 50, normalized size = 1.02

$$-\frac{\sqrt{9} \ln\left(\frac{(9x+3)\sqrt{9}}{9} + \sqrt{9x^2 + 6x - 8}\right)}{2} + \frac{(18x + 6)\sqrt{9x^2 + 6x - 8}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((9*x^2+6*x-8)^(1/2), x)

[Out] 1/36*(18*x+6)*(9*x^2+6*x-8)^(1/2)-1/2*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)

maxima [A] time = 2.87, size = 52, normalized size = 1.06

$$\frac{1}{2} \sqrt{9x^2 + 6x - 8} x + \frac{1}{6} \sqrt{9x^2 + 6x - 8} - \frac{3}{2} \log\left(18x + 6\sqrt{9x^2 + 6x - 8} + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x^2+6*x-8)^(1/2), x, algorithm="maxima")

[Out] 1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)

mupad [B] time = 0.21, size = 39, normalized size = 0.80

$$\left(\frac{x}{2} + \frac{1}{6}\right) \sqrt{9x^2 + 6x - 8} - \frac{3 \ln\left(3x + \sqrt{9x^2 + 6x - 8} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x + 9*x^2 - 8)^(1/2), x)

[Out] (x/2 + 1/6)*(6*x + 9*x^2 - 8)^(1/2) - (3*log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{9x^2 + 6x - 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9*x**2+6*x-8)**(1/2), x)

[Out] Integral(sqrt(9*x**2 + 6*x - 8), x)

3.106 $\int \sqrt{2 + 4x + 3x^2} dx$

Optimal. Leaf size=45

$$\frac{1}{6}\sqrt{3x^2 + 4x + 2}(3x + 2) + \frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

[Out] 1/9*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)+1/6*(2+3*x)*(3*x^2+4*x+2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 215}

$$\frac{1}{6}\sqrt{3x^2 + 4x + 2}(3x + 2) + \frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 4*x + 3*x^2], x]

[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c)], x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{2+4x+3x^2} dx &= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{1}{3} \int \frac{1}{\sqrt{2+4x+3x^2}} dx \\
&= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}} dx, x, 4+6x\right)}{6\sqrt{6}} \\
&= \frac{1}{6}(2+3x)\sqrt{2+4x+3x^2} + \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\sqrt{3x^2+4x+2} \left(\frac{x}{2} + \frac{1}{3}\right) + \frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 4*x + 3*x^2], x]

[Out] (1/3 + x/2)*Sqrt[2 + 4*x + 3*x^2] + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3*Sqrt[3])

fricas [A] time = 0.91, size = 58, normalized size = 1.29

$$\frac{1}{6} \sqrt{3x^2+4x+2} (3x+2) + \frac{1}{18} \sqrt{3} \log\left(-\sqrt{3} \sqrt{3x^2+4x+2} (3x+2) - 9x^2 - 12x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + 1/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)

giac [A] time = 0.46, size = 53, normalized size = 1.18

$$\frac{1}{6} \sqrt{3x^2+4x+2} (3x+2) - \frac{1}{9} \sqrt{3} \log\left(-\sqrt{3} \left(\sqrt{3}x - \sqrt{3x^2+4x+2}\right) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x+2)^(1/2), x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{3x^2 + 4x + 2}(3x + 2) - \frac{1}{9}\sqrt{3}\log(-\sqrt{3}(\sqrt{3}x - \sqrt{3x^2 + 4x + 2}) - 2)$

maple [A] time = 0.05, size = 35, normalized size = 0.78

$$\frac{\sqrt{3} \operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{9} + \frac{(6x+4)\sqrt{3x^2+4x+2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+4*x+2)^(1/2),x)`

[Out] $\frac{1}{12}(6x+4)(3x^2+4x+2)^{1/2} + \frac{1}{9}3^{1/2}\operatorname{arcsinh}\left(\frac{3}{2}\sqrt{2}\left(x+\frac{2}{3}\right)\right)$

maxima [A] time = 3.00, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{3x^2+4x+2}x + \frac{1}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+4*x+2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2}\sqrt{3x^2+4x+2}x + \frac{1}{9}\sqrt{3}\operatorname{arcsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right) + \frac{1}{3}\sqrt{3x^2+4x+2}$

mupad [B] time = 0.19, size = 48, normalized size = 1.07

$$\frac{\sqrt{3} \ln\left(\sqrt{3x^2+4x+2} + \frac{\sqrt{3}(3x+2)}{3}\right)}{9} + \left(\frac{x}{2} + \frac{1}{3}\right)\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 3*x^2 + 2)^(1/2),x)`

[Out] $\frac{3^{1/2}\log((4x+3x^2+2)^{1/2} + (3^{1/2}(3x+2))/3)}{9} + (x/2 + 1/3)(4x+3x^2+2)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2+4x+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+4*x+2)**(1/2),x)`

[Out] `Integral(sqrt(3*x**2 + 4*x + 2), x)`

3.107 $\int \sqrt{2 + 4x - 3x^2} dx$

Optimal. Leaf size=45

$$-\frac{1}{6}\sqrt{-3x^2 + 4x + 2}(2 - 3x) - \frac{5 \sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

[Out] $-5/9*\arcsin(1/10*(2-3*x)*10^{(1/2)})*3^{(1/2)}-1/6*(2-3*x)*(-3*x^2+4*x+2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 216}

$$-\frac{1}{6}\sqrt{-3x^2 + 4x + 2}(2 - 3x) - \frac{5 \sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 4*x - 3*x^2], x]

[Out] $-((2 - 3*x)*\text{Sqrt}[2 + 4*x - 3*x^2])/6 - (5*\text{ArcSin}[(2 - 3*x)/\text{Sqrt}[10]])/(3*\text{Sqrt}[3])$

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{2+4x-3x^2} dx &= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} + \frac{5}{3} \int \frac{1}{\sqrt{2+4x-3x^2}} dx \\
&= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{1}{6}\sqrt{\frac{5}{6}} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}} dx, x, 4-6x\right) \\
&= -\frac{1}{6}(2-3x)\sqrt{2+4x-3x^2} - \frac{5 \sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.02

$$\left(\frac{x}{2} - \frac{1}{3}\right)\sqrt{-3x^2 + 4x + 2} - \frac{5 \sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 4*x - 3*x^2], x]

[Out] (-1/3 + x/2)*Sqrt[2 + 4*x - 3*x^2] - (5*ArcSin[(2 - 3*x)/Sqrt[10]])/(3*Sqrt[3])

fricas [A] time = 0.76, size = 60, normalized size = 1.33

$$\frac{1}{6}\sqrt{-3x^2 + 4x + 2}(3x - 2) - \frac{5}{9}\sqrt{3} \arctan\left(\frac{\sqrt{3}\sqrt{-3x^2 + 4x + 2}(3x - 2)}{3(3x^2 - 4x - 2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))

giac [A] time = 0.47, size = 36, normalized size = 0.80

$$\frac{1}{6}\sqrt{-3x^2 + 4x + 2}(3x - 2) + \frac{5}{9}\sqrt{3} \arcsin\left(\frac{1}{10}\sqrt{10}(3x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x+2)^(1/2), x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{-3x^2 + 4x + 2}(3x - 2) + \frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x - 2)\right)$

maple [A] time = 0.05, size = 35, normalized size = 0.78

$$\frac{5\sqrt{3} \arcsin\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{9} - \frac{(-6x + 4)\sqrt{-3x^2 + 4x + 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((-3x^2+4x+2)^{(1/2)}, x)$

[Out] $-1/12*(-6*x+4)*(-3*x^2+4*x+2)^{(1/2)}+5/9*3^{(1/2)}*\arcsin(3/10*10^{(1/2)}*(x-2/3))$

maxima [A] time = 2.97, size = 46, normalized size = 1.02

$$\frac{1}{2}\sqrt{-3x^2 + 4x + 2}x - \frac{5}{9}\sqrt{3} \arcsin\left(-\frac{1}{10}\sqrt{10}(3x - 2)\right) - \frac{1}{3}\sqrt{-3x^2 + 4x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-3x^2+4x+2)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $\frac{1}{2}\sqrt{-3x^2 + 4x + 2}x - \frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x - 2)\right) - \frac{1}{3}\sqrt{-3x^2 + 4x + 2}$

mupad [B] time = 0.05, size = 35, normalized size = 0.78

$$\frac{5\sqrt{3} \operatorname{asin}\left(\frac{\sqrt{10}(3x-2)}{10}\right)}{9} + \left(\frac{x}{2} - \frac{1}{3}\right)\sqrt{-3x^2 + 4x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((4x - 3x^2 + 2)^{(1/2)}, x)$

[Out] $(5*3^{(1/2)}*\operatorname{asin}((10^{(1/2)}*(3*x - 2))/10))/9 + (x/2 - 1/3)*(4*x - 3*x^2 + 2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 4x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-3x**2+4*x+2)**(1/2), x)$

[Out] $\text{Integral}(\sqrt{-3x**2 + 4*x + 2}, x)$

3.108 $\int \sqrt{2 + 5x + 3x^2} dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x + 5)\sqrt{3x^2 + 5x + 2} - \frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

[Out] $-1/72*\operatorname{arctanh}(1/6*(5+6*x)*3^{(1/2)}/(3*x^2+5*x+2)^{(1/2)})*3^{(1/2)}+1/12*(5+6*x)*(3*x^2+5*x+2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x + 5)\sqrt{3x^2 + 5x + 2} - \frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 5*x + 3*x^2], x]

[Out] $((5 + 6*x)*\operatorname{Sqrt}[2 + 5*x + 3*x^2])/12 - \operatorname{ArcTanh}[(5 + 6*x)/(2*\operatorname{Sqrt}[3]*\operatorname{Sqrt}[2 + 5*x + 3*x^2])]/(24*\operatorname{Sqrt}[3])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{2+5x+3x^2} dx &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{24} \int \frac{1}{\sqrt{2+5x+3x^2}} dx \\ &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{1}{12} \text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}} \right) \\ &= \frac{1}{12}(5+6x)\sqrt{2+5x+3x^2} - \frac{\tanh^{-1} \left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}} \right)}{24\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.89

$$\frac{1}{72} \left(6(6x+5)\sqrt{3x^2+5x+2} - \sqrt{3} \log \left(2\sqrt{9x^2+15x+6} + 6x+5 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 5*x + 3*x^2], x]

[Out] (6*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*Log[5 + 6*x + 2*Sqrt[6 + 15*x + 9*x^2]])/72

fricas [A] time = 0.95, size = 58, normalized size = 0.94

$$\frac{1}{12} \sqrt{3x^2+5x+2}(6x+5) + \frac{1}{144} \sqrt{3} \log \left(-4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5) + 72x^2 + 120x + 49 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+5*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)

giac [A] time = 0.44, size = 54, normalized size = 0.87

$$\frac{1}{12} \sqrt{3x^2+5x+2}(6x+5) + \frac{1}{72} \sqrt{3} \log \left(\left| -2\sqrt{3} \left(\sqrt{3}x - \sqrt{3x^2+5x+2} \right) - 5 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+5*x+2)^(1/2), x, algorithm="giac")

[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))

maple [A] time = 0.05, size = 50, normalized size = 0.81

$$-\frac{\sqrt{3} \ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3} + \sqrt{3x^2 + 5x + 2}\right)}{72} + \frac{(6x + 5)\sqrt{3x^2 + 5x + 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+5*x+2)^(1/2),x)

[Out] 1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)

maxima [A] time = 2.98, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2 + 5x + 2}x - \frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2 + 5x + 2} + 6x + 5\right) + \frac{5}{12}\sqrt{3x^2 + 5x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+5*x+2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)

mupad [B] time = 0.20, size = 48, normalized size = 0.77

$$\left(\frac{x}{2} + \frac{5}{12}\right)\sqrt{3x^2 + 5x + 2} - \frac{\sqrt{3} \ln\left(\sqrt{3x^2 + 5x + 2} + \frac{\sqrt{3}\left(3x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x + 3*x^2 + 2)^(1/2),x)

[Out] (x/2 + 5/12)*(5*x + 3*x^2 + 2)^(1/2) - (3^(1/2)*log((5*x + 3*x^2 + 2)^(1/2) + (3^(1/2)*(3*x + 5/2))/3))/72

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 5x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+5*x+2)**(1/2),x)

[Out] Integral(sqrt(3*x**2 + 5*x + 2), x)

3.109 $\int \sqrt{2 + 5x - 3x^2} dx$

Optimal. Leaf size=43

$$-\frac{1}{12}\sqrt{-3x^2 + 5x + 2}(5 - 6x) - \frac{49 \sin^{-1}\left(\frac{1}{7}(5 - 6x)\right)}{24\sqrt{3}}$$

[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2 + 5x + 2}(5 - 6x) - \frac{49 \sin^{-1}\left(\frac{1}{7}(5 - 6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 5*x - 3*x^2], x]

[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{2+5x-3x^2} \, dx &= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} + \frac{49}{24} \int \frac{1}{\sqrt{2+5x-3x^2}} \, dx \\
&= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{7 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}} \, dx, x, 5-6x \right)}{24\sqrt{3}} \\
&= -\frac{1}{12}(5-6x)\sqrt{2+5x-3x^2} - \frac{49 \sin^{-1} \left(\frac{1}{7}(5-6x) \right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.02

$$\left(\frac{x}{2} - \frac{5}{12} \right) \sqrt{-3x^2 + 5x + 2} - \frac{49 \sin^{-1} \left(\frac{1}{7}(5-6x) \right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 5*x - 3*x^2], x]

[Out] (-5/12 + x/2)*Sqrt[2 + 5*x - 3*x^2] - (49*ArcSin[(5 - 6*x)/7])/(24*Sqrt[3])

fricas [A] time = 0.88, size = 60, normalized size = 1.40

$$\frac{1}{12} \sqrt{-3x^2 + 5x + 2} (6x - 5) - \frac{49}{72} \sqrt{3} \arctan \left(\frac{\sqrt{3} \sqrt{-3x^2 + 5x + 2} (6x - 5)}{6(3x^2 - 5x - 2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+5*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) - 49/72*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))

giac [A] time = 0.40, size = 31, normalized size = 0.72

$$\frac{1}{12} \sqrt{-3x^2 + 5x + 2} (6x - 5) + \frac{49}{72} \sqrt{3} \arcsin \left(\frac{6}{7}x - \frac{5}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+5*x+2)^(1/2), x, algorithm="giac")

[Out] $1/12*\sqrt{-3*x^2 + 5*x + 2}*(6*x - 5) + 49/72*\sqrt{3}*\arcsin(6/7*x - 5/7)$

maple [A] time = 0.04, size = 32, normalized size = 0.74

$$\frac{49\sqrt{3} \arcsin\left(\frac{6x}{7} - \frac{5}{7}\right)}{72} - \frac{(-6x + 5)\sqrt{-3x^2 + 5x + 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-3*x^2+5*x+2)^(1/2),x)`

[Out] $49/72*\arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2)$

maxima [A] time = 3.08, size = 41, normalized size = 0.95

$$\frac{1}{2}\sqrt{-3x^2 + 5x + 2}x - \frac{49}{72}\sqrt{3} \arcsin\left(-\frac{6}{7}x + \frac{5}{7}\right) - \frac{5}{12}\sqrt{-3x^2 + 5x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")`

[Out] $1/2*\sqrt{-3*x^2 + 5*x + 2}*x - 49/72*\sqrt{3}*\arcsin(-6/7*x + 5/7) - 5/12*\sqrt{-3*x^2 + 5*x + 2}$

mupad [B] time = 0.15, size = 30, normalized size = 0.70

$$\frac{49\sqrt{3} \operatorname{asin}\left(\frac{6x}{7} - \frac{5}{7}\right)}{72} + \left(\frac{x}{2} - \frac{5}{12}\right)\sqrt{-3x^2 + 5x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x - 3*x^2 + 2)^(1/2),x)`

[Out] $(49*3^(1/2)*\operatorname{asin}((6*x)/7 - 5/7))/72 + (x/2 - 5/12)*(5*x - 3*x^2 + 2)^(1/2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 5x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x**2+5*x+2)**(1/2),x)`

[Out] `Integral(sqrt(-3*x**2 + 5*x + 2), x)`

3.110 $\int \sqrt{-2 + 4x + 3x^2} dx$

Optimal. Leaf size=59

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2} - \frac{5 \tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

[Out] $-5/9*\operatorname{arctanh}(1/3*(2+3*x)*3^{(1/2)}/(3*x^2+4*x-2)^{(1/2)})*3^{(1/2)}+1/6*(2+3*x)*(3*x^2+4*x-2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 621, 206}

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2} - \frac{5 \tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2 + 4*x + 3*x^2], x]

[Out] $((2 + 3*x)*\operatorname{Sqrt}[-2 + 4*x + 3*x^2])/6 - (5*\operatorname{ArcTanh}[(2 + 3*x)/(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[-2 + 4*x + 3*x^2])])/(3*\operatorname{Sqrt}[3])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+4x+3x^2} dx &= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5}{3} \int \frac{1}{\sqrt{-2+4x+3x^2}} dx \\
&= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{10}{3} \operatorname{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}}\right) \\
&= \frac{1}{6}(2+3x)\sqrt{-2+4x+3x^2} - \frac{5 \tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.90

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2} - \frac{5 \log\left(\sqrt{9x^2+12x-6} + 3x+2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-2 + 4*x + 3*x^2], x]

[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*Log[2 + 3*x + Sqrt[-6 + 12*x + 9*x^2]])/(3*Sqrt[3])

fricas [A] time = 0.97, size = 58, normalized size = 0.98

$$\frac{1}{6} \sqrt{3x^2+4x-2}(3x+2) + \frac{5}{18} \sqrt{3} \log\left(-\sqrt{3} \sqrt{3x^2+4x-2}(3x+2) + 9x^2+12x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/18*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)

giac [A] time = 0.49, size = 54, normalized size = 0.92

$$\frac{1}{6} \sqrt{3x^2+4x-2}(3x+2) + \frac{5}{9} \sqrt{3} \log\left(\left|-\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2+4x-2}\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x-2)^(1/2), x, algorithm="giac")

[Out] $\frac{1}{6}\sqrt{3x^2 + 4x - 2}(3x + 2) + \frac{5}{9}\sqrt{3}\log(\text{abs}(-\sqrt{3})(\sqrt{3} * x - \sqrt{3x^2 + 4x - 2}) - 2)$

maple [A] time = 0.05, size = 50, normalized size = 0.85

$$-\frac{5\sqrt{3} \ln\left(\frac{(3x+2)\sqrt{3}}{3} + \sqrt{3x^2 + 4x - 2}\right)}{9} + \frac{(6x + 4)\sqrt{3x^2 + 4x - 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+4*x-2)^(1/2),x)`

[Out] $\frac{1}{12}(6x+4)(3x^2+4x-2)^{1/2} - \frac{5}{9}\ln\left(\frac{1}{3}(3x+2)3^{1/2} + (3x^2+4x-2)^{1/2}\right)3^{1/2}$

maxima [A] time = 2.98, size = 58, normalized size = 0.98

$$\frac{1}{2}\sqrt{3x^2 + 4x - 2}x - \frac{5}{9}\sqrt{3} \log\left(2\sqrt{3}\sqrt{3x^2 + 4x - 2} + 6x + 4\right) + \frac{1}{3}\sqrt{3x^2 + 4x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+4*x-2)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{2}\sqrt{3x^2 + 4x - 2}x - \frac{5}{9}\sqrt{3}\log(2\sqrt{3}\sqrt{3x^2 + 4x - 2} + 6x + 4) + \frac{1}{3}\sqrt{3x^2 + 4x - 2}$

mupad [B] time = 0.19, size = 48, normalized size = 0.81

$$\left(\frac{x}{2} + \frac{1}{3}\right)\sqrt{3x^2 + 4x - 2} - \frac{5\sqrt{3} \ln\left(\sqrt{3x^2 + 4x - 2} + \frac{\sqrt{3}(3x+2)}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 3*x^2 - 2)^(1/2),x)`

[Out] $\left(\frac{x}{2} + \frac{1}{3}\right)(4x + 3x^2 - 2)^{1/2} - \frac{(5\sqrt{3}\log((4x + 3x^2 - 2)^{1/2}) + (3^{1/2}(3x + 2))/3)}{9}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 4x - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+4*x-2)**(1/2),x)`

[Out] `Integral(sqrt(3*x**2 + 4*x - 2), x)`

3.111 $\int \sqrt{-2 + 4x - 3x^2} dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}} - \frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

[Out] 1/9*arctan(1/3*(2-3*x)*3^(1/2)/(-3*x^2+4*x-2)^(1/2))*3^(1/2)-1/6*(2-3*x)*(-3*x^2+4*x-2)^(1/2)

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 621, 204}

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}} - \frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2 + 4*x - 3*x^2], x]

[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/(3*Sqrt[3])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+4x-3x^2} dx &= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{1}{3} \int \frac{1}{\sqrt{-2+4x-3x^2}} dx \\
&= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-12-x^2} dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}} \right) \\
&= -\frac{1}{6}(2-3x)\sqrt{-2+4x-3x^2} + \frac{\tan^{-1} \left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}} \right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{-3x^2+4x-2}(3x-2) + \frac{\tan^{-1} \left(\frac{2-3x}{\sqrt{-9x^2+12x-6}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-2 + 4*x - 3*x^2], x]

[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/(3*Sqrt[3])

fricas [C] time = 0.85, size = 84, normalized size = 1.42

$$\frac{1}{6}\sqrt{-3x^2+4x-2}(3x-2) + \frac{1}{18}i\sqrt{3} \log \left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2} - 6x + 4}{x} \right) - \frac{1}{18}i\sqrt{3} \log \left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) + 1/18*I*sqrt(3)*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) - 1/18*I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2+4x-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-3*x^2 + 4*x - 2), x)

maple [A] time = 0.05, size = 46, normalized size = 0.78

$$-\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{9} - \frac{(-6x+4)\sqrt{-3x^2+4x-2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2+4*x-2)^(1/2),x)

[Out] -1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))

maxima [C] time = 2.89, size = 46, normalized size = 0.78

$$\frac{1}{2}\sqrt{-3x^2+4x-2}x + \frac{1}{9}i\sqrt{3} \operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right) - \frac{1}{3}\sqrt{-3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x - 2)

mupad [B] time = 0.05, size = 36, normalized size = 0.61

$$\frac{\sqrt{3} \operatorname{asin}\left(\frac{\sqrt{2}(3x-2)i}{2}\right)}{9} + \left(\frac{x}{2} - \frac{1}{3}\right)\sqrt{-3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - 3*x^2 - 2)^(1/2),x)

[Out] (3^(1/2)*asin((2^(1/2)*(3*x - 2)*i)/2))/9 + (x/2 - 1/3)*(4*x - 3*x^2 - 2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 4x - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x - 2), x)
```

3.112 $\int \sqrt{-2 + 5x + 3x^2} dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x + 5)\sqrt{3x^2 + 5x - 2} - \frac{49 \tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

[Out] $-49/72*\operatorname{arctanh}(1/6*(5+6*x)*3^{(1/2)}/(3*x^2+5*x-2)^{(1/2)})*3^{(1/2)}+1/12*(5+6*x)*(3*x^2+5*x-2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 621, 206}

$$\frac{1}{12}(6x + 5)\sqrt{3x^2 + 5x - 2} - \frac{49 \tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2 + 5*x + 3*x^2], x]

[Out] $((5 + 6*x)*\operatorname{Sqrt}[-2 + 5*x + 3*x^2])/12 - (49*\operatorname{ArcTanh}[(5 + 6*x)/(2*\operatorname{Sqrt}[3]*\operatorname{Sqrt}[-2 + 5*x + 3*x^2])])/(24*\operatorname{Sqrt}[3])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{-2+5x+3x^2} \, dx &= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49}{24} \int \frac{1}{\sqrt{-2+5x+3x^2}} \, dx \\
&= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49}{12} \operatorname{Subst}\left(\int \frac{1}{12-x^2} \, dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}}\right) \\
&= \frac{1}{12}(5+6x)\sqrt{-2+5x+3x^2} - \frac{49 \tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.89

$$\frac{1}{72} \left(6(6x+5)\sqrt{3x^2+5x-2} - 49\sqrt{3} \log\left(2\sqrt{9x^2+15x-6} + 6x+5\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-2 + 5*x + 3*x^2], x]

[Out] (6*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*Log[5 + 6*x + 2*Sqrt[-6 + 15*x + 9*x^2]])/72

fricas [A] time = 0.97, size = 58, normalized size = 0.94

$$\frac{1}{12} \sqrt{3x^2+5x-2}(6x+5) + \frac{49}{144} \sqrt{3} \log\left(-4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5) + 72x^2 + 120x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+5*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/144*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)

giac [A] time = 0.44, size = 54, normalized size = 0.87

$$\frac{1}{12} \sqrt{3x^2+5x-2}(6x+5) + \frac{49}{72} \sqrt{3} \log\left(\left|-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2+5x-2}\right) - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+5*x-2)^(1/2), x, algorithm="giac")

[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))

maple [A] time = 0.05, size = 50, normalized size = 0.81

$$-\frac{49\sqrt{3} \ln\left(\frac{\left(3x+\frac{5}{2}\right)\sqrt{3}}{3} + \sqrt{3x^2 + 5x - 2}\right)}{72} + \frac{(6x + 5)\sqrt{3x^2 + 5x - 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+5*x-2)^(1/2), x)`

[Out] `1/12*(6*x+5)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)`

maxima [A] time = 3.00, size = 58, normalized size = 0.94

$$\frac{1}{2}\sqrt{3x^2 + 5x - 2}x - \frac{49}{72}\sqrt{3} \log\left(2\sqrt{3}\sqrt{3x^2 + 5x - 2} + 6x + 5\right) + \frac{5}{12}\sqrt{3x^2 + 5x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+5*x-2)^(1/2), x, algorithm="maxima")`

[Out] `1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)`

mupad [B] time = 0.22, size = 48, normalized size = 0.77

$$\left(\frac{x}{2} + \frac{5}{12}\right)\sqrt{3x^2 + 5x - 2} - \frac{49\sqrt{3} \ln\left(\sqrt{3x^2 + 5x - 2} + \frac{\sqrt{3}\left(3x+\frac{5}{2}\right)}{3}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + 3*x^2 - 2)^(1/2), x)`

[Out] `(x/2 + 5/12)*(5*x + 3*x^2 - 2)^(1/2) - (49*3^(1/2)*log((5*x + 3*x^2 - 2)^(1/2) + (3^(1/2)*(3*x + 5/2))/3))/72`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x^2 + 5x - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+5*x-2)**(1/2), x)`

[Out] `Integral(sqrt(3*x**2 + 5*x - 2), x)`

3.113 $\int \sqrt{-2 + 5x - 3x^2} dx$

Optimal. Leaf size=39

$$-\frac{1}{12}\sqrt{-3x^2 + 5x - 2}(5 - 6x) - \frac{\sin^{-1}(5 - 6x)}{24\sqrt{3}}$$

[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {612, 619, 216}

$$-\frac{1}{12}\sqrt{-3x^2 + 5x - 2}(5 - 6x) - \frac{\sin^{-1}(5 - 6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2 + 5*x - 3*x^2], x]

[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt[3])

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{-2 + 5x - 3x^2} \, dx &= -\frac{1}{12}(5 - 6x)\sqrt{-2 + 5x - 3x^2} + \frac{1}{24} \int \frac{1}{\sqrt{-2 + 5x - 3x^2}} \, dx \\
&= -\frac{1}{12}(5 - 6x)\sqrt{-2 + 5x - 3x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 5 - 6x\right)}{24\sqrt{3}} \\
&= -\frac{1}{12}(5 - 6x)\sqrt{-2 + 5x - 3x^2} - \frac{\sin^{-1}(5 - 6x)}{24\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.03

$$\left(\frac{x}{2} - \frac{5}{12}\right)\sqrt{-3x^2 + 5x - 2} - \frac{\sin^{-1}(5 - 6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-2 + 5*x - 3*x^2], x]

[Out] (-5/12 + x/2)*Sqrt[-2 + 5*x - 3*x^2] - ArcSin[5 - 6*x]/(24*Sqrt[3])

fricas [A] time = 0.92, size = 60, normalized size = 1.54

$$\frac{1}{12} \sqrt{-3x^2 + 5x - 2}(6x - 5) - \frac{1}{72} \sqrt{3} \arctan\left(\frac{\sqrt{3} \sqrt{-3x^2 + 5x - 2}(6x - 5)}{6(3x^2 - 5x + 2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+5*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) - 1/72*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))

giac [A] time = 0.53, size = 31, normalized size = 0.79

$$\frac{1}{12} \sqrt{-3x^2 + 5x - 2}(6x - 5) + \frac{1}{72} \sqrt{3} \arcsin(6x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+5*x-2)^(1/2), x, algorithm="giac")

[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x - 5)

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{\sqrt{3} \arcsin(6x - 5)}{72} - \frac{(-6x + 5) \sqrt{-3x^2 + 5x - 2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2+5*x-2)^(1/2),x)

[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(-6*x+5)*(-3*x^2+5*x-2)^(1/2)

maxima [A] time = 3.08, size = 41, normalized size = 1.05

$$\frac{1}{2} \sqrt{-3x^2 + 5x - 2} x + \frac{1}{72} \sqrt{3} \arcsin(6x - 5) - \frac{5}{12} \sqrt{-3x^2 + 5x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/12*sqrt(-3*x^2 + 5*x - 2)

mupad [B] time = 0.05, size = 30, normalized size = 0.77

$$\frac{\sqrt{3} \operatorname{asin}(6x - 5)}{72} + \left(\frac{x}{2} - \frac{5}{12} \right) \sqrt{-3x^2 + 5x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x - 3*x^2 - 2)^(1/2),x)

[Out] (3^(1/2)*asin(6*x - 5))/72 + (x/2 - 5/12)*(5*x - 3*x^2 - 2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-3x^2 + 5x - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**2+5*x-2)**(1/2),x)

[Out] Integral(sqrt(-3*x**2 + 5*x - 2), x)

$$3.114 \quad \int \frac{1}{\sqrt{5-6x+9x^2}} dx$$

Optimal. Leaf size=14

$$\frac{1}{3} \sinh^{-1} \left(\frac{1}{2}(3x-1) \right)$$

[Out] 1/3*arcsinh(-1/2+3/2*x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 215}

$$\frac{1}{3} \sinh^{-1} \left(\frac{1}{2}(3x-1) \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[5 - 6*x + 9*x^2], x]

[Out] ArcSinh[(-1 + 3*x)/2]/3

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{5-6x+9x^2}} dx &= \frac{1}{36} \text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{144}}} dx, x, -6 + 18x \right) \\ &= \frac{1}{3} \sinh^{-1} \left(\frac{1}{2}(-1 + 3x) \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3} \sinh^{-1} \left(\frac{1}{2} (3x - 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[5 - 6*x + 9*x^2], x]

[Out] ArcSinh[(-1 + 3*x)/2]/3

fricas [B] time = 0.90, size = 20, normalized size = 1.43

$$-\frac{1}{3} \log \left(-3x + \sqrt{9x^2 - 6x + 5} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-6*x+5)^(1/2), x, algorithm="fricas")

[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)

giac [B] time = 0.63, size = 20, normalized size = 1.43

$$-\frac{1}{3} \log \left(-3x + \sqrt{9x^2 - 6x + 5} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-6*x+5)^(1/2), x, algorithm="giac")

[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)

maple [A] time = 0.04, size = 9, normalized size = 0.64

$$\frac{\operatorname{arcsinh} \left(\frac{3x}{2} - \frac{1}{2} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2-6*x+5)^(1/2), x)

[Out] 1/3*arcsinh(3/2*x-1/2)

maxima [A] time = 2.95, size = 8, normalized size = 0.57

$$\frac{1}{3} \operatorname{arsinh} \left(\frac{3}{2} x - \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2-6*x+5)^(1/2),x, algorithm="maxima")

[Out] 1/3*arcsinh(3/2*x - 1/2)

mupad [B] time = 0.20, size = 20, normalized size = 1.43

$$\frac{\ln\left(3x + \sqrt{9x^2 - 6x + 5} - 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2 - 6*x + 5)^(1/2),x)

[Out] log(3*x + (9*x^2 - 6*x + 5)^(1/2) - 1)/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^2 - 6x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x**2-6*x+5)**(1/2),x)

[Out] Integral(1/sqrt(9*x**2 - 6*x + 5), x)

$$3.115 \quad \int \frac{1}{\sqrt{3-4x-4x^2}} dx$$

Optimal. Leaf size=10

$$\frac{1}{2} \sin^{-1} \left(x + \frac{1}{2} \right)$$

[Out] 1/2*arcsin(1/2+x)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$\frac{1}{2} \sin^{-1} \left(x + \frac{1}{2} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3 - 4*x - 4*x^2], x]

[Out] ArcSin[1/2 + x]/2

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3-4x-4x^2}} dx &= - \left(\frac{1}{16} \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} dx, x, -4-8x \right) \right) \\ &= \frac{1}{2} \sin^{-1} \left(\frac{1}{2} + x \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-\frac{1}{2} \sin^{-1} \left(\frac{1}{2}(-2x-1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]

[Out] -1/2*ArcSin[(-1 - 2*x)/2]

fricas [B] time = 1.05, size = 33, normalized size = 3.30

$$-\frac{1}{2} \arctan\left(\frac{\sqrt{-4x^2 - 4x + 3}(2x + 1)}{4x^2 + 4x - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="fricas")

[Out] -1/2*arctan(sqrt(-4*x^2 - 4*x + 3)*(2*x + 1)/(4*x^2 + 4*x - 3))

giac [A] time = 0.52, size = 6, normalized size = 0.60

$$\frac{1}{2} \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="giac")

[Out] 1/2*arcsin(x + 1/2)

maple [A] time = 0.05, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4*x^2-4*x+3)^(1/2),x)

[Out] 1/2*arcsin(x+1/2)

maxima [A] time = 3.01, size = 8, normalized size = 0.80

$$-\frac{1}{2} \arcsin\left(-x - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-4*x^2-4*x+3)^(1/2),x, algorithm="maxima")

[Out] $-1/2 \cdot \arcsin(-x - 1/2)$

mupad [B] time = 0.13, size = 6, normalized size = 0.60

$$\frac{\arcsin\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3 - 4*x^2 - 4*x)^(1/2), x)`

[Out] $\arcsin(x + 1/2)/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^2 - 4x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4*x**2-4*x+3)**(1/2), x)`

[Out] `Integral(1/sqrt(-4*x**2 - 4*x + 3), x)`

$$3.116 \quad \int \frac{1}{\sqrt{-8+6x+9x^2}} dx$$

Optimal. Leaf size=25

$$\frac{1}{3} \tanh^{-1} \left(\frac{3x+1}{\sqrt{9x^2+6x-8}} \right)$$

[Out] 1/3*arctanh((1+3*x)/(9*x^2+6*x-8)^(1/2))

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {621, 206}

$$\frac{1}{3} \tanh^{-1} \left(\frac{3x+1}{\sqrt{9x^2+6x-8}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-8 + 6*x + 9*x^2], x]

[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-8+6x+9x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{36-x^2} dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}} \right) \\ &= \frac{1}{3} \tanh^{-1} \left(\frac{1+3x}{\sqrt{-8+6x+9x^2}} \right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{1}{3} \log \left(\sqrt{9x^2+6x-8} + 3x + 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]

[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3

fricas [A] time = 0.94, size = 20, normalized size = 0.80

$$-\frac{1}{3} \log\left(-3x + \sqrt{9x^2 + 6x - 8} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")

[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)

giac [A] time = 0.57, size = 21, normalized size = 0.84

$$-\frac{1}{3} \log\left(\left|-3x + \sqrt{9x^2 + 6x - 8} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")

[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))

maple [A] time = 0.04, size = 30, normalized size = 1.20

$$\frac{\sqrt{9} \ln\left(\frac{(9x+3)\sqrt{9}}{9} + \sqrt{9x^2 + 6x - 8}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(9*x^2+6*x-8)^(1/2),x)

[Out] 1/9*9^(1/2)*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))

maxima [A] time = 2.99, size = 22, normalized size = 0.88

$$\frac{1}{3} \log\left(18x + 6\sqrt{9x^2 + 6x - 8} + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")

[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)

mupad [B] time = 0.22, size = 20, normalized size = 0.80

$$\frac{\ln(3x + \sqrt{9x^2 + 6x - 8} + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(6*x + 9*x^2 - 8)^(1/2), x)`

[Out] `log(3*x + (6*x + 9*x^2 - 8)^(1/2) + 1)/3`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{9x^2 + 6x - 8}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(9*x**2+6*x-8)**(1/2), x)`

[Out] `Integral(1/sqrt(9*x**2 + 6*x - 8), x)`

$$3.117 \quad \int \frac{1}{\sqrt{2+4x+3x^2}} dx$$

Optimal. Leaf size=18

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

[Out] 1/3*arcsinh(1/2*(2+3*x)*2^(1/2))*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 4*x + 3*x^2], x]

[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{2+4x+3x^2}} dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{8}}} dx, x, 4+6x\right)}{2\sqrt{6}} = \frac{\sinh^{-1}\left(\frac{2+3x}{\sqrt{2}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + 4*x + 3*x^2], x]

[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]

fricas [B] time = 0.96, size = 38, normalized size = 2.11

$$\frac{1}{6} \sqrt{3} \log\left(-\sqrt{3} \sqrt{3x^2 + 4x + 2} (3x + 2) - 9x^2 - 12x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 9*x^2 - 12*x - 5)

giac [B] time = 0.62, size = 33, normalized size = 1.83

$$-\frac{1}{3} \sqrt{3} \log\left(-\sqrt{3} \left(\sqrt{3}x - \sqrt{3x^2 + 4x + 2}\right) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2)^(1/2), x, algorithm="giac")

[Out] -1/3*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2)

maple [A] time = 0.05, size = 15, normalized size = 0.83

$$\frac{\sqrt{3} \operatorname{arcsinh}\left(\frac{3\sqrt{2}\left(x+\frac{2}{3}\right)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+4*x+2)^(1/2), x)

[Out] 1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))

maxima [A] time = 2.88, size = 16, normalized size = 0.89

$$\frac{1}{3} \sqrt{3} \operatorname{arsinh} \left(\frac{1}{2} \sqrt{2} (3x + 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x+2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))

mupad [B] time = 0.22, size = 26, normalized size = 1.44

$$\frac{\sqrt{3} \ln \left(\sqrt{3} \left(x + \frac{2}{3} \right) + \sqrt{3x^2 + 4x + 2} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 3*x^2 + 2)^(1/2),x)

[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 + 2)^(1/2)))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 4x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+4*x+2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**2 + 4*x + 2), x)

$$3.118 \quad \int \frac{1}{\sqrt{2+4x-3x^2}} dx$$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

[Out] -1/3*arcsin(1/10*(2-3*x)*10^(1/2))*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 4*x - 3*x^2], x]

[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{2+4x-3x^2}} dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{40}}} dx, x, 4-6x\right)}{2\sqrt{30}} = -\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + 4*x - 3*x^2], x]

[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])

fricas [B] time = 0.96, size = 40, normalized size = 2.11

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+4x+2}(3x-2)}{3(3x^2-4x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^(1/2), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2)/(3*x^2 - 4*x - 2))

giac [A] time = 0.82, size = 16, normalized size = 0.84

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^(1/2), x, algorithm="giac")

[Out] 1/3*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2))

maple [A] time = 0.05, size = 15, normalized size = 0.79

$$\frac{\sqrt{3}\arcsin\left(\frac{3\sqrt{10}\left(x-\frac{2}{3}\right)}{10}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+4*x+2)^(1/2), x)

[Out] 1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))

maxima [A] time = 2.93, size = 16, normalized size = 0.84

$$-\frac{1}{3} \sqrt{3} \arcsin\left(-\frac{1}{10} \sqrt{10} (3x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x+2)^(1/2),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))

mupad [B] time = 0.14, size = 16, normalized size = 0.84

$$\frac{\sqrt{3} \operatorname{asin}\left(\frac{\sqrt{40} (6x-4)}{40}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 3*x^2 + 2)^(1/2),x)

[Out] (3^(1/2)*asin((40^(1/2)*(6*x - 4))/40))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2 + 4x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+4*x+2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**2 + 4*x + 2), x)

$$3.119 \quad \int \frac{1}{\sqrt{2+5x+3x^2}} dx$$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x+2)^(1/2))*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 5*x + 3*x^2], x]

[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+5x+3x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{5+6x}{\sqrt{2+5x+3x^2}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{2+5x+3x^2}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{\log\left(2\sqrt{9x^2 + 15x + 6} + 6x + 5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + 5*x + 3*x^2], x]

[Out] Log[5 + 6*x + 2*Sqrt[6 + 15*x + 9*x^2]]/Sqrt[3]

fricas [A] time = 0.90, size = 38, normalized size = 1.09

$$\frac{1}{6} \sqrt{3} \log\left(4 \sqrt{3} \sqrt{3x^2 + 5x + 2} (6x + 5) + 72x^2 + 120x + 49\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 72*x^2 + 120*x + 49)

giac [A] time = 0.80, size = 34, normalized size = 0.97

$$-\frac{1}{3} \sqrt{3} \log\left(\left|-2 \sqrt{3} \left(\sqrt{3} x - \sqrt{3x^2 + 5x + 2}\right) - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2)^(1/2), x, algorithm="giac")

[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))

maple [A] time = 0.04, size = 30, normalized size = 0.86

$$\frac{\sqrt{3} \ln\left(\frac{\left(\frac{3x+5}{2}\right)\sqrt{3}}{3} + \sqrt{3x^2 + 5x + 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+5*x+2)^(1/2), x)

[Out] 1/3*3^(1/2)*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x+2)^(1/2))

maxima [A] time = 3.01, size = 28, normalized size = 0.80

$$\frac{1}{3} \sqrt{3} \log \left(2 \sqrt{3} \sqrt{3x^2 + 5x + 2} + 6x + 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x+2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)

mupad [B] time = 0.24, size = 26, normalized size = 0.74

$$\frac{\sqrt{3} \ln \left(\sqrt{3} \left(x + \frac{5}{6} \right) + \sqrt{3x^2 + 5x + 2} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x + 3*x^2 + 2)^(1/2),x)

[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 + 2)^(1/2)))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 5x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+5*x+2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**2 + 5*x + 2), x)

$$3.120 \quad \int \frac{1}{\sqrt{2+5x-3x^2}} dx$$

Optimal. Leaf size=17

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

[Out] 1/3*arcsin(-5/7+6/7*x)*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 5*x - 3*x^2], x]

[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+5x-3x^2}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{49}}} dx, x, 5-6x\right)}{7\sqrt{3}} \\ &= -\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + 5*x - 3*x^2], x]

[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])

fricas [B] time = 0.85, size = 40, normalized size = 2.35

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)}{6(3x^2-5x-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2)^(1/2), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5)/(3*x^2 - 5*x - 2))

giac [A] time = 0.66, size = 11, normalized size = 0.65

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{6}{7}x - \frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2)^(1/2), x, algorithm="giac")

[Out] 1/3*sqrt(3)*arcsin(6/7*x - 5/7)

maple [A] time = 0.05, size = 12, normalized size = 0.71

$$\frac{\sqrt{3}\arcsin\left(\frac{6x}{7} - \frac{5}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+5*x+2)^(1/2), x)

[Out] 1/3*3^(1/2)*arcsin(6/7*x-5/7)

maxima [A] time = 3.00, size = 11, normalized size = 0.65

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{6}{7}x + \frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x+2)^(1/2),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arcsin(-6/7*x + 5/7)

mupad [B] time = 0.17, size = 11, normalized size = 0.65

$$\frac{\sqrt{3}\operatorname{asin}\left(\frac{6x}{7} - \frac{5}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x - 3*x^2 + 2)^(1/2),x)

[Out] (3^(1/2)*asin((6*x)/7 - 5/7))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2 + 5x + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+5*x+2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**2 + 5*x + 2), x)

$$3.121 \quad \int \frac{1}{\sqrt{-2+4x+3x^2}} dx$$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(1/3*(2+3*x)*3^(1/2)/(3*x^2+4*x-2)^(1/2))*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 + 4*x + 3*x^2], x]

[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2+4x+3x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{4+6x}{\sqrt{-2+4x+3x^2}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{2+3x}{\sqrt{3}\sqrt{-2+4x+3x^2}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.81

$$\frac{\log\left(\sqrt{9x^2 + 12x - 6} + 3x + 2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2 + 4*x + 3*x^2], x]

[Out] Log[2 + 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3]

fricas [A] time = 0.87, size = 37, normalized size = 1.16

$$\frac{1}{6} \sqrt{3} \log\left(\sqrt{3} \sqrt{3x^2 + 4x - 2} (3x + 2) + 9x^2 + 12x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 9*x^2 + 12*x - 1)

giac [A] time = 0.56, size = 34, normalized size = 1.06

$$-\frac{1}{3} \sqrt{3} \log\left(\left|-\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2 + 4x - 2}\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x-2)^(1/2), x, algorithm="giac")

[Out] -1/3*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2))

maple [A] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\sqrt{3} \ln\left(\frac{(3x+2)\sqrt{3}}{3} + \sqrt{3x^2 + 4x - 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+4*x-2)^(1/2), x)

[Out] 1/3*3^(1/2)*ln(1/3*(3*x+2)*3^(1/2)+(3*x^2+4*x-2)^(1/2))

maxima [A] time = 2.99, size = 28, normalized size = 0.88

$$\frac{1}{3} \sqrt{3} \log\left(2 \sqrt{3} \sqrt{3x^2 + 4x - 2} + 6x + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+4*x-2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)

mupad [B] time = 0.23, size = 26, normalized size = 0.81

$$\frac{\sqrt{3} \ln\left(\sqrt{3} \left(x + \frac{2}{3}\right) + \sqrt{3x^2 + 4x - 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 3*x^2 - 2)^(1/2),x)

[Out] (3^(1/2)*log(3^(1/2)*(x + 2/3) + (4*x + 3*x^2 - 2)^(1/2)))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 4x - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+4*x-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**2 + 4*x - 2), x)

$$3.122 \quad \int \frac{1}{\sqrt{-2+4x-3x^2}} dx$$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

[Out] $-1/3*\arctan(1/3*(2-3*x)*3^{(1/2)/(-3*x^2+4*x-2)^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {621, 204}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 + 4*x - 3*x^2], x]

[Out] $-(\text{ArcTan}[(2 - 3*x)/(\text{Sqrt}[3]*\text{Sqrt}[-2 + 4*x - 3*x^2])]/\text{Sqrt}[3])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2+4x-3x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{-12-x^2} dx, x, \frac{4-6x}{\sqrt{-2+4x-3x^2}} \right) \\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-2+4x-3x^2}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2 + 4*x - 3*x^2], x]

[Out] -(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])

fricas [C] time = 0.93, size = 65, normalized size = 1.97

$$-\frac{1}{6}i\sqrt{3}\log\left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)+\frac{1}{6}i\sqrt{3}\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x-2)^(1/2), x, algorithm="fricas")

[Out] -1/6*I*sqrt(3)*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) + 1/6*I*sqrt(3)*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2+4x-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x-2)^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(-3*x^2 + 4*x - 2), x)

maple [A] time = 0.04, size = 26, normalized size = 0.79

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x-\frac{2}{3}\right)}{\sqrt{-3x^2+4x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+4*x-2)^(1/2), x)

[Out] 1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))

maxima [C] time = 2.91, size = 16, normalized size = 0.48

$$-\frac{1}{3}i\sqrt{3} \operatorname{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+4*x-2)^(1/2),x, algorithm="maxima")

[Out] -1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))

mupad [B] time = 0.14, size = 17, normalized size = 0.52

$$-\frac{\sqrt{3} \operatorname{asin}\left(\sqrt{2}\left(\frac{3x}{2}-1\right)1i\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 3*x^2 - 2)^(1/2),x)

[Out] -(3^(1/2)*asin(2^(1/2)*((3*x)/2 - 1)*1i))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2 + 4x - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+4*x-2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**2 + 4*x - 2), x)

$$3.123 \quad \int \frac{1}{\sqrt{-2+5x+3x^2}} dx$$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

[Out] 1/3*arctanh(1/6*(5+6*x)*3^(1/2)/(3*x^2+5*x-2)^(1/2))*3^(1/2)

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {621, 206}

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 + 5*x + 3*x^2], x]

[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2+5x+3x^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{5+6x}{\sqrt{-2+5x+3x^2}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{5+6x}{2\sqrt{3}\sqrt{-2+5x+3x^2}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{\log\left(2\sqrt{9x^2 + 15x - 6} + 6x + 5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2 + 5*x + 3*x^2], x]

[Out] Log[5 + 6*x + 2*Sqrt[-6 + 15*x + 9*x^2]]/Sqrt[3]

fricas [A] time = 1.03, size = 38, normalized size = 1.09

$$\frac{1}{6} \sqrt{3} \log\left(4 \sqrt{3} \sqrt{3x^2 + 5x - 2} (6x + 5) + 72x^2 + 120x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x-2)^(1/2), x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 72*x^2 + 120*x + 1)

giac [A] time = 0.73, size = 34, normalized size = 0.97

$$-\frac{1}{3} \sqrt{3} \log\left(\left|-2 \sqrt{3} \left(\sqrt{3} x - \sqrt{3x^2 + 5x - 2}\right) - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x-2)^(1/2), x, algorithm="giac")

[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))

maple [A] time = 0.04, size = 30, normalized size = 0.86

$$\frac{\sqrt{3} \ln\left(\frac{\left(\frac{3x+5}{2}\right)\sqrt{3}}{3} + \sqrt{3x^2 + 5x - 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x^2+5*x-2)^(1/2), x)

[Out] 1/3*3^(1/2)*ln(1/3*(3*x+5/2)*3^(1/2)+(3*x^2+5*x-2)^(1/2))

maxima [A] time = 2.93, size = 28, normalized size = 0.80

$$\frac{1}{3} \sqrt{3} \log \left(2 \sqrt{3} \sqrt{3x^2 + 5x - 2} + 6x + 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^2+5*x-2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)

mupad [B] time = 0.23, size = 26, normalized size = 0.74

$$\frac{\sqrt{3} \ln \left(\sqrt{3} \left(x + \frac{5}{6} \right) + \sqrt{3x^2 + 5x - 2} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x + 3*x^2 - 2)^(1/2),x)

[Out] (3^(1/2)*log(3^(1/2)*(x + 5/6) + (5*x + 3*x^2 - 2)^(1/2)))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^2 + 5x - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**2+5*x-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**2 + 5*x - 2), x)

$$3.124 \quad \int \frac{1}{\sqrt{-2+5x-3x^2}} dx$$

Optimal. Leaf size=13

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

[Out] 1/3*arcsin(-5+6*x)*3^(1/2)

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-2 + 5*x - 3*x^2], x]

[Out] -(ArcSin[5 - 6*x]/Sqrt[3])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-2+5x-3x^2}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 5-6x\right)}{\sqrt{3}} \\ &= -\frac{\sin^{-1}(5-6x)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]

[Out] -(ArcSin[5 - 6*x]/Sqrt[3])

fricas [B] time = 1.02, size = 40, normalized size = 3.08

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2+5x-2}(6x-5)}{6(3x^2-5x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5)/(3*x^2 - 5*x + 2))

giac [A] time = 0.64, size = 11, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arcsin(6x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arcsin(6*x - 5)

maple [A] time = 0.04, size = 12, normalized size = 0.92

$$\frac{\sqrt{3}\arcsin(6x-5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+5*x-2)^(1/2),x)

[Out] 1/3*3^(1/2)*arcsin(6*x-5)

maxima [A] time = 3.07, size = 11, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arcsin(6x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+5*x-2)^(1/2),x, algorithm="maxima")

[Out] $1/3*\text{sqrt}(3)*\text{arcsin}(6*x - 5)$

mupad [B] time = 0.16, size = 11, normalized size = 0.85

$$\frac{\sqrt{3} \operatorname{asin}(6x - 5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5*x - 3*x^2 - 2)^(1/2), x)`

[Out] $(3^{1/2}*\text{asin}(6*x - 5))/3$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^2 + 5x - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+5*x-2)**(1/2), x)`

[Out] `Integral(1/sqrt(-3*x**2 + 5*x - 2), x)`

$$3.125 \quad \int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}} dx$$

Optimal. Leaf size=22

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

[Out] arcsinh(1/2*(2*c*x+b)/c^(1/2))/c^(1/2)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {619, 215}

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2], x]

[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{\frac{b^2+4c}{4c} + bx + cx^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{4c}}} dx, x, b+2cx \right)}{2c}$$

$$= \frac{\sinh^{-1} \left(\frac{b+2cx}{2\sqrt{c}} \right)}{\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{\sinh^{-1} \left(\frac{b+2cx}{2\sqrt{c}} \right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2], x]

[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]

fricas [B] time = 0.89, size = 137, normalized size = 6.23

$$\left[\frac{\log \left(-4c^2x^2 - 4bcx - b^2 - (2cx + b)\sqrt{c} \sqrt{\frac{4c^2x^2 + 4bcx + b^2 + 4c}{c}} - 2c \right)}{2\sqrt{c}}, -\frac{\sqrt{-c} \arctan \left(\frac{(2cx+b)\sqrt{-c} \sqrt{\frac{4c^2x^2 + 4bcx + b^2 + 4c}{c}}}{4c^2x^2 + 4bcx + b^2 + 4c} \right)}{c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*log(-4*c^2*x^2 - 4*b*c*x - b^2 - (2*c*x + b)*sqrt(c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c) - 2*c)/sqrt(c), -sqrt(-c)*arctan((2*c*x + b)*sqrt(-c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c)/(4*c^2*x^2 + 4*b*c*x + b^2 + 4*c))/c]

giac [B] time = 0.75, size = 46, normalized size = 2.09

$$\frac{\log \left(\left(2\sqrt{c}x - \sqrt{4cx^2 + 4bx + \frac{b^2+4c}{c}} \right) \sqrt{c} + b \right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")

[Out] -log(abs((2*sqrt(c)*x - sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c))*sqrt(c) + b)/sqrt(c)

maple [B] time = 0.08, size = 51, normalized size = 2.32

$$\frac{\sqrt{4} \ln\left(\frac{(4cx+2b)\sqrt{4}}{4\sqrt{c}} + \sqrt{4cx^2 + 4bx + \frac{b^2+4c}{c}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x)

[Out] 1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2))*4^(1/2)/c^(1/2)

maxima [A] time = 1.40, size = 16, normalized size = 0.73

$$\frac{\operatorname{arsinh}\left(\frac{2cx+b}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="maxima")

[Out] arcsinh(1/2*(2*c*x + b)/sqrt(c))/sqrt(c)

mupad [B] time = 0.42, size = 40, normalized size = 1.82

$$\frac{\ln\left(\frac{b+2cx}{\sqrt{c}} + \sqrt{\frac{b^2+4c}{c} + 4bx + 4cx^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2),x)

[Out] log((b + 2*c*x)/c^(1/2) + ((4*c + b^2)/c + 4*b*x + 4*c*x^2)^(1/2))/c^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int \frac{1}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4), x)
```


$$3.126 \quad \int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}} dx$$

Optimal. Leaf size=23

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

[Out] $-\arcsin(1/2*(-2*c*x+b)/c^{(1/2)})/c^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[(-b^2 + 4*c)/(4*c) + b*x - c*x^2], x]$

[Out] $-(\text{ArcSin}[(b - 2*c*x)/(2*\text{Sqrt}[c]])/\text{Sqrt}[c])$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p)}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c} + bx - cx^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4c}}} dx, x, b-2cx \right)}{2c}$$

$$= \frac{\sin^{-1} \left(\frac{b-2cx}{2\sqrt{c}} \right)}{\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{\sin^{-1} \left(\frac{b-2cx}{2\sqrt{c}} \right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2], x]

[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c]])/Sqrt[c])

fricas [B] time = 1.00, size = 141, normalized size = 6.13

$$\left[\frac{\sqrt{-c} \log \left(4c^2x^2 - 4bcx + b^2 - (2cx - b)\sqrt{-c} \sqrt{\frac{4c^2x^2 - 4bcx + b^2 - 4c}{c}} - 2c \right)}{2c}, -\frac{\arctan \left(\frac{(2cx-b)\sqrt{c} \sqrt{\frac{4c^2x^2 - 4bcx + b^2 - 4c}{c}}}{4c^2x^2 - 4bcx + b^2 - 4c} \right)}{\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/2*sqrt(-c)*log(4*c^2*x^2 - 4*b*c*x + b^2 - (2*c*x - b)*sqrt(-c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c) - 2*c)/c, -arctan((2*c*x - b)*sqrt(c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c)/(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c))/sqrt(c)]

giac [B] time = 1.08, size = 53, normalized size = 2.30

$$\frac{\log \left(\left(2\sqrt{-c}x - \sqrt{-4cx^2 + 4bx - \frac{b^2-4c}{c}} \right) \sqrt{-c} + b \right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")`

[Out] $-\log(\text{abs}((2*\sqrt{-c})*x - \sqrt{-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c})*\sqrt{-c} + b))/\sqrt{-c}$

maple [B] time = 0.09, size = 44, normalized size = 1.91

$$\frac{\arctan\left(\frac{2\left(x-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-4cx^2+4bx-\frac{b^2-4c}{c}}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x)`

[Out] $1/c^{(1/2)}*\arctan(2*c^{(1/2)}*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/c)^{(1/2)})$

maxima [A] time = 3.08, size = 19, normalized size = 0.83

$$\frac{\arcsin\left(-\frac{2cx-b}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")`

[Out] $-\arcsin(-1/2*(2*c*x - b)/\sqrt{c}))/\sqrt{c}$

mupad [B] time = 0.41, size = 46, normalized size = 2.00

$$\frac{\ln\left(\frac{b-2cx}{\sqrt{-c}} + \sqrt{4bx + \frac{4c-b^2}{c} - 4cx^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2/(4*b*x + (4*c - b^2)/c - 4*c*x^2)^(1/2),x)`

[Out] $\log((b - 2*c*x)/(-c)^{(1/2)} + (4*b*x + (4*c - b^2)/c - 4*c*x^2)^{(1/2)})/(-c)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int \frac{1}{\sqrt{-\frac{b^2}{c} + 4bx - 4cx^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 4), x)
```

$$3.127 \quad \int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}} dx$$

Optimal. Leaf size=20

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

[Out] $-\arcsin((-2*c*x+b)/c^{(1/2)})/c^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[(-b^2 + c)/(4*c) + b*x - c*x^2], x]$

[Out] $-(\text{ArcSin}[(b - 2*c*x)/\text{Sqrt}[c]]/\text{Sqrt}[c])$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p)}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^{(p)}, x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b^2+c}{4c} + bx - cx^2}} dx = \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{c}}} dx, x, b-2cx \right)}{c}$$

$$= -\frac{\sin^{-1} \left(\frac{b-2cx}{\sqrt{c}} \right)}{\sqrt{c}}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{\sin^{-1} \left(\frac{b-2cx}{\sqrt{c}} \right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2], x]

[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])

fricas [B] time = 0.94, size = 143, normalized size = 7.15

$$\left[\frac{\sqrt{-c} \log \left(8c^2x^2 - 8bcx + 2b^2 - 2(2cx - b)\sqrt{-c} \sqrt{-\frac{4c^2x^2 - 4bcx + b^2 - c}{c}} - c \right)}{2c}, \arctan \left(\frac{(2cx - b)\sqrt{c} \sqrt{-\frac{4c^2x^2 - 4bcx + b^2 - c}{c}}}{4c^2x^2 - 4bcx + b^2 - c} \right) \right] / \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2), x, algorithm="fricas")

[Out] [-1/2*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + 2*b^2 - 2*(2*c*x - b)*sqrt(-c)*sqrt(-4*c^2*x^2 - 4*b*c*x + b^2 - c)/c) - c)/c, -arctan((2*c*x - b)*sqrt(c)*sqrt(-4*c^2*x^2 - 4*b*c*x + b^2 - c)/(4*c^2*x^2 - 4*b*c*x + b^2 - c))/sqrt(c)]

giac [B] time = 0.82, size = 53, normalized size = 2.65

$$\frac{\log \left(\left(2\sqrt{-c}x - \sqrt{-4cx^2 + 4bx - \frac{b^2-c}{c}} \right) \sqrt{-c} + b \right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="giac")

[Out] -log(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c))*sqrt(-c) + b))/sqrt(-c)

maple [B] time = 0.08, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{2\left(x-\frac{b}{2c}\right)\sqrt{c}}{\sqrt{-4cx^2+4bx-\frac{b^2-c}{c}}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x)

[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)^(1/2))

maxima [A] time = 2.89, size = 19, normalized size = 0.95

$$\frac{\arcsin\left(-\frac{2cx-b}{\sqrt{c}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-(2*c*x - b)/sqrt(c))/sqrt(c)

mupad [B] time = 0.40, size = 44, normalized size = 2.20

$$\frac{\ln\left(\frac{b-2cx}{\sqrt{-c}} + \sqrt{\frac{c-b^2}{c} + 4bx - 4cx^2}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2),x)

[Out] log((b - 2*c*x)/(-c)^(1/2) + ((c - b^2)/c + 4*b*x - 4*c*x^2)^(1/2))/(-c)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int \frac{1}{\sqrt{-\frac{b^2}{c} + 4bx - 4cx^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 1), x)
```


$$3.128 \quad \int \frac{1}{(2+3x+x^2)^{3/2}} dx$$

Optimal. Leaf size=19

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

[Out] $-2*(3+2*x)/(x^2+3*x+2)^{(1/2)}$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {613}

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + 3*x + x^2)^{-3/2}, x]$

[Out] $(-2*(3 + 2*x))/\text{Sqrt}[2 + 3*x + x^2]$

Rule 613

$\text{Int}[(a_. + (b_.)*(x_) + (c_.)*(x_)^2)^{-3/2}, x_Symbol] :> \text{Simp}[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[a + b*x + c*x^2]), x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\int \frac{1}{(2+3x+x^2)^{3/2}} dx = -\frac{2(3+2x)}{\sqrt{2+3x+x^2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(2 + 3*x + x^2)^{-3/2}, x]$

[Out] $(-2*(3 + 2*x))/\text{Sqrt}[2 + 3*x + x^2]$

fricas [B] time = 0.88, size = 38, normalized size = 2.00

$$\frac{2(2x^2 + \sqrt{x^2 + 3x + 2}(2x + 3) + 6x + 4)}{x^2 + 3x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="fricas")

[Out] -2*(2*x^2 + sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)/(x^2 + 3*x + 2)

giac [A] time = 0.83, size = 17, normalized size = 0.89

$$\frac{2(2x + 3)}{\sqrt{x^2 + 3x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="giac")

[Out] -2*(2*x + 3)/sqrt(x^2 + 3*x + 2)

maple [A] time = 0.05, size = 24, normalized size = 1.26

$$\frac{2(x + 2)(x + 1)(2x + 3)}{(x^2 + 3x + 2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+3*x+2)^(3/2),x)

[Out] -2*(x+2)*(x+1)*(2*x+3)/(x^2+3*x+2)^(3/2)

maxima [A] time = 1.34, size = 26, normalized size = 1.37

$$-\frac{4x}{\sqrt{x^2 + 3x + 2}} - \frac{6}{\sqrt{x^2 + 3x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+3*x+2)^(3/2),x, algorithm="maxima")

[Out] -4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)

mupad [B] time = 0.05, size = 15, normalized size = 0.79

$$\frac{4\left(x + \frac{3}{2}\right)}{\sqrt{x^2 + 3x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x + x^2 + 2)^(3/2), x)`

[Out] `-(4*(x + 3/2))/(3*x + x^2 + 2)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 3x + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+3*x+2)**(3/2), x)`

[Out] `Integral((x**2 + 3*x + 2)**(-3/2), x)`

$$3.129 \quad \int \frac{1}{(27-24x+4x^2)^{3/2}} dx$$

Optimal. Leaf size=23

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

[Out] 1/9*(3-x)/(4*x^2-24*x+27)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {613}

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

[In] Int[(27 - 24*x + 4*x^2)^(-3/2), x]

[Out] (3 - x)/(9*sqrt[27 - 24*x + 4*x^2])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{(27-24x+4x^2)^{3/2}} dx = \frac{3-x}{9\sqrt{27-24x+4x^2}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

[In] Integrate[(27 - 24*x + 4*x^2)^(-3/2), x]

[Out] (3 - x)/(9*sqrt[27 - 24*x + 4*x^2])

fricas [B] time = 1.19, size = 41, normalized size = 1.78

$$-\frac{4x^2 + 2\sqrt{4x^2 - 24x + 27}(x - 3) - 24x + 27}{18(4x^2 - 24x + 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="fricas")

[Out] -1/18*(4*x^2 + 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)/(4*x^2 - 24*x + 27)

giac [A] time = 0.77, size = 17, normalized size = 0.74

$$-\frac{x - 3}{9\sqrt{4x^2 - 24x + 27}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="giac")

[Out] -1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)

maple [A] time = 0.05, size = 28, normalized size = 1.22

$$-\frac{(2x - 3)(2x - 9)(x - 3)}{9(4x^2 - 24x + 27)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2-24*x+27)^(3/2),x)

[Out] -1/9*(2*x-3)*(2*x-9)*(x-3)/(4*x^2-24*x+27)^(3/2)

maxima [A] time = 1.24, size = 30, normalized size = 1.30

$$-\frac{x}{9\sqrt{4x^2 - 24x + 27}} + \frac{1}{3\sqrt{4x^2 - 24x + 27}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^2-24*x+27)^(3/2),x, algorithm="maxima")

[Out] -1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)

mupad [B] time = 0.06, size = 17, normalized size = 0.74

$$-\frac{x-3}{9\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x^2 - 24*x + 27)^(3/2), x)`

[Out] `-(x - 3)/(9*(4*x^2 - 24*x + 27)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4x^2 - 24x + 27)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x**2-24*x+27)**(3/2), x)`

[Out] `Integral((4*x**2 - 24*x + 27)**(-3/2), x)`

$$3.130 \quad \int \frac{x}{(5-4x-x^2)^{3/2}} dx$$

Optimal. Leaf size=23

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

[Out] 1/9*(5-2*x)/(-x^2-4*x+5)^(1/2)

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {636}

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

[In] Int[x/(5 - 4*x - x^2)^(3/2), x]

[Out] (5 - 2*x)/(9*sqrt[5 - 4*x - x^2])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{x}{(5-4x-x^2)^{3/2}} dx = \frac{5-2x}{9\sqrt{5-4x-x^2}}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 1.00

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(5 - 4*x - x^2)^(3/2), x]

[Out] (5 - 2*x)/(9*sqrt[5 - 4*x - x^2])

fricas [A] time = 0.86, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2 - 4x + 5}(2x - 5)}{9(x^2 + 4x - 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="fricas")

[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)

giac [A] time = 0.61, size = 29, normalized size = 1.26

$$\frac{\sqrt{-x^2 - 4x + 5}(2x - 5)}{9(x^2 + 4x - 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="giac")

[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)

maple [A] time = 0.05, size = 26, normalized size = 1.13

$$\frac{(x + 5)(x - 1)(2x - 5)}{9(-x^2 - 4x + 5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^2-4*x+5)^(3/2),x)

[Out] 1/9*(x+5)*(x-1)*(2*x-5)/(-x^2-4*x+5)^(3/2)

maxima [A] time = 1.34, size = 30, normalized size = 1.30

$$-\frac{2x}{9\sqrt{-x^2 - 4x + 5}} + \frac{5}{9\sqrt{-x^2 - 4x + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2-4*x+5)^(3/2),x, algorithm="maxima")

[Out] -2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)

mupad [B] time = 0.05, size = 19, normalized size = 0.83

$$-\frac{2x - 5}{9\sqrt{-x^2 - 4x + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(5 - x^2 - 4*x)^(3/2), x)`

[Out] `-(2*x - 5)/(9*(5 - x^2 - 4*x)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(-(x-1)(x+5))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x**2-4*x+5)**(3/2), x)`

[Out] `Integral(x/(-(x - 1)*(x + 5))**(3/2), x)`

$$3.131 \quad \int \frac{1}{(5-4x-x^2)^{5/2}} dx$$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}} + \frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

[Out] 1/27*(2+x)/(-x^2-4*x+5)^(3/2)+2/243*(2+x)/(-x^2-4*x+5)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {614, 613}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}} + \frac{x+2}{27(-x^2-4x+5)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x - x^2)^(-5/2), x]

[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rubi steps

$$\int \frac{1}{(5-4x-x^2)^{5/2}} dx = \frac{2+x}{27(5-4x-x^2)^{3/2}} + \frac{2}{27} \int \frac{1}{(5-4x-x^2)^{3/2}} dx$$

$$= \frac{2+x}{27(5-4x-x^2)^{3/2}} + \frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$-\frac{(x+2)(2x^2+8x-19)}{243(-x^2-4x+5)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 4*x - x^2)^(-5/2), x]

[Out] -1/243*((2 + x)*(-19 + 8*x + 2*x^2))/(5 - 4*x - x^2)^(3/2)

fricas [A] time = 0.89, size = 49, normalized size = 1.14

$$-\frac{(2x^3 + 12x^2 - 3x - 38)\sqrt{-x^2 - 4x + 5}}{243(x^4 + 8x^3 + 6x^2 - 40x + 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-4*x+5)^(5/2), x, algorithm="fricas")

[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^2 - 40*x + 25)

giac [A] time = 0.78, size = 36, normalized size = 0.84

$$-\frac{((2(x+6)x-3)x-38)\sqrt{-x^2-4x+5}}{243(x^2+4x-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-4*x+5)^(5/2), x, algorithm="giac")

[Out] -1/243*((2*(x+6)*x-3)*x-38)*sqrt(-x^2-4*x+5)/(x^2+4*x-5)^2

maple [A] time = 0.04, size = 36, normalized size = 0.84

$$\frac{(x+5)(x-1)(2x^3+12x^2-3x-38)}{243(-x^2-4x+5)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-x^2-4*x+5)^(5/2),x)`

[Out] `1/243*(x+5)*(x-1)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)`

maxima [A] time = 1.41, size = 59, normalized size = 1.37

$$\frac{2x}{243\sqrt{-x^2-4x+5}} + \frac{4}{243\sqrt{-x^2-4x+5}} + \frac{x}{27(-x^2-4x+5)^{\frac{3}{2}}} + \frac{2}{27(-x^2-4x+5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")`

[Out] `2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 - 4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)`

mupad [B] time = 0.03, size = 29, normalized size = 0.67

$$-\frac{(4x+8)(8x^2+32x-76)}{3888(-x^2-4x+5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5 - x^2 - 4*x)^(5/2),x)`

[Out] `-((4*x + 8)*(32*x + 8*x^2 - 76))/(3888*(5 - x^2 - 4*x)^(3/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^2-4x+5)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**2-4*x+5)**(5/2),x)`

[Out] `Integral((-x**2 - 4*x + 5)**(-5/2), x)`

3.132 $\int (a + bx + cx^2)^p dx$

Optimal. Leaf size=122

$$\frac{2^{p+1} \left(-\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}} \right)^{-p-1} (a+bx+cx^2)^{p+1} {}_2F_1 \left(-p, p+1; p+2; \frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}} \right)}{(p+1)\sqrt{b^2-4ac}}$$

[Out] $-2^{(1+p)}*(c*x^2+b*x+a)^{(1+p)}*\text{hypergeom}([-p, 1+p], [2+p], 1/2*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})*((-b-2*c*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(-1-p)}/(1+p)/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {624}

$$\frac{2^{p+1} \left(-\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}} \right)^{-p-1} (a+bx+cx^2)^{p+1} {}_2F_1 \left(-p, p+1; p+2; \frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}} \right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)^p, x]

[Out] $-((2^{(1+p)}*((b - \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]))^{(-1-p)}*(a + b*x + c*x^2)^{(1+p)}*\text{Hypergeometric2F1}[-p, 1+p, 2+p, (b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/(2*\text{Sqrt}[b^2 - 4*a*c])]/(\text{Sqrt}[b^2 - 4*a*c]*(1+p)))$

Rule 624

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p+1)*Hypergeometric2F1[-p, p+1, p+2, (b+q+2*c*x)/(2*q)]/(q*(p+1)*((q-b-2*c*x)/(2*q))^(p+1)), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]

Rubi steps

$$\int (a + bx + cx^2)^p dx = -\frac{2^{1+p} \left(-\frac{b-\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}} \right)^{-1-p} (a+bx+cx^2)^{1+p} {}_2F_1 \left(-p, 1+p; 2+p; \frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}} \right)}{\sqrt{b^2-4ac}(1+p)}$$

Mathematica [A] time = 0.14, size = 126, normalized size = 1.03

$$\frac{2^{p-1} \left(-\sqrt{b^2 - 4ac} + b + 2cx \right) \left(\frac{\sqrt{b^2 - 4ac} + b + 2cx}{\sqrt{b^2 - 4ac}} \right)^{-p} (a + x(b + cx))^p {}_2F_1 \left(-p, p + 1; p + 2; \frac{-b - 2cx + \sqrt{b^2 - 4ac}}{2\sqrt{b^2 - 4ac}} \right)}{c(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)^p, x]

[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c])^p)

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^2 + bx + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^p,x, algorithm="fricas")

[Out] integral((c*x^2 + b*x + a)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^p,x, algorithm="giac")

[Out] integrate((c*x^2 + b*x + a)^p, x)

maple [F] time = 1.23, size = 0, normalized size = 0.00

$$\int (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)^p,x)

[Out] int((c*x^2+b*x+a)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)^p,x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x + a)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x + c*x^2)^p,x)

[Out] int((a + b*x + c*x^2)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)**p,x)

[Out] Integral((a + b*x + c*x**2)**p, x)

3.133 $\int (3 + 4x + 5x^2)^p dx$

Optimal. Leaf size=37

$$5^{-p-1}11^p(5x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{11}(5x+2)^2\right)$$

[Out] $5^{(-1-p)}*11^p*(2+5*x)*\text{hypergeom}([1/2, -p], [3/2], -1/11*(2+5*x)^2)$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$5^{-p-1}11^p(5x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + 5*x^2)^p, x]

[Out] $5^{(-1-p)}*11^p*(2+5*x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, -(2+5*x)^2/11]$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x + 5x^2)^p dx &= \frac{1}{2} (5^{-1-p}11^p) \text{Subst}\left(\int \left(1 + \frac{x^2}{44}\right)^p dx, x, 4 + 10x\right) \\ &= 5^{-1-p}11^p(2 + 5x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{11}(2 + 5x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$5^{-p-1}11^p(5x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + 5*x^2)^p, x]

[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/11*(2 + 5*x)^2]

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(5x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((5*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((5*x^2 + 4*x + 3)^p, x)

maple [F] time = 3.09, size = 0, normalized size = 0.00

$$\int (5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+4*x+3)^p,x)

[Out] int((5*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((5*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 5*x^2 + 3)^p,x)

[Out] int((4*x + 5*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+4*x+3)**p,x)

[Out] Integral((5*x**2 + 4*x + 3)**p, x)

$$3.134 \quad \int (3 + 4x + 4x^2)^p dx$$

Optimal. Leaf size=32

$$2^{p-1}(2x+1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{2}(2x+1)^2\right)$$

[Out] $2^{(-1+p)*(1+2*x)} \text{hypergeom}([1/2, -p], [3/2], -1/2*(1+2*x)^2)$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$2^{p-1}(2x+1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{2}(2x+1)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + 4*x^2)^p, x]

[Out] $2^{(-1+p)*(1+2*x)} \text{Hypergeometric2F1}[1/2, -p, 3/2, -(1+2*x)^2/2]$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c)], x]^p, x], x, b + 2*c*x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x + 4x^2)^p dx &= 2^{-3+p} \text{Subst}\left(\int \left(1 + \frac{x^2}{32}\right)^p dx, x, 4 + 8x\right) \\ &= 2^{-1+p}(1 + 2x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{2}(1 + 2x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2^{p-3}(8x+4) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{32}(8x+4)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + 4*x^2)^p, x]

[Out] 2^(-3 + p)*(4 + 8*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/32*(4 + 8*x)^2]

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(4x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((4*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((4*x^2 + 4*x + 3)^p, x)

maple [F] time = 3.28, size = 0, normalized size = 0.00

$$\int (4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+4*x+3)^p,x)

[Out] int((4*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((4*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 3)^p,x)

[Out] int((4*x + 4*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+4*x+3)**p,x)

[Out] Integral((4*x**2 + 4*x + 3)**p, x)

$$3.135 \quad \int (3 + 4x + 3x^2)^p dx$$

Optimal. Leaf size=37

$$3^{-p-1}5^p(3x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{5}(3x+2)^2\right)$$

[Out] $3^{(-1-p)}*5^p*(2+3*x)*\text{hypergeom}([1/2, -p], [3/2], -1/5*(2+3*x)^2)$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$3^{-p-1}5^p(3x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + 3*x^2)^p, x]

[Out] $3^{(-1-p)}*5^p*(2+3*x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, -(2+3*x)^2/5]$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x + 3x^2)^p dx &= \frac{1}{2} (3^{-1-p}5^p) \text{Subst}\left(\int \left(1 + \frac{x^2}{20}\right)^p dx, x, 4 + 6x\right) \\ &= 3^{-1-p}5^p(2 + 3x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{5}(2 + 3x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$3^{-p-1}5^p(3x+2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + 3*x^2)^p, x]

[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -1/5*(2 + 3*x)^2]

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((3*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((3*x^2 + 4*x + 3)^p, x)

maple [F] time = 2.55, size = 0, normalized size = 0.00

$$\int (3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+4*x+3)^p,x)

[Out] int((3*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((3*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 3*x^2 + 3)^p,x)

[Out] int((4*x + 3*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+4*x+3)**p,x)

[Out] Integral((3*x**2 + 4*x + 3)**p, x)

$$3.136 \quad \int (3 + 4x + 2x^2)^p dx$$

Optimal. Leaf size=21

$$(x+1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -2(x+1)^2\right)$$

[Out] (1+x)*hypergeom([1/2, -p], [3/2], -2*(1+x)^2)

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$(x+1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -2(x+1)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + 2*x^2)^p, x]

[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x + 2x^2)^p dx &= \frac{1}{4} \text{Subst}\left(\int \left(1 + \frac{x^2}{8}\right)^p dx, x, 4 + 4x\right) \\ &= (1+x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -2(1+x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$(x+1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -2(x+1)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + 2*x^2)^p, x]

[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((2*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((2*x^2 + 4*x + 3)^p, x)

maple [F] time = 3.22, size = 0, normalized size = 0.00

$$\int (2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+4*x+3)^p,x)

[Out] int((2*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((2*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int (2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 2*x^2 + 3)^p,x)

[Out] int((4*x + 2*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+4*x+3)**p,x)

[Out] Integral((2*x**2 + 4*x + 3)**p, x)

$$3.137 \quad \int (3 + 4x + x^2)^p dx$$

Optimal. Leaf size=54

$$\frac{2^{2p+1}(-2x-2)^{-p-1}(x^2+4x+3)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{x+3}{2}\right)}{p+1}$$

[Out] $-2^{(1+2*p)}*(-2-2*x)^{(-1-p)}*(x^2+4*x+3)^{(1+p)}*\text{hypergeom}([-p, 1+p], [2+p], 3/2+1/2*x)/(1+p)$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {624}

$$\frac{2^{2p+1}(-2x-2)^{-p-1}(x^2+4x+3)^{p+1} {}_2F_1\left(-p, p+1; p+2; \frac{x+3}{2}\right)}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x + x^2)^p, x]

[Out] $-((2^{(1+2*p)}*(-2-2*x)^{(-1-p)}*(3+4*x+x^2)^{(1+p)}*\text{Hypergeometric2F1}[-p, 1+p, 2+p, (3+x)/2])/(1+p))$

Rule 624

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]

Rubi steps

$$\int (3 + 4x + x^2)^p dx = -\frac{2^{1+2p}(-2-2x)^{-1-p}(3+4x+x^2)^{1+p} {}_2F_1\left(-p, 1+p; 2+p; \frac{3+x}{2}\right)}{1+p}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.89

$$\frac{2^p(x+1)(x+3)^{-p}(x^2+4x+3)^p {}_2F_1\left(-p, p+1; p+2; \frac{1}{2}(-x-1)\right)}{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x + x^2)^p,x]

[Out] (2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-1 - x)/2])/((1 + p)*(3 + x)^p)

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((x^2 + 4*x + 3)^p, x)

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int (x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+4*x+3)^p,x)

[Out] int((x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + x^2 + 3)^p, x)`

[Out] `int((4*x + x^2 + 3)^p, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+4*x+3)**p, x)`

[Out] `Integral((x**2 + 4*x + 3)**p, x)`

3.138 $\int (3 + 4x)^p dx$

Optimal. Leaf size=18

$$\frac{(4x + 3)^{p+1}}{4(p + 1)}$$

[Out] $1/4*(3+4*x)^{(1+p)/(1+p)}$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(4x + 3)^{p+1}}{4(p + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 + 4*x)^p, x]$

[Out] $(3 + 4*x)^{(1 + p)/(4*(1 + p))}$

Rule 32

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)/(b*(m + 1))}, x] /; \text{FreeQ}[\{a, b, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\int (3 + 4x)^p dx = \frac{(3 + 4x)^{1+p}}{4(1 + p)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{(4x + 3)^{p+1}}{4p + 4}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(3 + 4*x)^p, x]$

[Out] $(3 + 4*x)^{(1 + p)/(4 + 4*p)}$

fricas [A] time = 0.88, size = 19, normalized size = 1.06

$$\frac{(4x + 3)^p(4x + 3)}{4(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)^p,x, algorithm="fricas")

[Out] 1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)

giac [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{(4x + 3)^{p+1}}{4(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)^p,x, algorithm="giac")

[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)

maple [A] time = 0.04, size = 17, normalized size = 0.94

$$\frac{(4x + 3)^{p+1}}{4p + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+4*x)^p,x)

[Out] 1/4*(3+4*x)^(p+1)/(p+1)

maxima [A] time = 1.25, size = 16, normalized size = 0.89

$$\frac{(4x + 3)^{p+1}}{4(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)^p,x, algorithm="maxima")

[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)

mupad [B] time = 0.39, size = 32, normalized size = 1.78

$$\begin{cases} \frac{\ln(4x+3)}{4} & \text{if } p = -1 \\ \frac{(4x+3)^{p+1}}{4(p+1)} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x + 3)^p,x)`

[Out] `piecewise(p == -1, log(4*x + 3)/4, p != -1, (4*x + 3)^(p + 1)/(4*(p + 1)))`

sympy [A] time = 0.06, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(4x+3)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(4x + 3) & \text{otherwise} \end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+4*x)**p,x)`

[Out] `Piecewise(((4*x + 3)**(p + 1))/(p + 1), Ne(p, -1)), (log(4*x + 3), True))/4`

$$3.139 \quad \int (3 + 4x - x^2)^p dx$$

Optimal. Leaf size=31

$$-7^p(2-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{7}(2-x)^2\right)$$

[Out] $-7^p(2-x)*\text{hypergeom}([1/2, -p], [3/2], 1/7*(2-x)^2)$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$-7^p(2-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{7}(2-x)^2\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 + 4*x - x^2)^p, x]$

[Out] $-(7^p(2-x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, (2-x)^2/7])$

Rule 245

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

Rule 619

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{(p)}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rubi steps

$$\begin{aligned} \int (3 + 4x - x^2)^p dx &= -\left(\frac{1}{2}7^p \text{Subst}\left(\int \left(1 - \frac{x^2}{28}\right)^p dx, x, 4 - 2x\right)\right) \\ &= -7^p(2-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{7}(2-x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.84

$$7^p(x-2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{7}(x-2)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x - x^2)^p, x]

[Out] 7^p*(-2 + x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 + x)^2/7]

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((-x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-x^2 + 4*x + 3)^p, x)

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int (-x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+4*x+3)^p,x)

[Out] int((-x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((-x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (-x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - x^2 + 3)^p,x)

[Out] int((4*x - x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+4*x+3)**p,x)

[Out] Integral((-x**2 + 4*x + 3)**p, x)

$$3.140 \quad \int (3 + 4x - 2x^2)^p dx$$

Optimal. Leaf size=31

$$-5^p(1-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{2}{5}(1-x)^2\right)$$

[Out] $-5^p(1-x) \text{hypergeom}([1/2, -p], [3/2], 2/5*(1-x)^2)$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$-5^p(1-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{2}{5}(1-x)^2\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 + 4*x - 2*x^2)^p, x]$

[Out] $-(5^p*(1-x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, (2*(1-x)^2)/5])$

Rule 245

$\text{Int}[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] \rightarrow \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

Rule 619

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(p_), x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rubi steps

$$\begin{aligned} \int (3 + 4x - 2x^2)^p dx &= -\left(\frac{1}{4}5^p \text{Subst}\left(\int \left(1 - \frac{x^2}{40}\right)^p dx, x, 4 - 4x\right)\right) \\ &= -5^p(1-x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{2}{5}(1-x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.84

$$5^p(x-1) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{2}{5}(x-1)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x - 2*x^2)^p, x]

[Out] 5^p*(-1 + x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(-1 + x)^2)/5]

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-2x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((-2*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-2*x^2 + 4*x + 3)^p, x)

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int (-2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^2+4*x+3)^p,x)

[Out] int((-2*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((-2*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (-2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - 2*x^2 + 3)^p,x)

[Out] int((4*x - 2*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-2x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**2+4*x+3)**p,x)

[Out] Integral((-2*x**2 + 4*x + 3)**p, x)

$$3.141 \quad \int (3 + 4x - 3x^2)^p dx$$

Optimal. Leaf size=38

$$-3^{-p-1}13^p(2-3x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{13}(2-3x)^2\right)$$

[Out] $-3^{(-1-p)}*13^p*(2-3*x)*\text{hypergeom}([1/2, -p], [3/2], 1/13*(2-3*x)^2)$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$-3^{-p-1}13^p(2-3x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x - 3*x^2)^p, x]

[Out] $-(3^{(-1-p)}*13^p*(2-3*x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, (2-3*x)^2/13])$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x - 3x^2)^p dx &= -\left(\frac{1}{2} (3^{-1-p}13^p) \text{Subst}\left(\int \left(1 - \frac{x^2}{52}\right)^p dx, x, 4 - 6x\right)\right) \\ &= -3^{-1-p}13^p(2-3x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{13}(2-3x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.97

$$3^{-p-1}13^p(3x-2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x - 3*x^2)^p, x]

[Out] 3^(-1 - p)*13^p*(-2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 3*x)^2/13]

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-3x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((-3*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-3*x^2 + 4*x + 3)^p, x)

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int (-3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^2+4*x+3)^p,x)

[Out] int((-3*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((-3*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (-3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - 3*x^2 + 3)^p,x)

[Out] int((4*x - 3*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**2+4*x+3)**p,x)

[Out] Integral((-3*x**2 + 4*x + 3)**p, x)

$$3.142 \quad \int (3 + 4x - 4x^2)^p dx$$

Optimal. Leaf size=35

$$-2^{2p-1}(1-2x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{4}(1-2x)^2\right)$$

[Out] $-2^{(-1+2*p)*(1-2*x)} \text{hypergeom}([1/2, -p], [3/2], 1/4*(1-2*x)^2)$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$-2^{2p-1}(1-2x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{4}(1-2x)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x - 4*x^2)^p, x]

[Out] $-(2^{(-1+2*p)*(1-2*x)} \text{Hypergeometric2F1}[1/2, -p, 3/2, (1-2*x)^2/4])$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c)], x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x - 4x^2)^p dx &= -\left(2^{-3+2p} \text{Subst}\left(\int \left(1 - \frac{x^2}{64}\right)^p dx, x, 4 - 8x\right)\right) \\ &= -2^{-1+2p}(1-2x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{4}(1-2x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$-2^{2p-3}(4-8x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{64}(4-8x)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x - 4*x^2)^p, x]

[Out] -(2^(-3 + 2*p)*(4 - 8*x)*Hypergeometric2F1[1/2, -p, 3/2, (4 - 8*x)^2/64])

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-4x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((-4*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-4*x^2 + 4*x + 3)^p, x)

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int (-4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2+4*x+3)^p,x)

[Out] int((-4*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((-4*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (-4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - 4*x^2 + 3)^p,x)

[Out] int((4*x - 4*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x**2+4*x+3)**p,x)

[Out] Integral((-4*x**2 + 4*x + 3)**p, x)

$$3.143 \quad \int (3 + 4x - 5x^2)^p dx$$

Optimal. Leaf size=38

$$-5^{-p-1}19^p(2-5x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{19}(2-5x)^2\right)$$

[Out] $-5^{(-1-p)}*19^p*(2-5*x)*\text{hypergeom}([1/2, -p], [3/2], 1/19*(2-5*x)^2)$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {619, 245}

$$-5^{-p-1}19^p(2-5x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x - 5*x^2)^p, x]

[Out] $-(5^{(-1-p)}*19^p*(2-5*x)*\text{Hypergeometric2F1}[1/2, -p, 3/2, (2-5*x)^2/19])$

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int (3 + 4x - 5x^2)^p dx &= -\left(\frac{1}{2} (5^{-1-p}19^p) \text{Subst}\left(\int \left(1 - \frac{x^2}{76}\right)^p dx, x, 4 - 10x\right)\right) \\ &= -5^{-1-p}19^p(2-5x) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{19}(2-5x)^2\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.97

$$5^{-p-1}19^p(5x-2) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x - 5*x^2)^p, x]

[Out] 5^(-1 - p)*19^p*(-2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 5*x)^2/19]

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-5x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="fricas")

[Out] integral((-5*x^2 + 4*x + 3)^p, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="giac")

[Out] integrate((-5*x^2 + 4*x + 3)^p, x)

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int (-5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5*x^2+4*x+3)^p,x)

[Out] int((-5*x^2+4*x+3)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+4*x+3)^p,x, algorithm="maxima")

[Out] integrate((-5*x^2 + 4*x + 3)^p, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (-5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x - 5*x^2 + 3)^p,x)

[Out] int((4*x - 5*x^2 + 3)^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-5x^2 + 4x + 3)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x**2+4*x+3)**p,x)

[Out] Integral((-5*x**2 + 4*x + 3)**p, x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```